Kinematics – analytical methods

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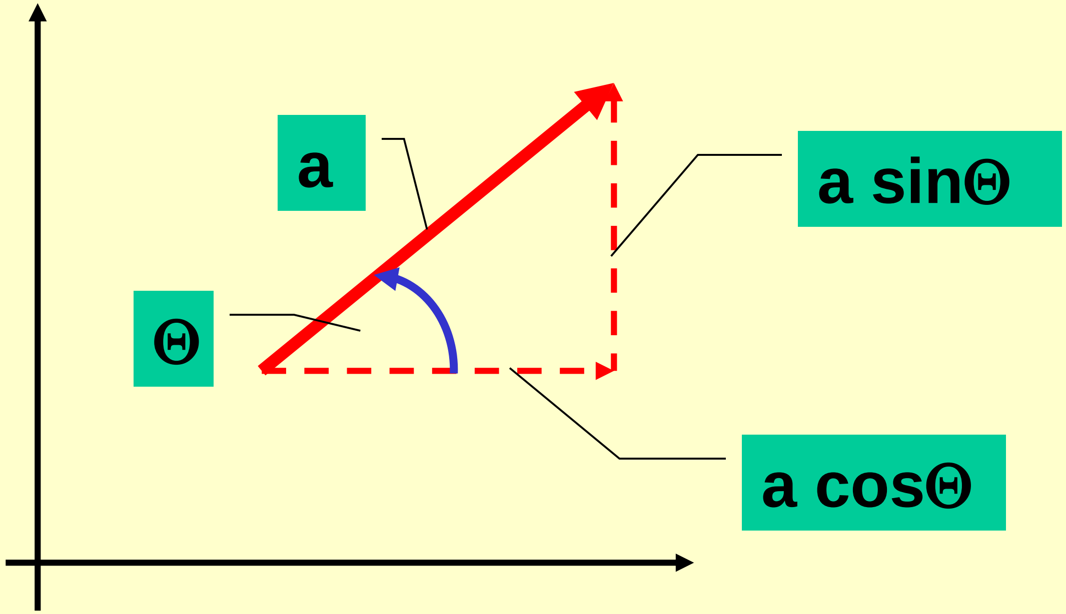
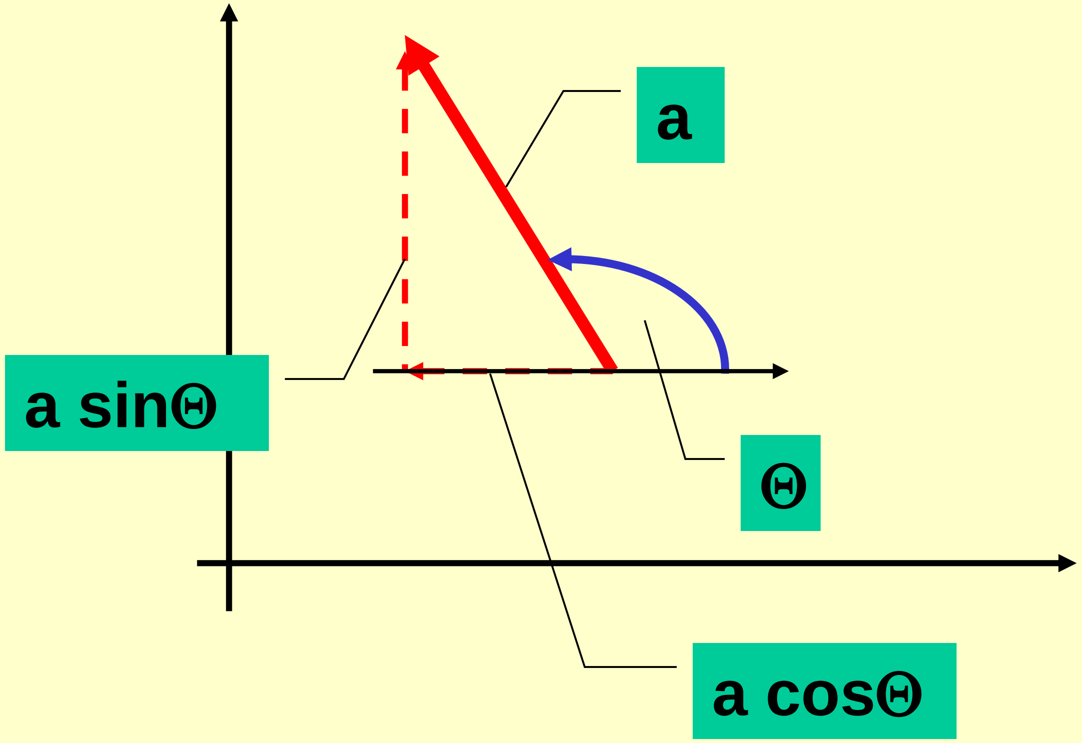
To find vel & acc using analytical method we need:

position equations and derive first (velocity) and second (acceleration) time derivative!

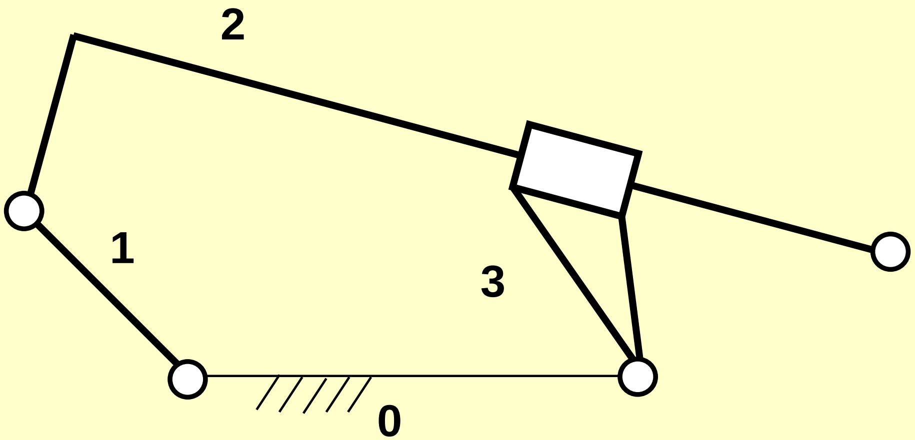
Any mechanism can be considered as a chain of vectors projections

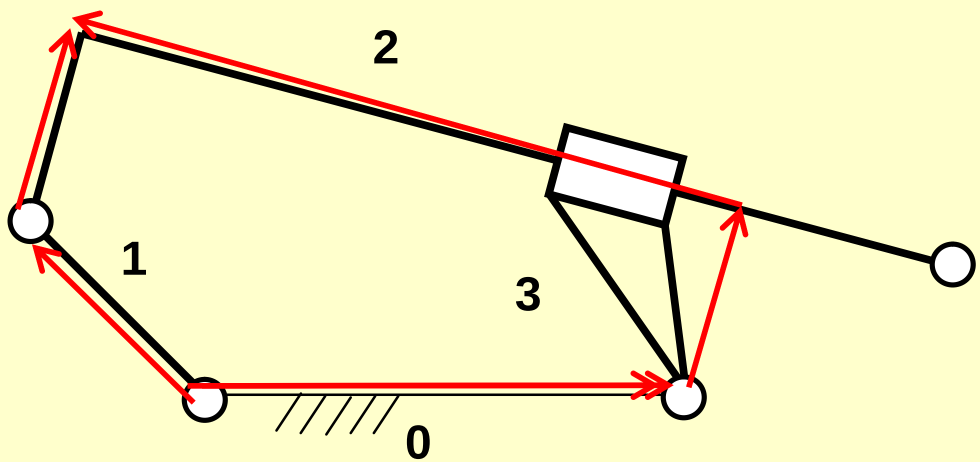


projections

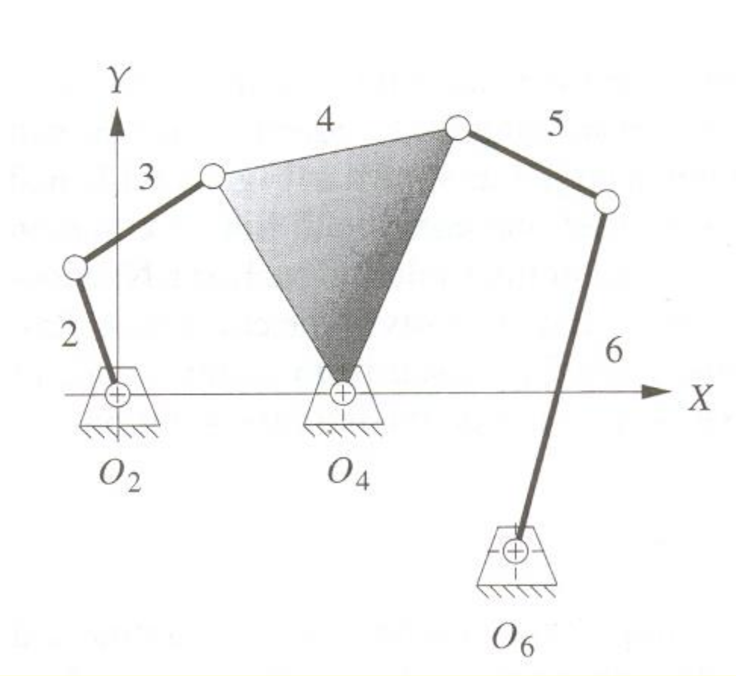


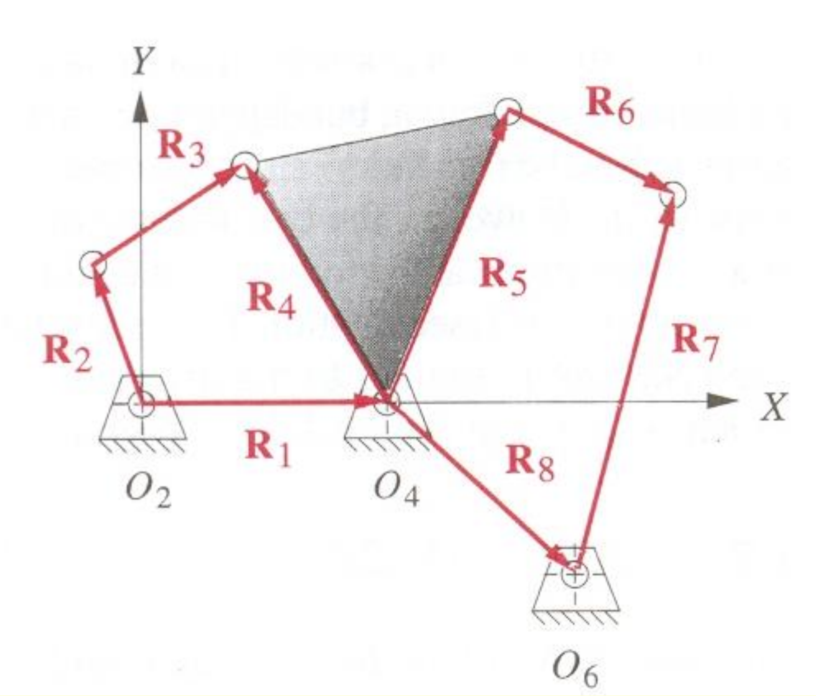
MECHANISM → POLYGON OF VECTORS





What is the chain of vectors ?





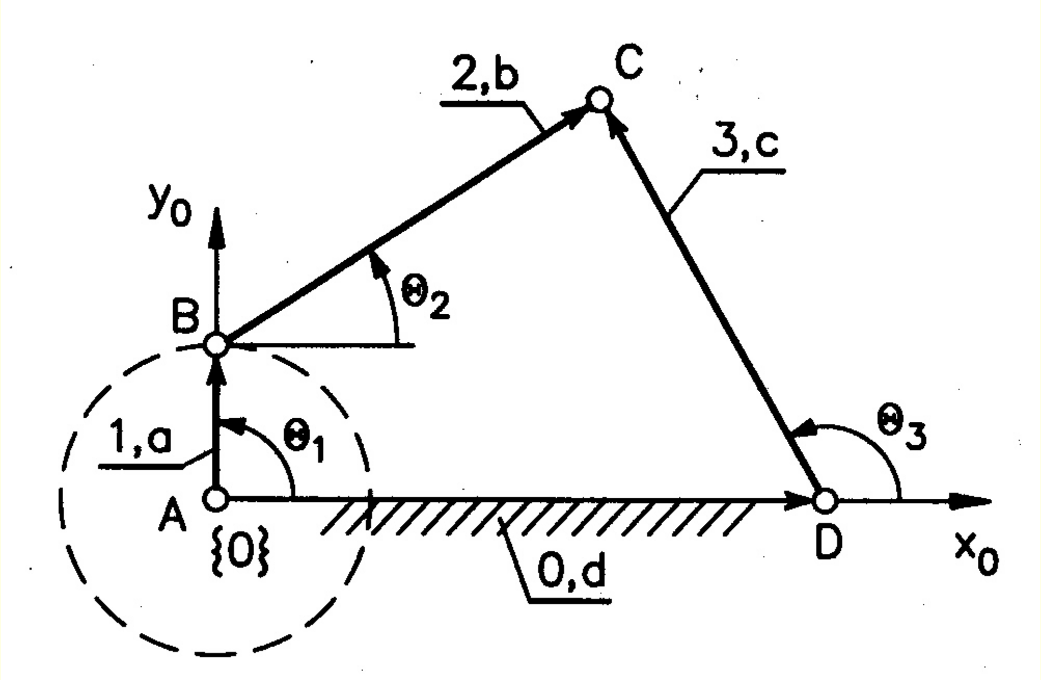
2 loops:

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_1 - \mathbf{R}_4 = \mathbf{0}$$

$$\mathbf{R}_5 + \mathbf{R}_6 - \mathbf{R}_8 - \mathbf{R}_7 = \mathbf{0}$$

→ 4 projections (equations)

Example – 4R VELOCITY → ACCELERATION



$$\mathbf{a} + \mathbf{b} - \mathbf{d} - \mathbf{c} = \mathbf{0} \longrightarrow$$

2 eq.: x, y projections

x & y projections → position equations:

$$a\cos\Theta_1 + b\cos\Theta_2 - d - c\cos\Theta_3 = 0$$
$$a\sin\Theta_1 + b\sin\Theta_2 - c\sin\Theta_3 = 0$$

Variables:

$$\Theta_1(t),\ \Theta_2(t),\ \Theta_3(t) \longleftarrow$$

1 independent variable (driver), assume ω_1

Vel equations – 1-st time derivative

$$-a\dot{\theta}_1 \sin\theta_1 - b\dot{\theta}_2 \sin\theta_2 + c\dot{\theta}_3 \sin\theta_3 = 0$$
$$a\dot{\theta}_1 \cos\theta_1 + b\dot{\theta}_2 \cos\theta_2 - c\dot{\theta}_3 \cos\theta_3 = 0$$

$$\dot{\theta}_i = \frac{d\theta_i}{dt} = \omega_i \quad i = 1,2,3$$

Matrix form

$$\begin{bmatrix} -a\sin\theta_1 \\ a\cos\theta_1 \end{bmatrix}\dot{\theta}_1 + \begin{bmatrix} -b\sin\theta_2 & c\sin\theta_3 \\ b\cos\theta_2 & -c\cos\theta_3 \end{bmatrix}\begin{bmatrix} \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = 0$$

For given ω_1

$$\begin{bmatrix} \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = -\begin{bmatrix} -b\sin\theta_2 & c\sin\theta_3 \\ b\cos\theta_2 & -c\cos\theta_3 \end{bmatrix}^{-1}\begin{bmatrix} a\sin\theta_1 \\ a\cos\theta_1 \end{bmatrix}\dot{\theta}_1 = 0$$

known numbers, depend on position

Vel equations $\rightarrow$ accel eguations

$$-a\dot{\theta}_1 \sin\theta_1 - b\dot{\theta}_2 \sin\theta_2 + c\dot{\theta}_3 \sin\theta_3 = 0$$
$$a\dot{\theta}_1 \cos\theta_1 + b\dot{\theta}_2 \cos\theta_2 - c\dot{\theta}_3 \cos\theta_3 = 0$$

Accel equations – 2-nd time derivative

$$-a\ddot{\theta}_1 \sin\theta_1 - a\dot{\theta}_1^2 \cos\theta_1 - b\ddot{\theta}_2 \sin\theta_2 - b\dot{\theta}_2^2 \cos\theta_2 +$$
$$+c\ddot{\theta}_3 \sin\theta_3 + c\dot{\theta}_3^2 \cos\theta_3 = 0$$

$$\ddot{\theta}_i = \frac{d^2\theta_i}{dt^2} = \frac{d\omega_i}{dt} = \varepsilon_i \quad i = 1,2,3$$

Vel equations $\rightarrow$ accel eguations

$$-a\dot{\theta}_1 \sin\theta_1 - b\dot{\theta}_2 \sin\theta_2 + c\dot{\theta}_3 \sin\theta_3 = 0$$
$$a\dot{\theta}_1 \cos\theta_1 + b\dot{\theta}_2 \cos\theta_2 - c\dot{\theta}_3 \cos\theta_3 = 0$$

$$a\ddot{\theta}_1 \cos\theta_1 - a\dot{\theta}_1^2 \sin\theta_1 + b\ddot{\theta}_2 \cos\theta_2 - b\dot{\theta}_2^2 \sin\theta_2 +$$
$$- c\ddot{\theta}_3 \cos\theta_3 + c\dot{\theta}_3^2 \sin\theta_3 = 0$$

Accel in matrix form

$$-\begin{bmatrix} a\sin\theta_1 & a\cos\theta_1 \\ -a\cos\theta_1 & a\sin\theta_1 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \dot{\theta}_1^2 \end{bmatrix} + \begin{bmatrix} -b\sin\theta_2 & c\sin\theta_3 \\ b\cos\theta_2 & -c\cos\theta_3 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} +$$

$$+\begin{bmatrix} -b\cos\theta_2 & +c\cos\theta_3 \\ -b\sin\theta_2 & +c\sin\theta_3 \end{bmatrix}\begin{bmatrix} \dot{\theta}_2^2 \\ \dot{\theta}_3^2 \end{bmatrix} = 0$$

For given ω_1 and ε_1

$$\begin{bmatrix} \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} = \begin{bmatrix} -b\sin\theta_2 & c\sin\theta_3 \\ b\cos\theta_2 & -c\cos\theta_3 \end{bmatrix}^{-1} \left(\begin{bmatrix} b\cos\theta_2 & -c\cos\theta_2 \\ b\sin\theta_2 & -c\sin\theta_2 \end{bmatrix}\begin{bmatrix} \dot{\theta}_2^2 \\ \dot{\theta}_3^2 \end{bmatrix} + + \begin{bmatrix} a\sin\theta_1 & a\cos\theta_1 \\ -a\cos\theta_1 & a\sin\theta_1 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \dot{\theta}_1^2 \end{bmatrix} \right)$$