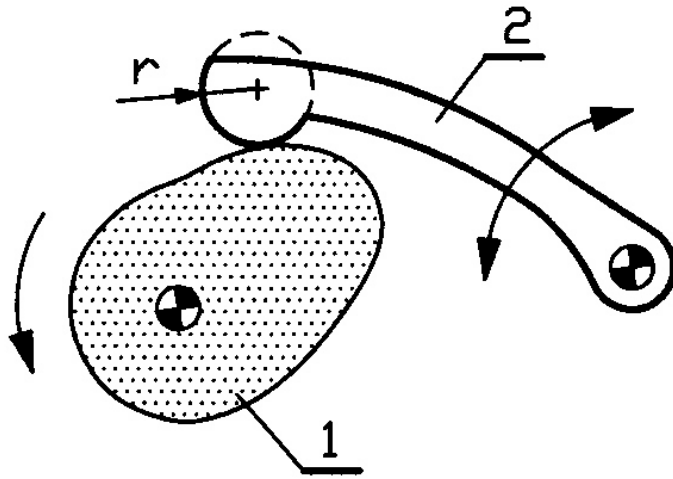


a)



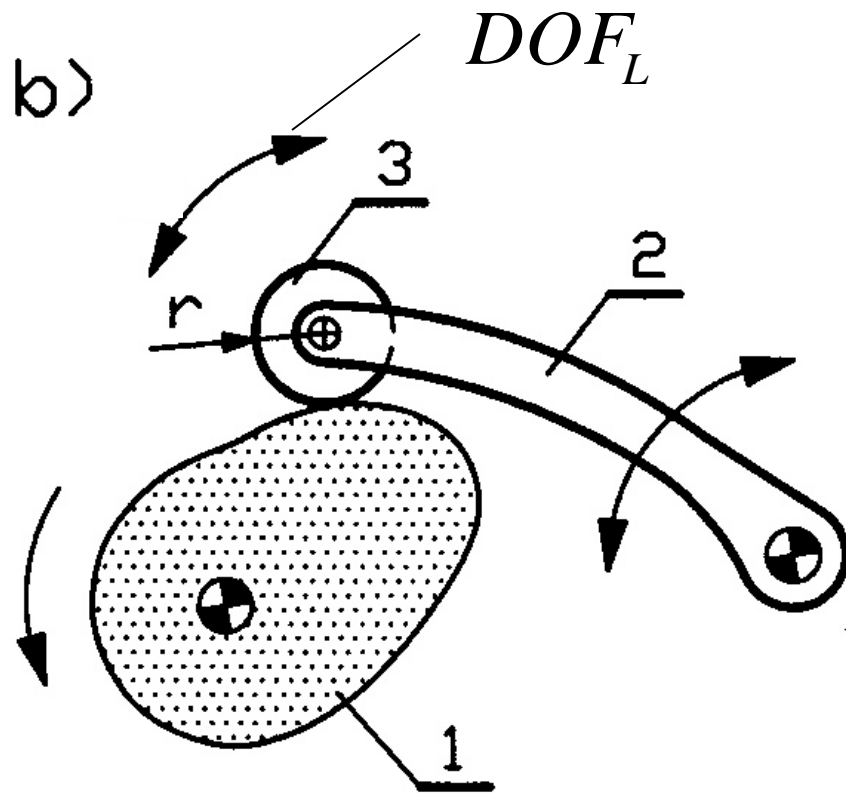
$$k = 2$$

$$p_1 = 2$$

$$p_2 = 1$$

$$W_T = 3(n-1) - 2p_1 - 1p_2 =$$

$$W_T = 1 \quad \overset{?}{\longleftrightarrow} \quad W_{REAL} = 1$$



$$k = 3$$

$$p_1 = 3$$

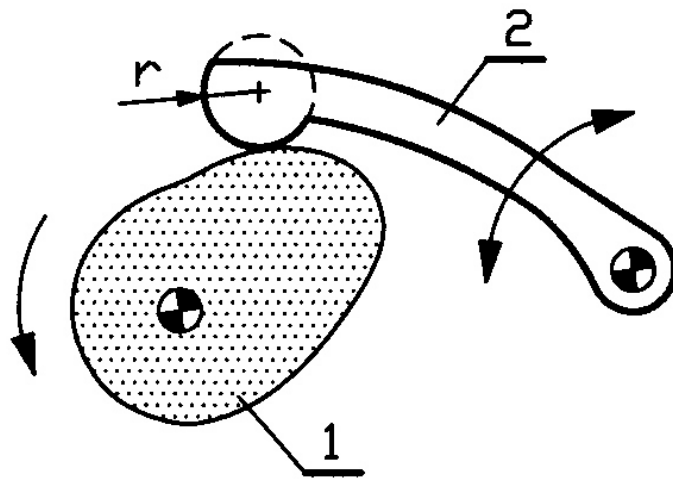
$$p_2 = 1$$

$$W_T = 2$$

$$W_{REAL} = 1$$

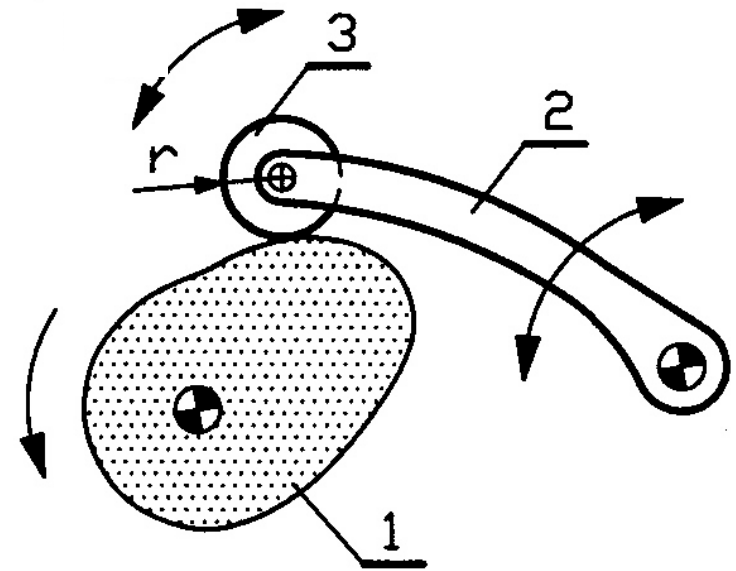
COMPARISON

a)

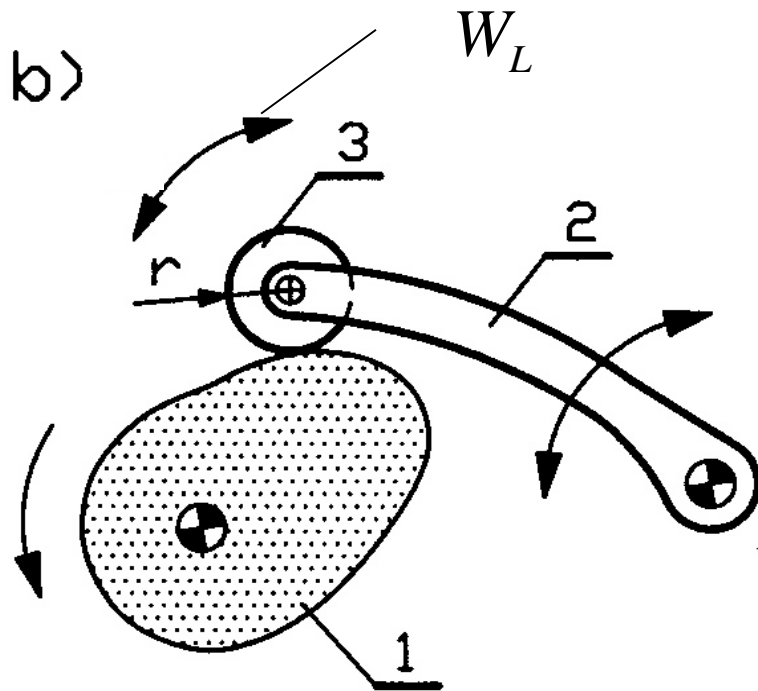


$$W_T = 1$$

b)



$$W_T = 2$$



$$k = 3$$

$$p_1 = 3$$

$$p_2 = 1$$

$$W_T = 2$$

IDLE (LOCAL MOBILITY) OF LINK 3 – ROLLER 3 CAN ROTATE AROUND ITS CENTER

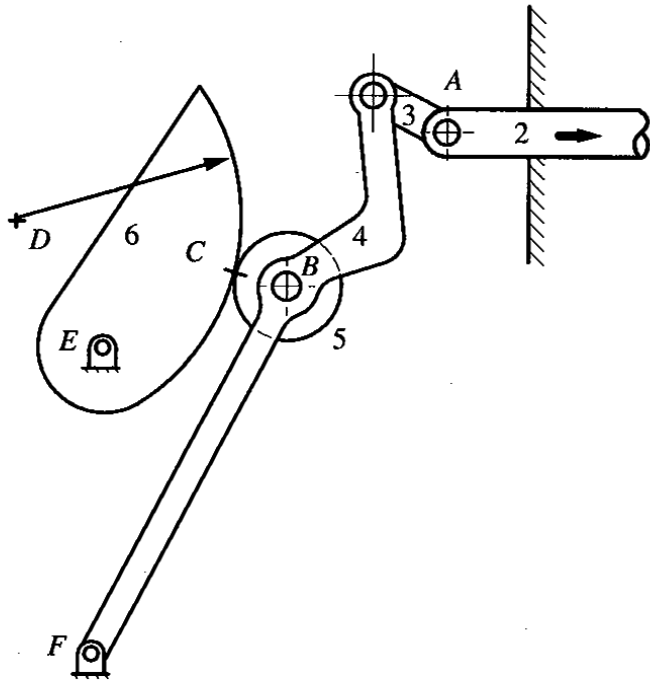
$$W_L = 1$$

$$W_R = 1$$

idle = inactive

REAL MOBILITY:

$$W_R = W_T - W_L$$

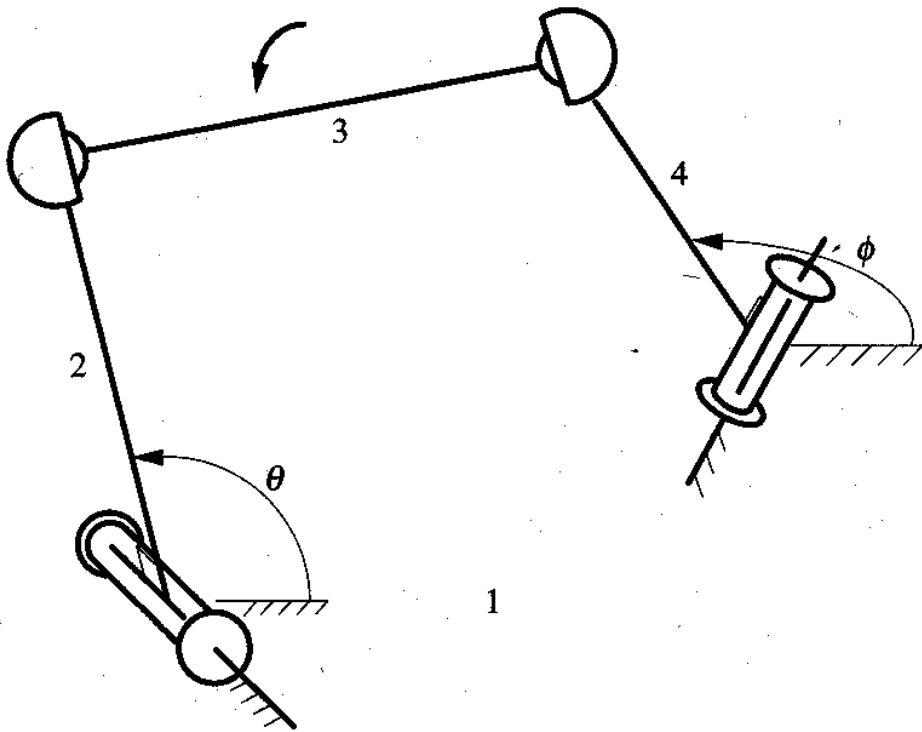


$$W_R =$$

$$W_T =$$

$$W_L =$$

Figure 1.30 Planar mechanism with an idle degree of freedom.

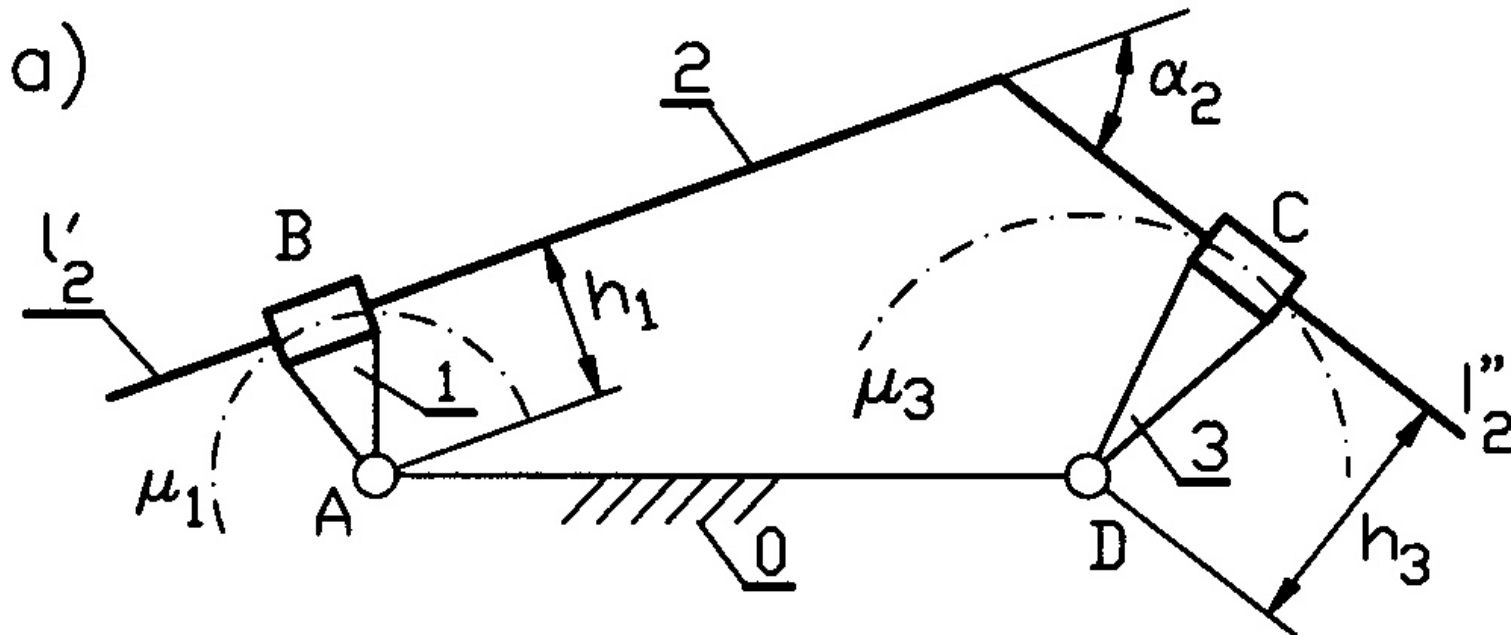


$$W_R = \dots$$

$$W_T = \dots$$

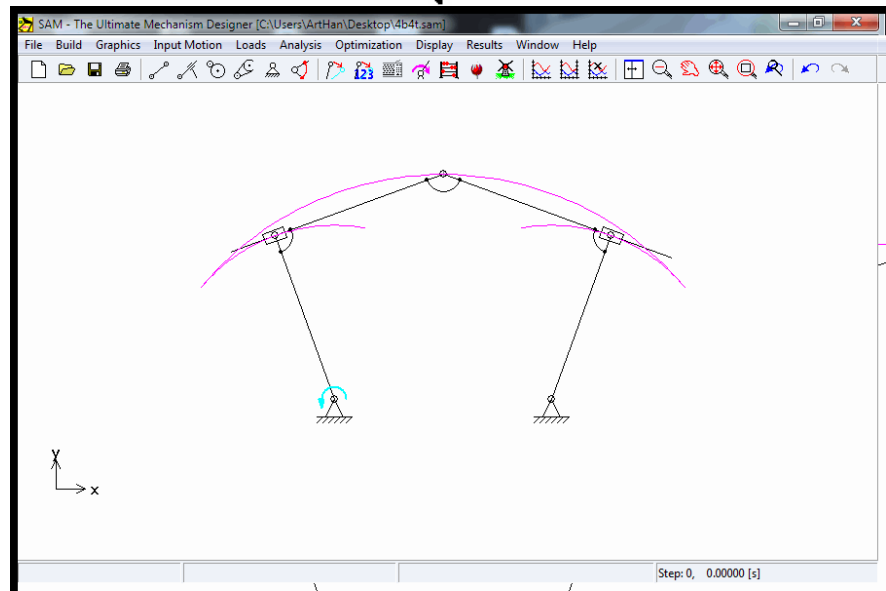
$$W_L = \dots$$

Figure 1.28 A spatial four-member, four-joint linkage. Two of the joints are revolute. The other two are spherical joints. θ is the input joint angle and ϕ is the output joint angle. The linkage has an idle degree of freedom since member 3 can spin about the line joining the centers of the spherical joints without affecting the relationship between θ and ϕ .



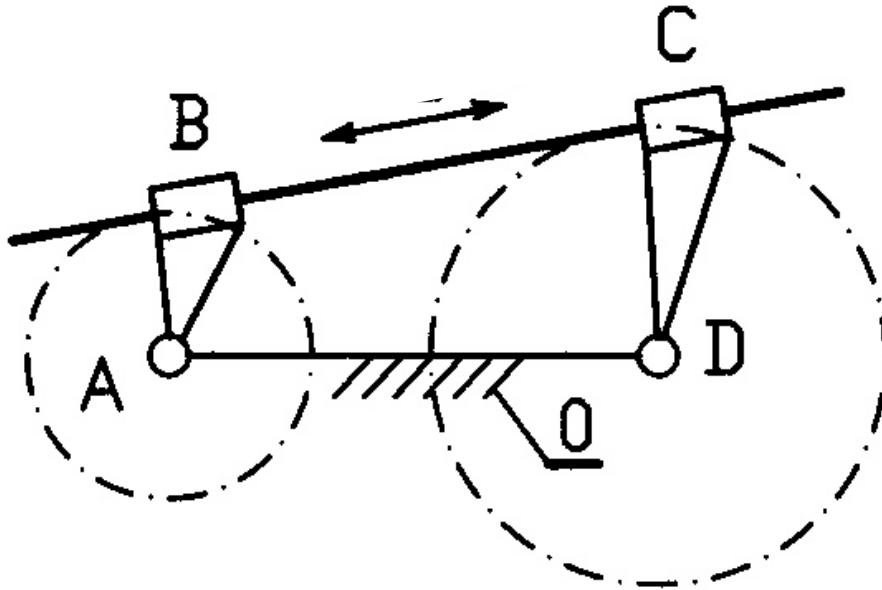
$$W_R =$$

$$W_R = W_T - W_L$$



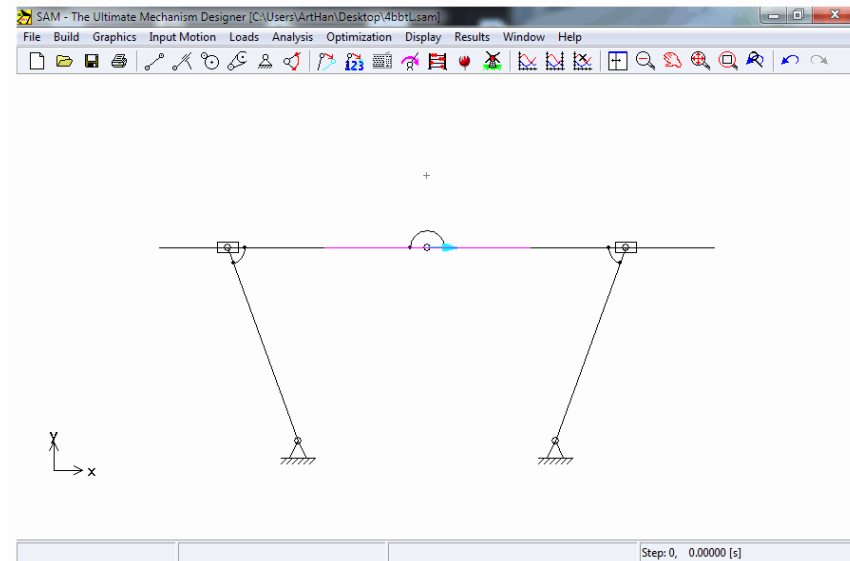
Special geometry:

Link BC = straight line



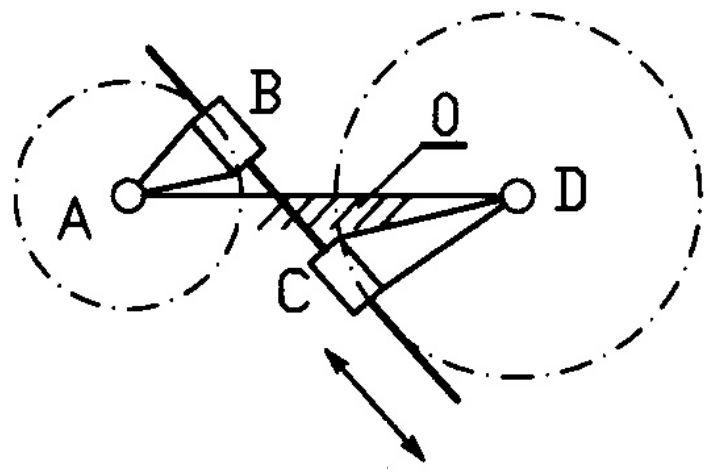
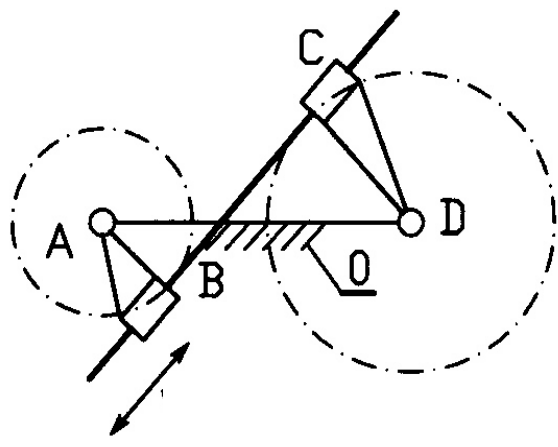
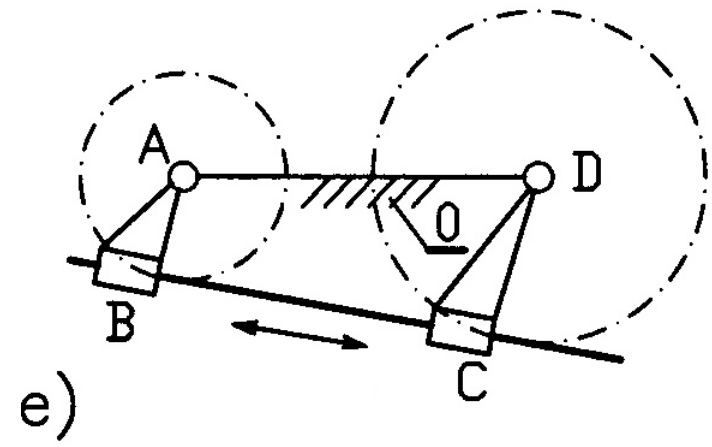
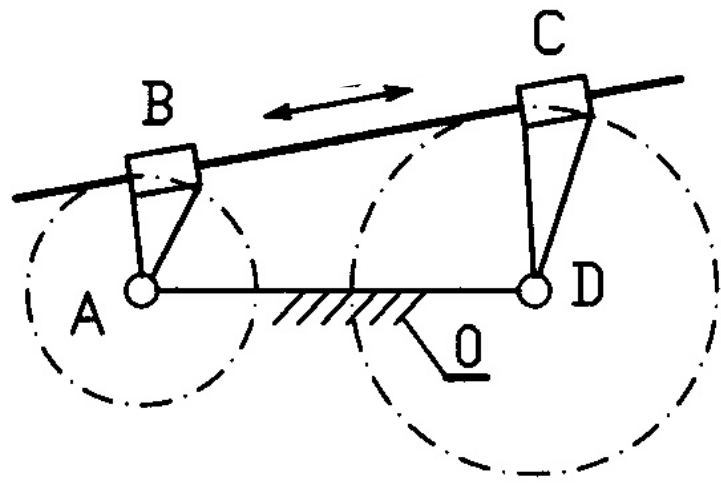
$$W_R = 0$$

$$W_R = W_T - W_L = 1 - 1$$



This mechanism can only be assembled in 4 configurations !

This mechanism can only be assembled in 4 configurations !



IDLE MOBILITY ALWAYS REDUCES
TOPOLOGICAL (THEORETICAL) MOBILITY

$$W_R = W_T - W_L$$

IDLE MOBILITY IS THE RESULT OF SPECIAL
GEOMETRY OF LINKS

HOMEWORK

Draw a kinematic system (a real one) having at least 4 links

Number the links, classify joints, calculate mobility

SUMMARY:

Link:

solid body which can move in relation to other links

Joint (kinematic pair):

class I, II, III, IV and V (according to number of DOF)

lower and **higher** joints (type of contact: surface, point, line)

Kinematic system (mechanism, machine)

Mobility:

DOF number of independent coordinates (parameters) to define a system position (motion)

W_T - topological (theoretical) mobility

Planar systems (2D)

$$W_T = 3(n-1) - 2p_1 - 1p_2$$

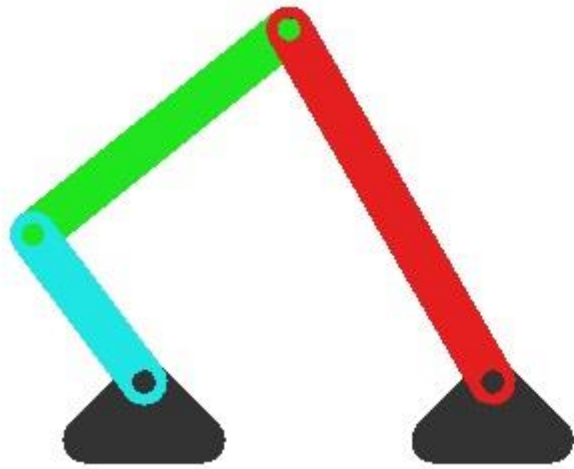
Spatial systems (3D)

$$W_T = 6(n-1) - 5p_1 - 4p_2 - 3p_3 - 2p_4 - 1p_5$$

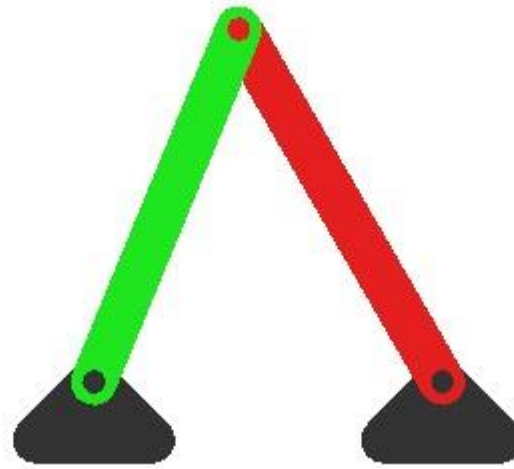
W_R - real (practical) mobility

W_L - idle (local) mobility (in case of special geometry)

$$W_R = W_T - W_L$$



$$W_T = +1$$

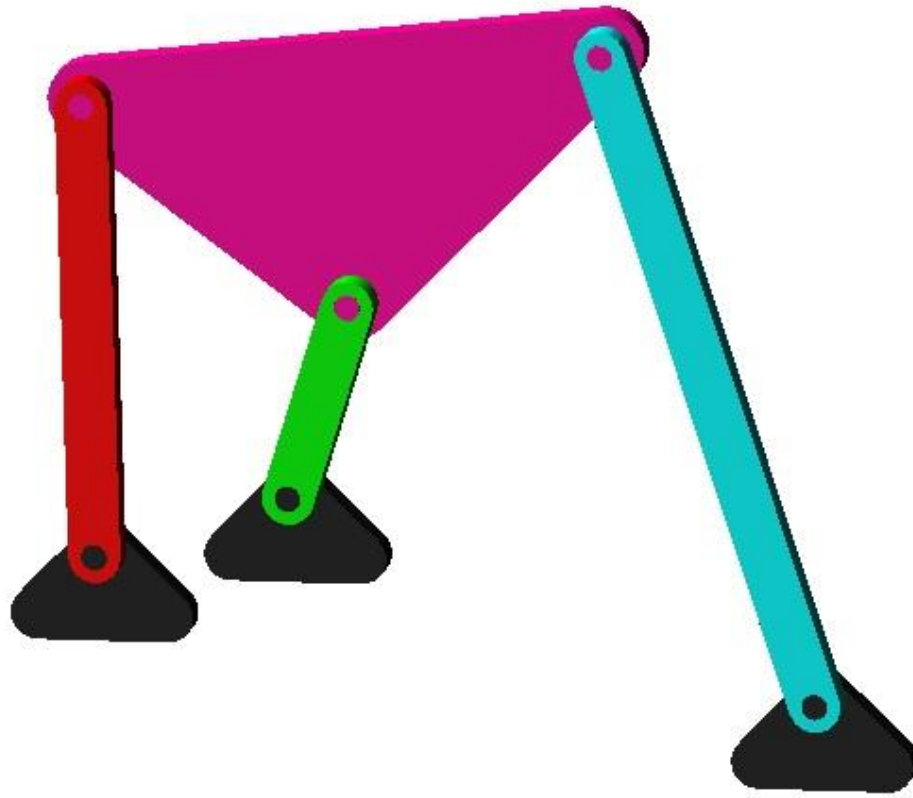


$$W_T = 0$$



$$W_T = -1$$

Because mobility criterion pays no attention to link sizes or shapes it can give misleading results in the face of unique geometric conditions

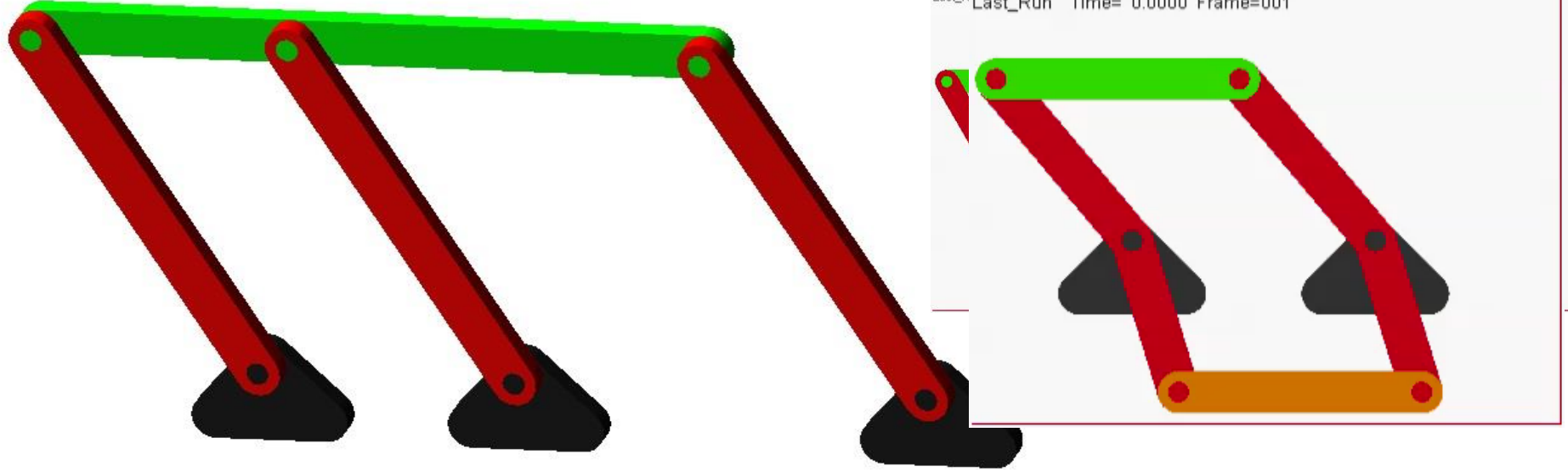


$$W_T = 0$$



$$W_R = 0$$

In general if $W_T = 0$ relative motion of links is not allowed !



$$W_T=0$$



$$W_R=1$$

the results (W) disagree - motion is possible due to unique geometry:

- ternary links are straight and parallel and with equispaced nodes,
- the three binaries are equal in length

Kinematic systems having:

$$W_T \leq 0 \quad \text{and} \quad W_R = 1 \text{ (or more)}$$

are called

- mechanisms with redundant constraints (passive constraints)

or

- paradoxical mechanisms

Number of redundant constraints (R_C) must satisfy equation:

$$W_R = W_T - W_L + R_C$$

Transmission of angular motion between two wheels

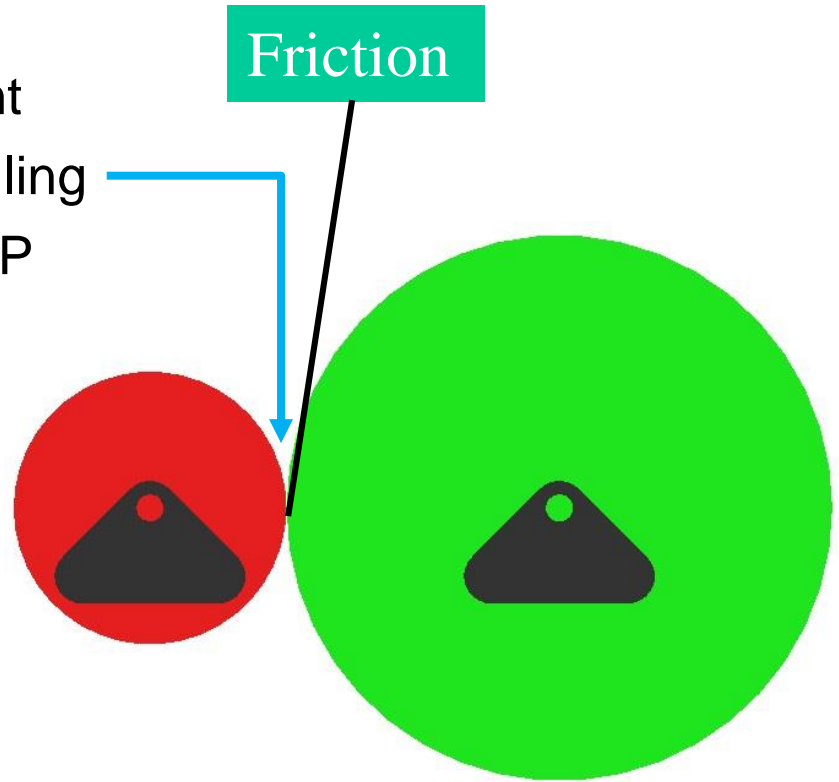
$$k = 2$$

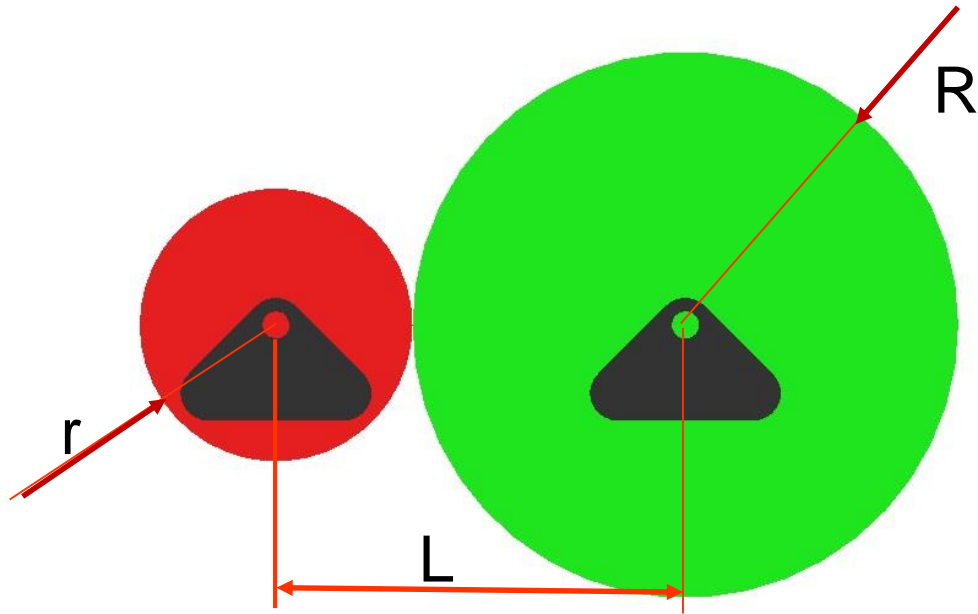
$$p_1 = 3, p_2 = 0$$



$$W_T = 0 \quad \text{and} \quad W_R = 1$$

Full joint
pure rolling
NO SLIP



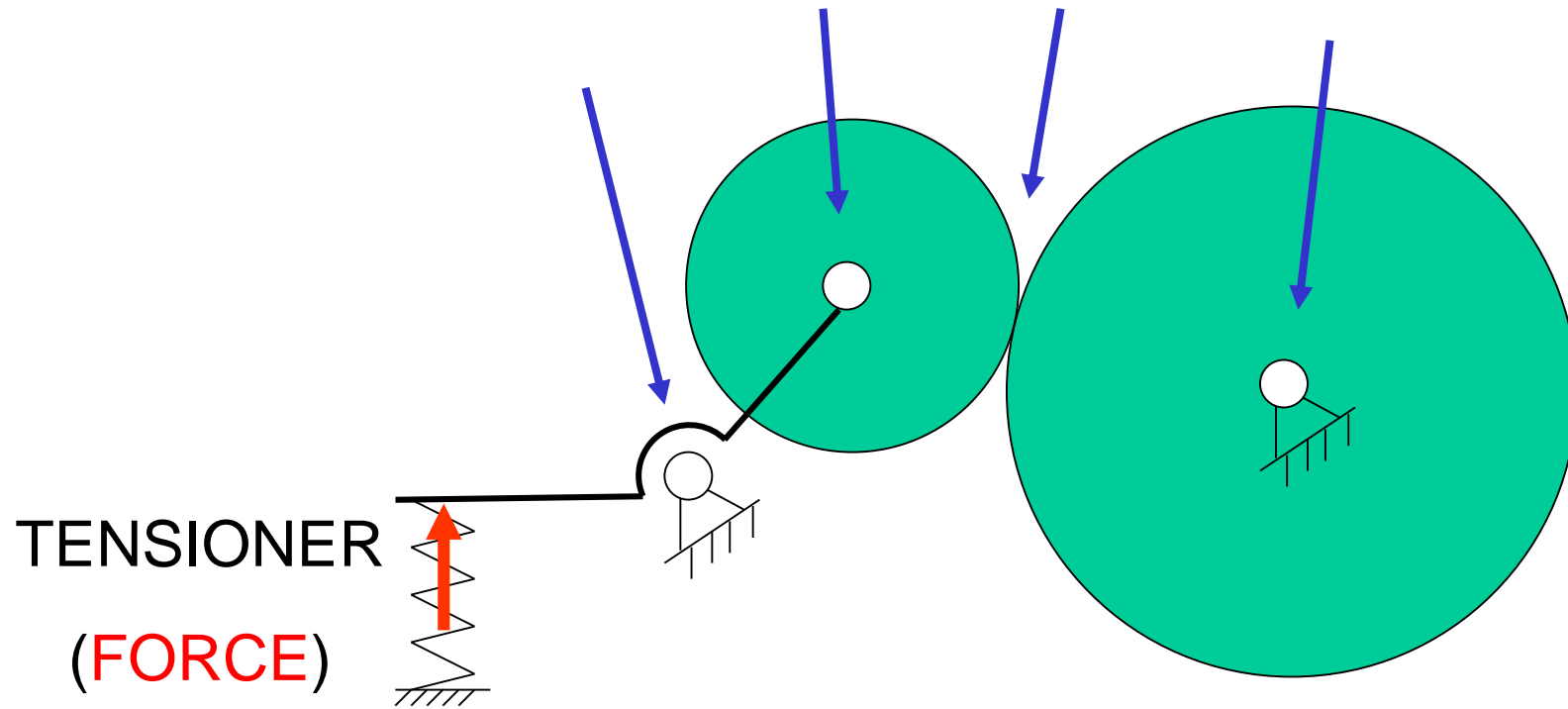


Motion allowed if: $L = R + r$

If $L > R + r$ joint disappears (no contact)

If $L < R + r$ mechanism can not be assembled

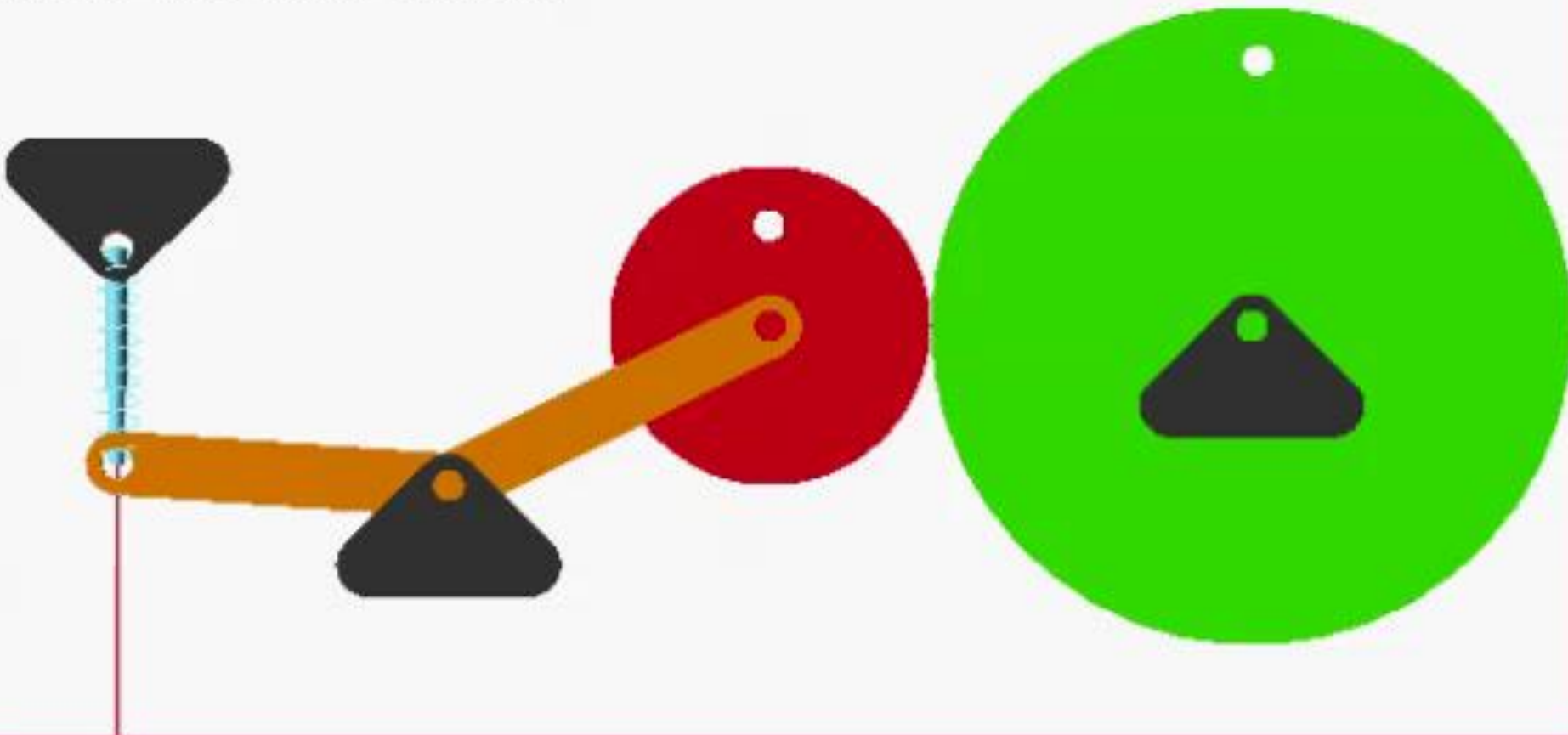
4 x R joints



$$W_R = 1$$

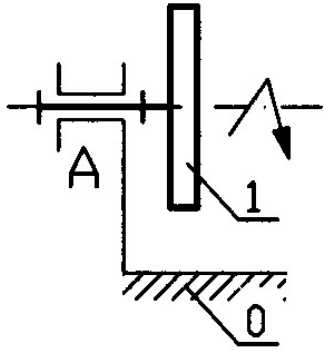
$$R_C = 0$$

Last_Run Time= 0.0000 Frame=001



Rotating disc

a)



$$W_R = 1$$

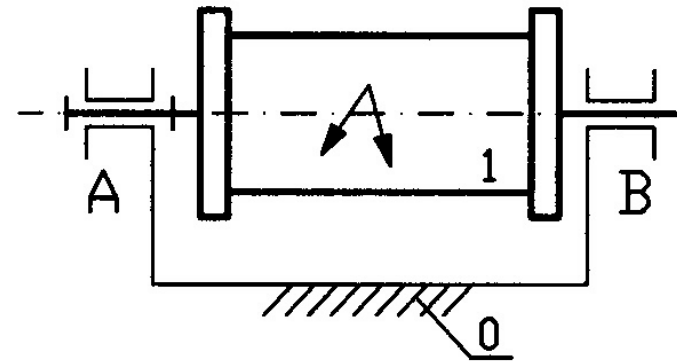
$$W_T = 1$$

$$W_L = 0$$

$$R_C = 0$$

Rotor

b)



$$W_R = 1$$

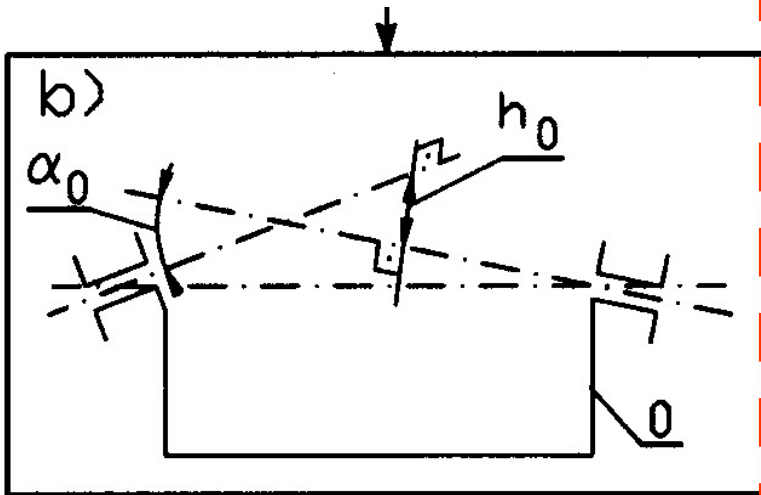
$$W_T = -3$$

$$W_L = 0$$

$$R_C = 4$$

Geometrical conditions of motion

frame

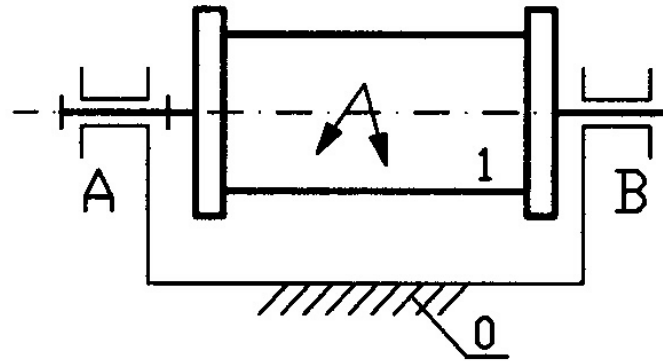


$$\alpha_0 = 0$$

$$h_0 = 0$$

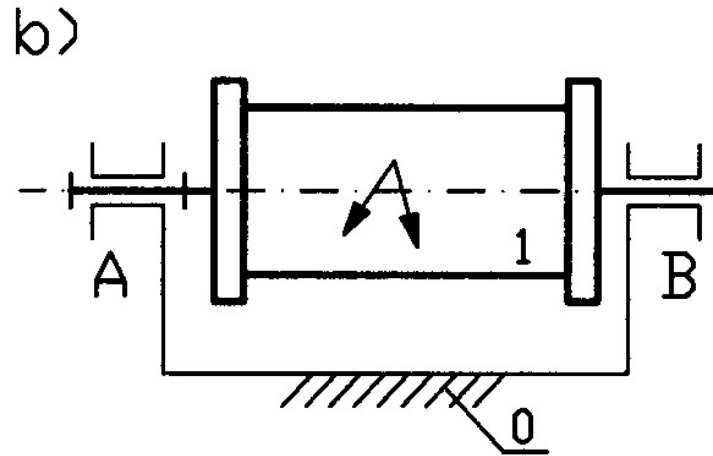
Rotor

b)



What can be modified to improve topology ???

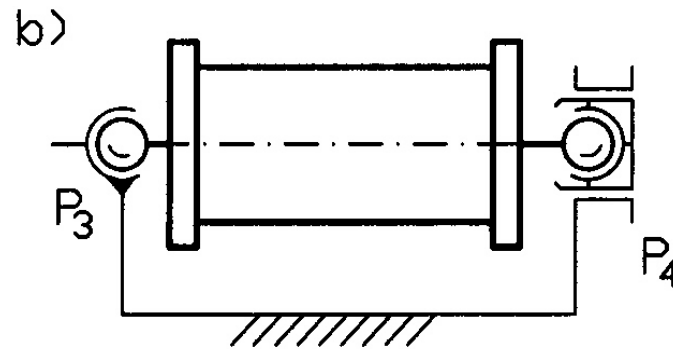
Improvement by changing joints A and B (joint classes)



The goal is:

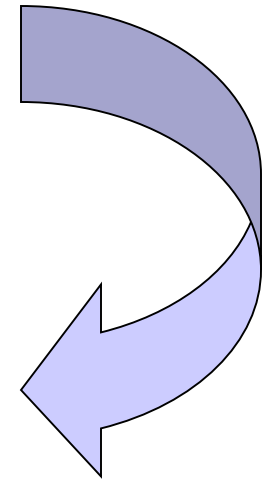
$$W_T = W_R = 1; \quad R_C = 0; \quad W_L = 0$$

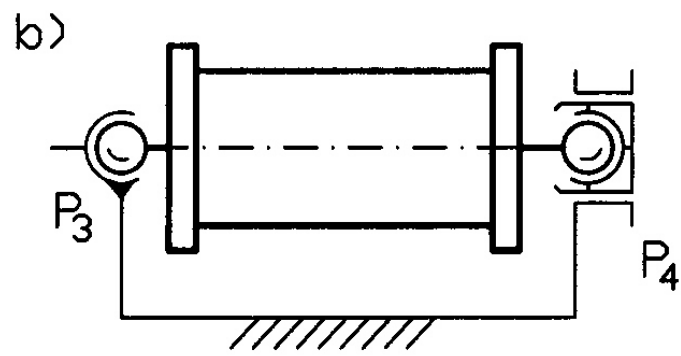
W_T	=	$6k$	$-5p_1$	$-4p_2$	$-3p_3$	$-2p_4$	$-1p_5$
1	=	$6*1$	$-5*0$	$-4*1$	$-3*0$	$-2*0$	$-1*1$
1	=	$6*1$	$-5*0$	$-4*0$	$-3*1$	$-2*1$	$-1*0$



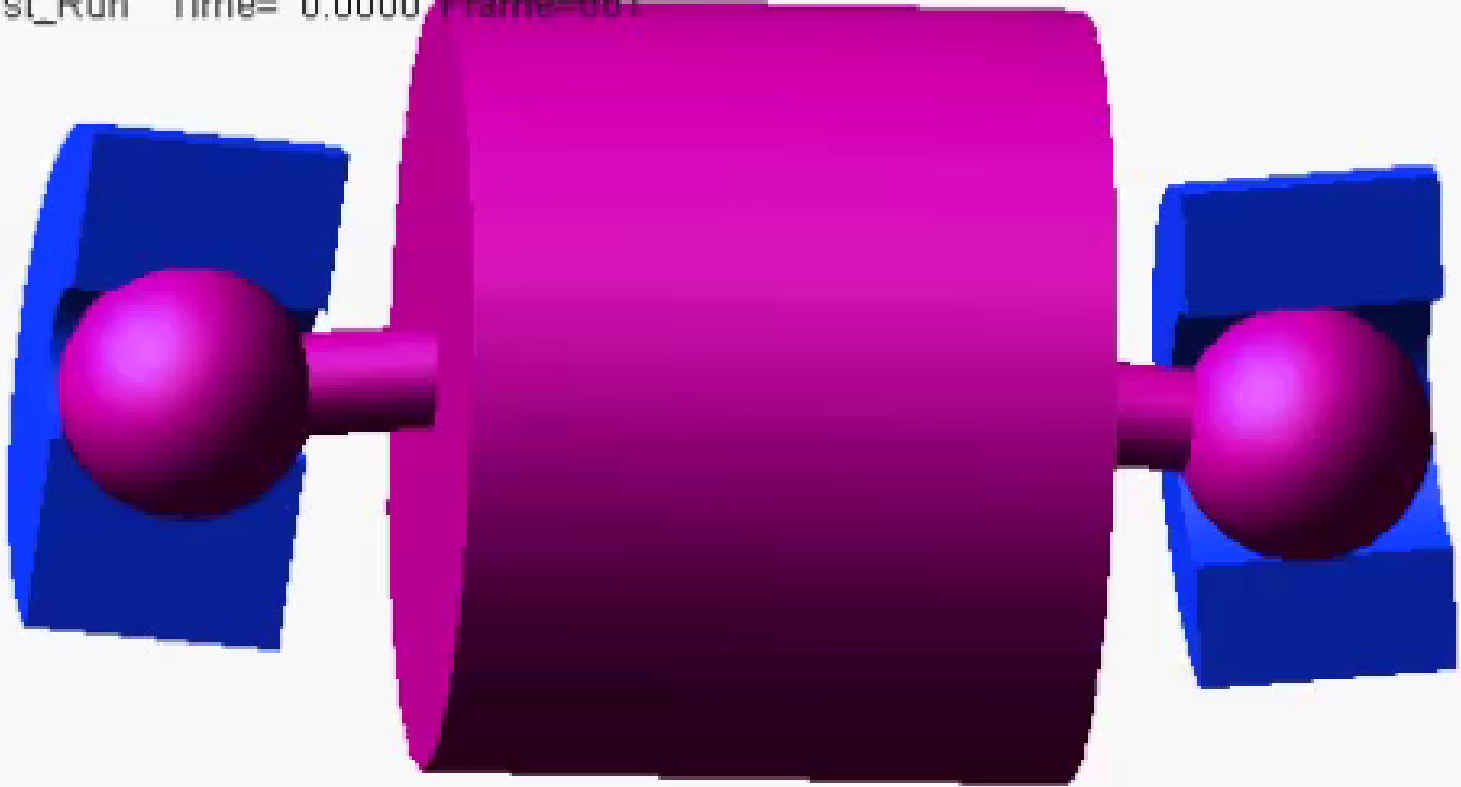
Spherical joint (self-aligning bearing)

Spherical joint with translation allowed





Last_Run Time= 0.0000 Frame=001



For mechanism with redundant constraints:

because of manufacturing errors

assembly needs forces (elastic deformations)

→ friction in joints increases

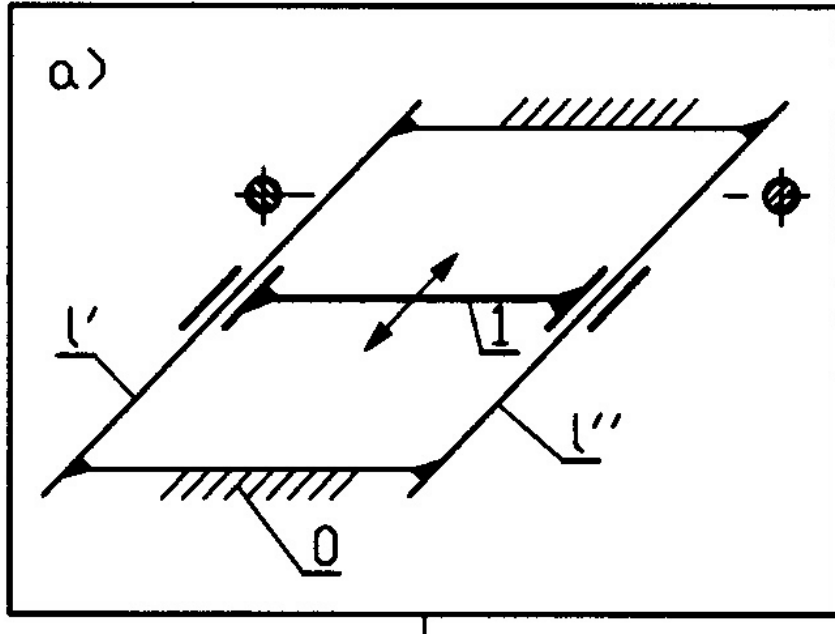
→ resistance of motion increases

→ both efficiency and reliability decreases

Mechanisms with redundant constraints are called

IRRATIONAL KINEMATIC SYSTEMS

SYSTEMS WITH WRONG TOPOLOGY



$$W_R = 1$$

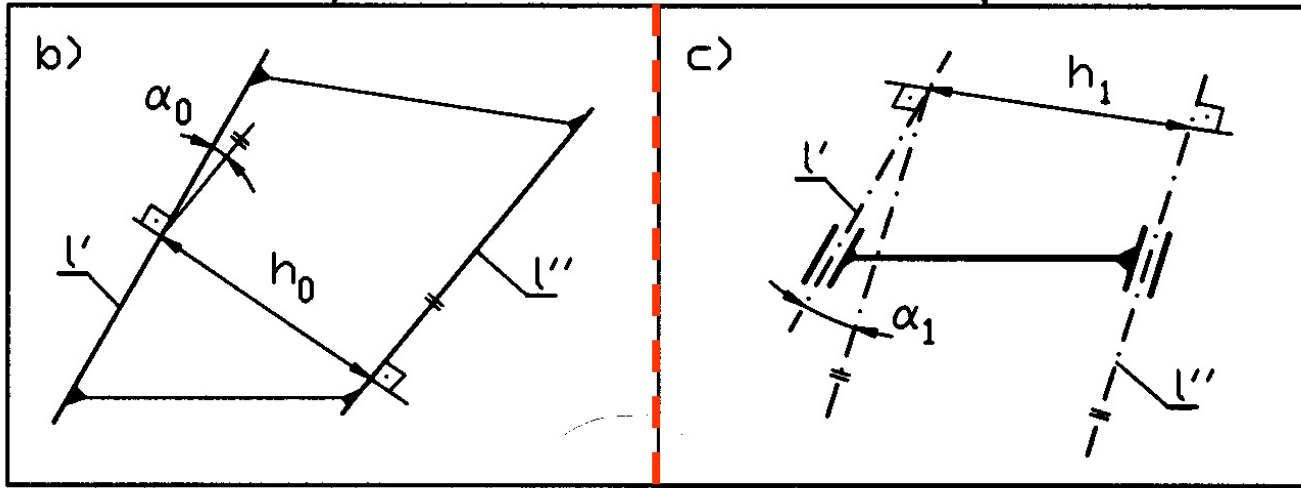
$$W_T = -2$$

$$W_L = 0$$

$$R_C = 3$$

frame 0

slider 1



$$\alpha_0 = 0$$

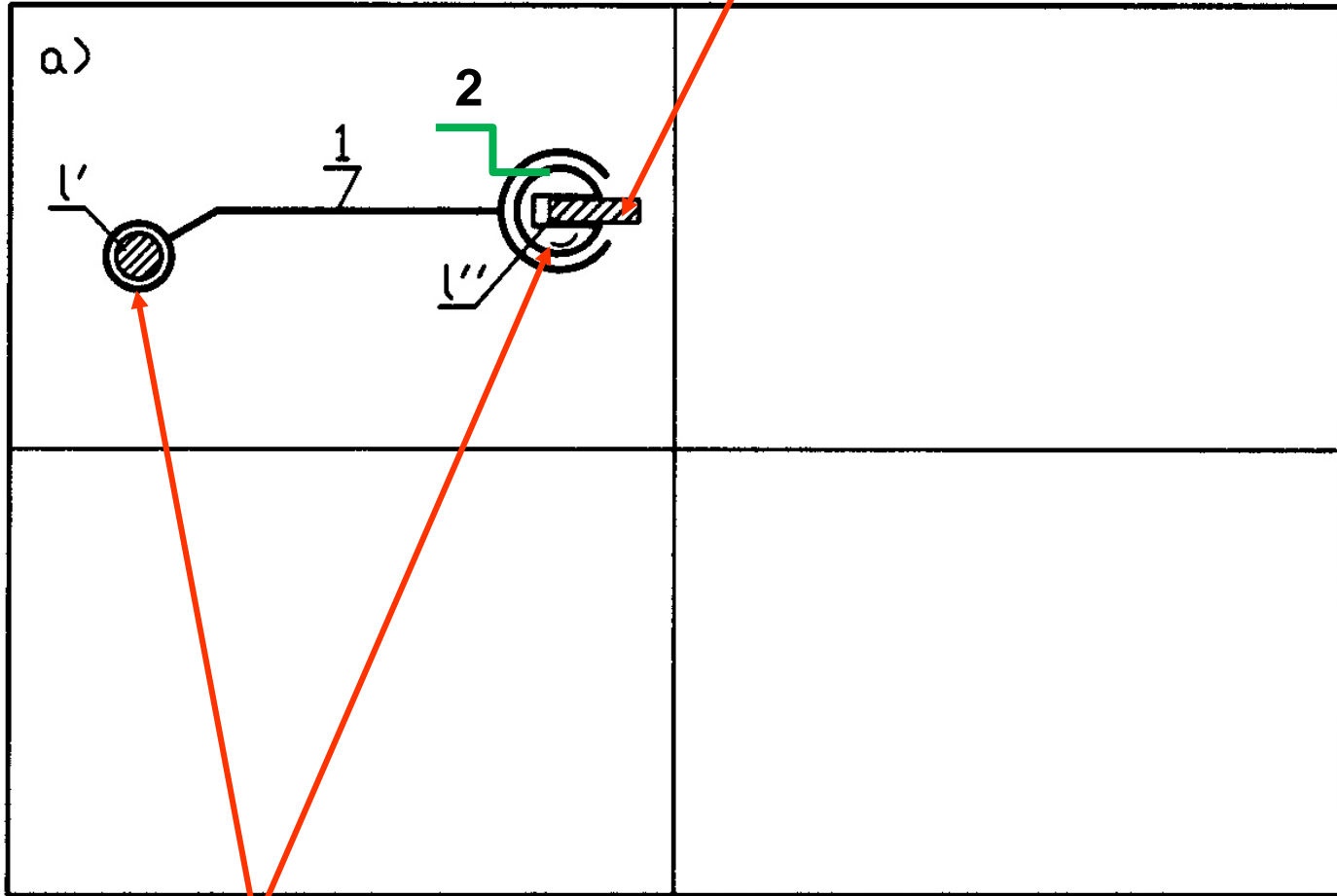
$$\alpha_1 = 0$$

$$h_0 = h_1$$

Rational topology - examples

Planar joint

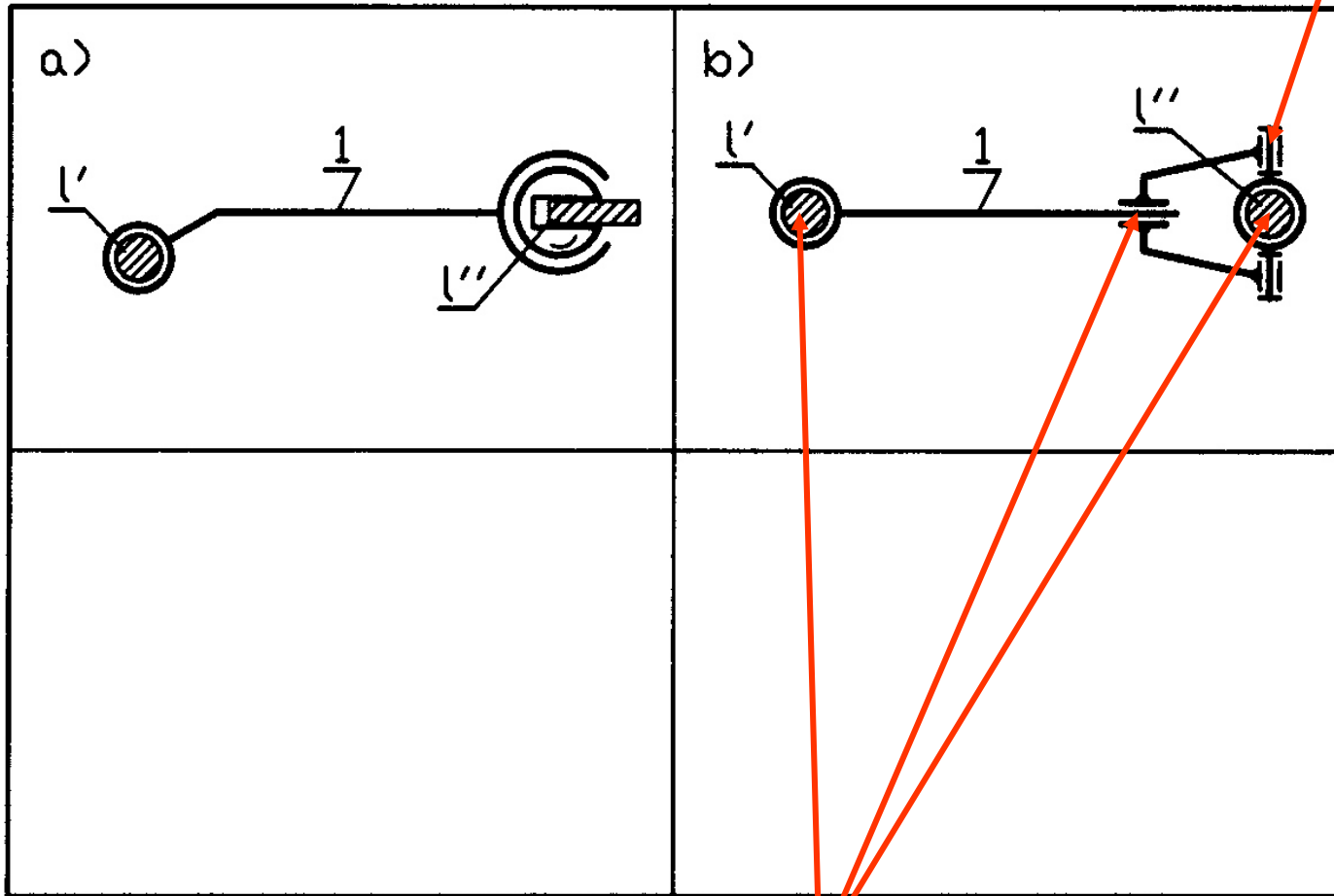
$k = 2$
 $p_2 = 2$
 $p_3 = 1$



Cylindrical joint (linear bearing)

Rational topology - examples

$k = 2$
 $p_2 = 2$
 $p_3 = 1$



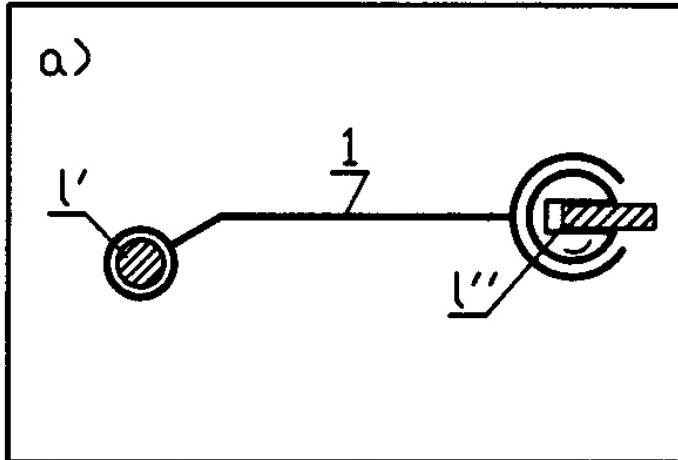
$k = 3$
 $p_1 = 1$
 $p_2 = 2$

Revolute joint

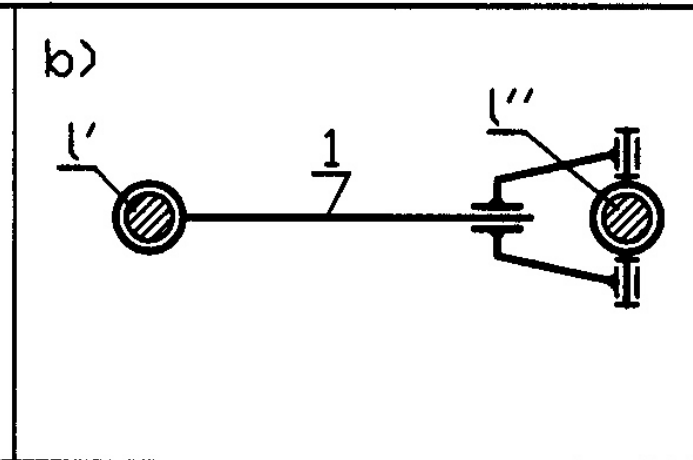
Cylindrical joint (linear bearing)

Rational topology - examples

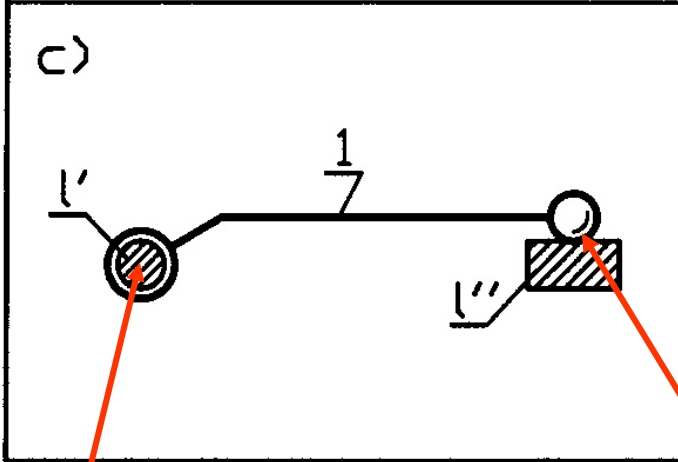
$k = 2$
 $p_2 = 2$
 $p_3 = 1$



$k = 3$
 $p_1 = 1$
 $p_2 = 2$



$p_2 = 1$
 $p_5 = 1$

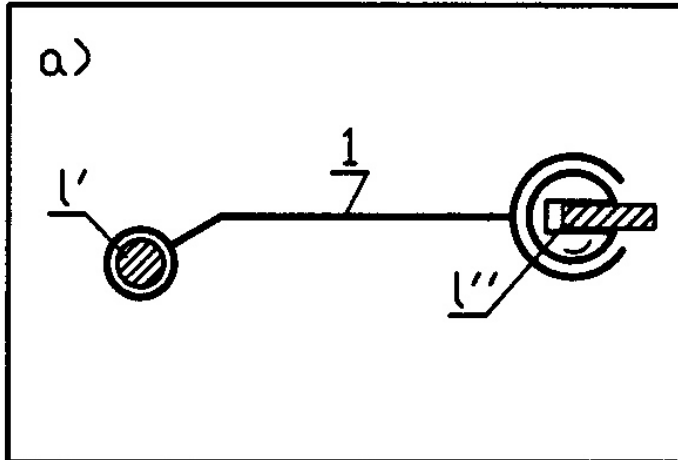


Cylindrical joint (linear bearing)

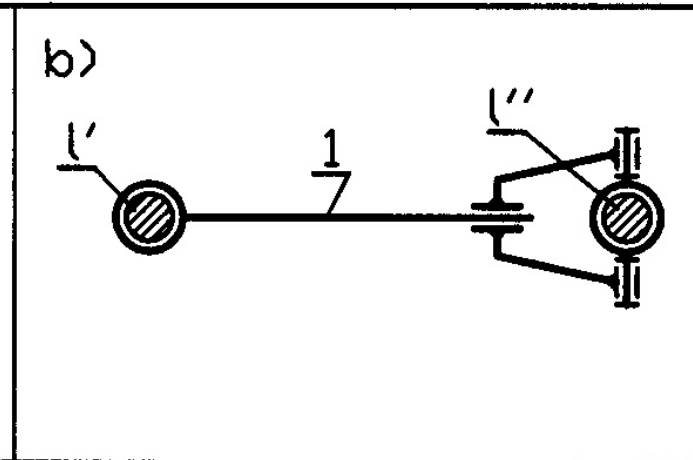
Point to surface joint

Rational topology - examples

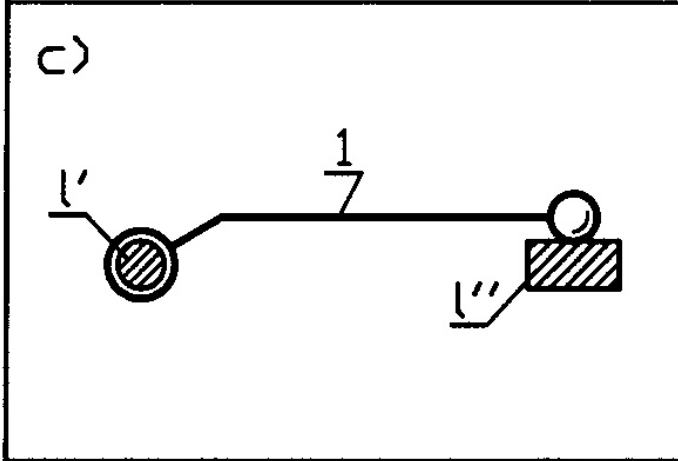
$k = 2$
 $p_2 = 2$
 $p_3 = 1$



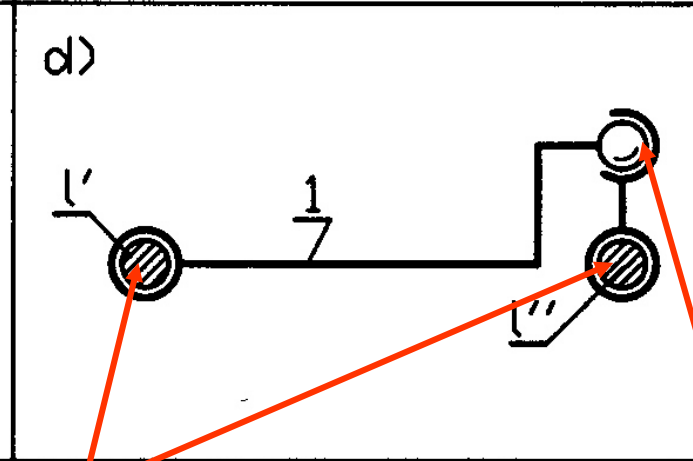
$k = 3$
 $p_1 = 1$
 $p_2 = 2$



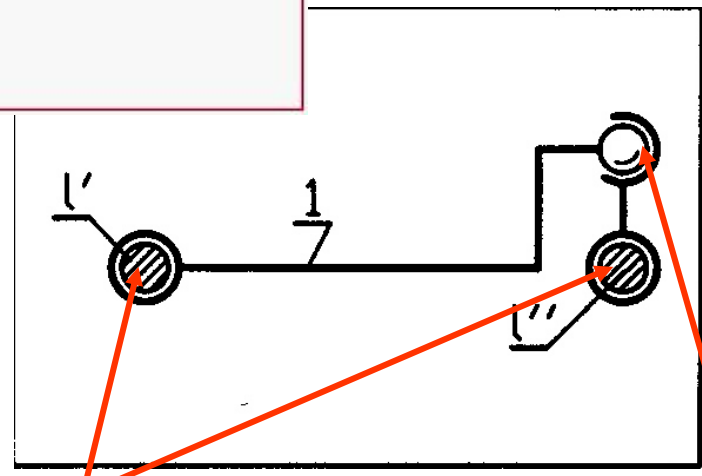
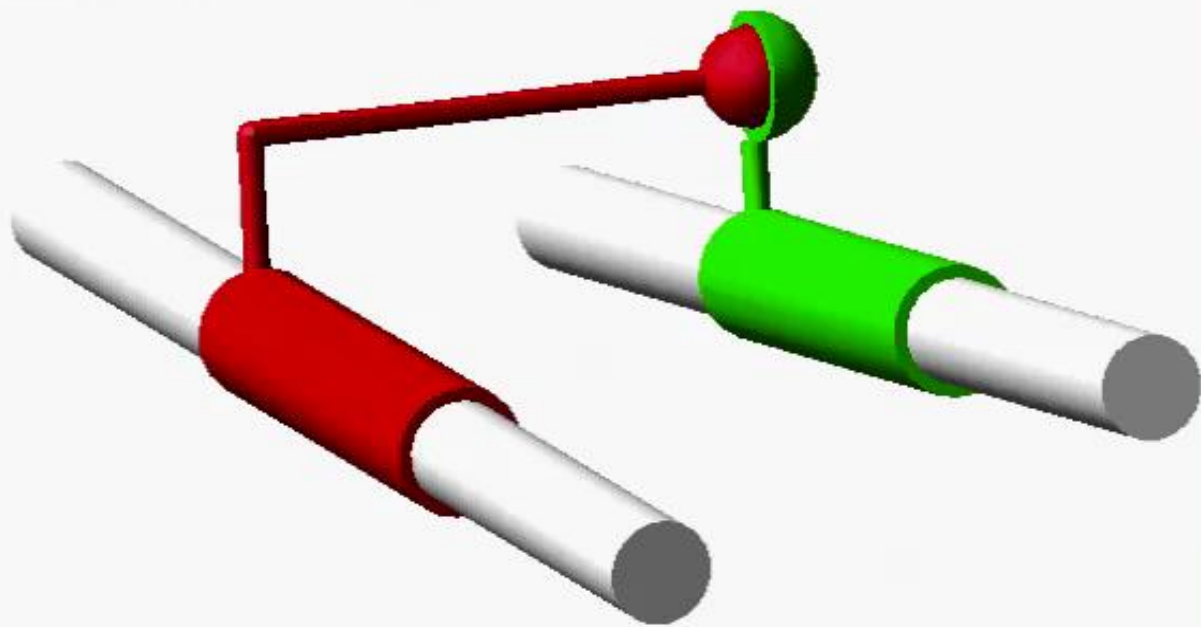
$p_2 = 1$
 $p_5 = 1$



$k = 2$
 $p_2 = 2$
 $p_3 = 1$

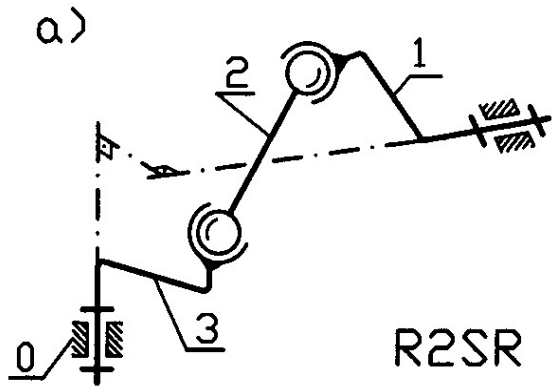


Cylindrical joint (linear bearing) Spherical joint

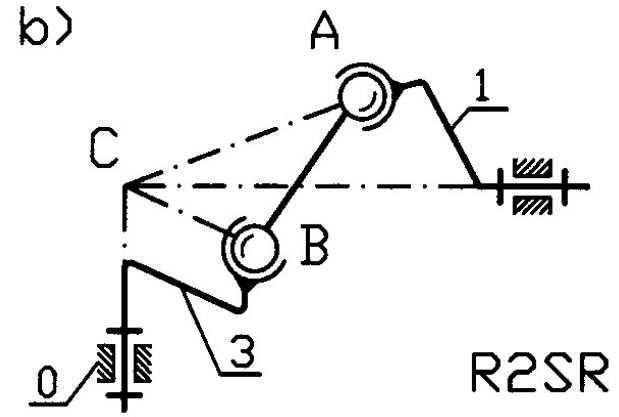


$k = 2$
 $p_2 = 2$
 $p_3 = 1$

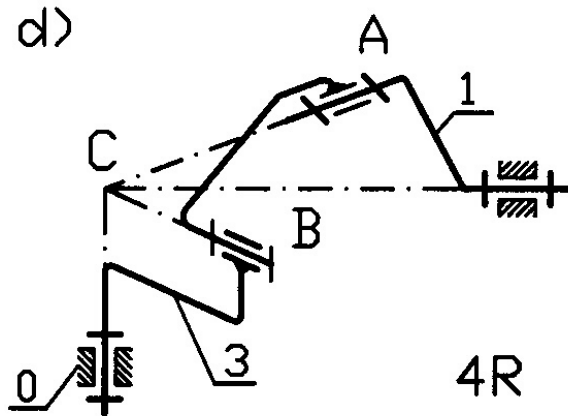
Cylindrical joint (linear bearing) Spherical joint



3D 4 bar (general)



3D 4 bar (input (1) & output (3) axes intersect)



3D 4 bar (all joint axes intersect at point)