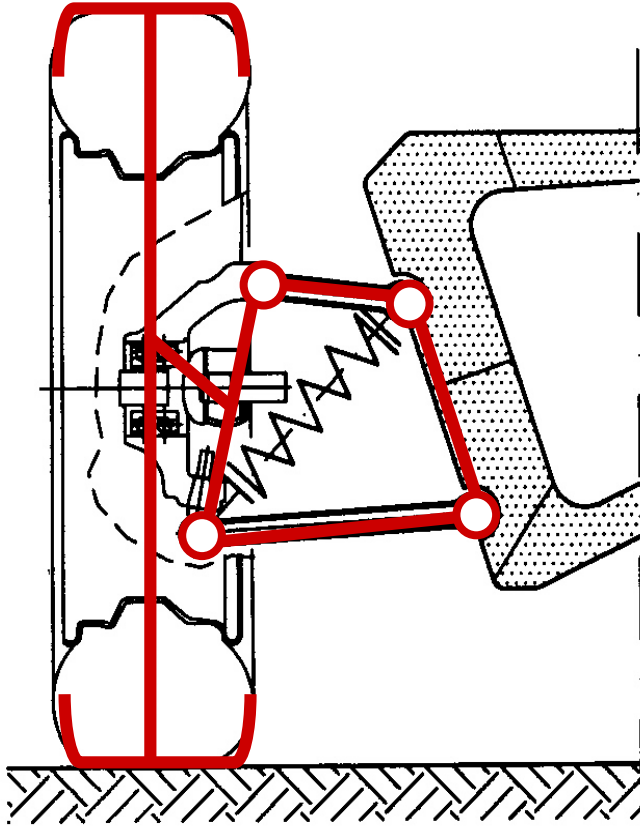
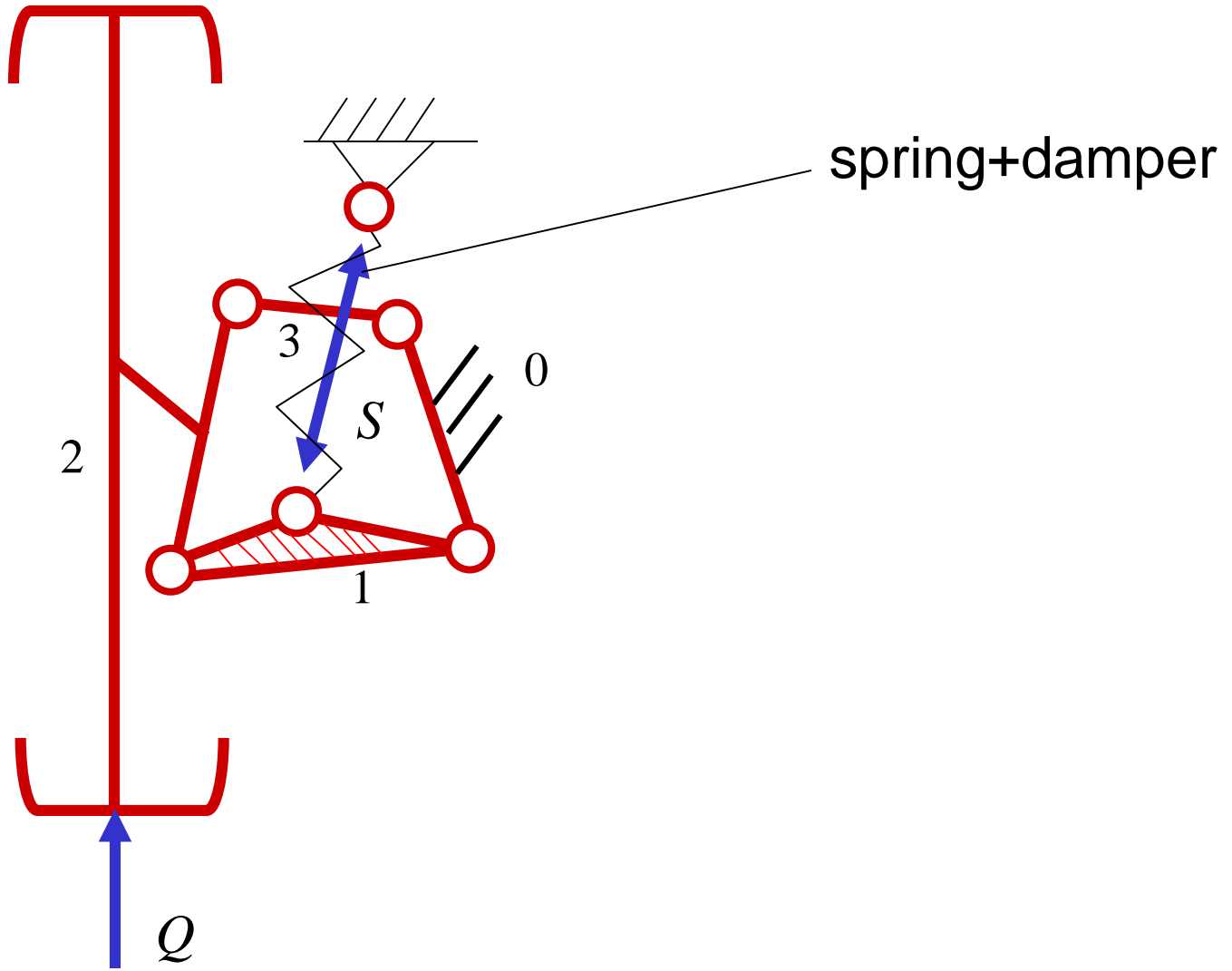
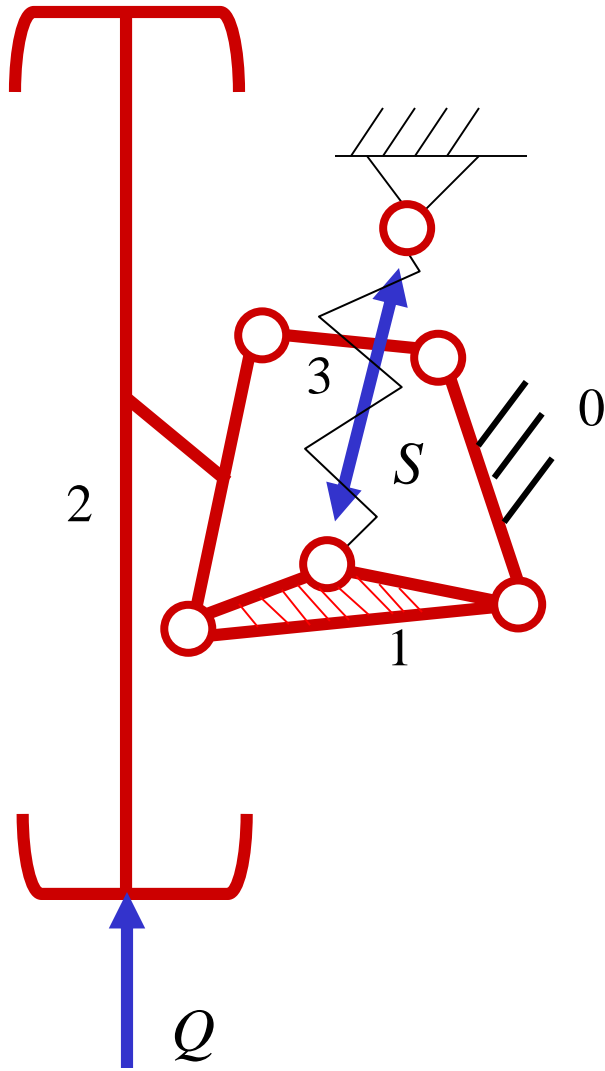


Suspension system



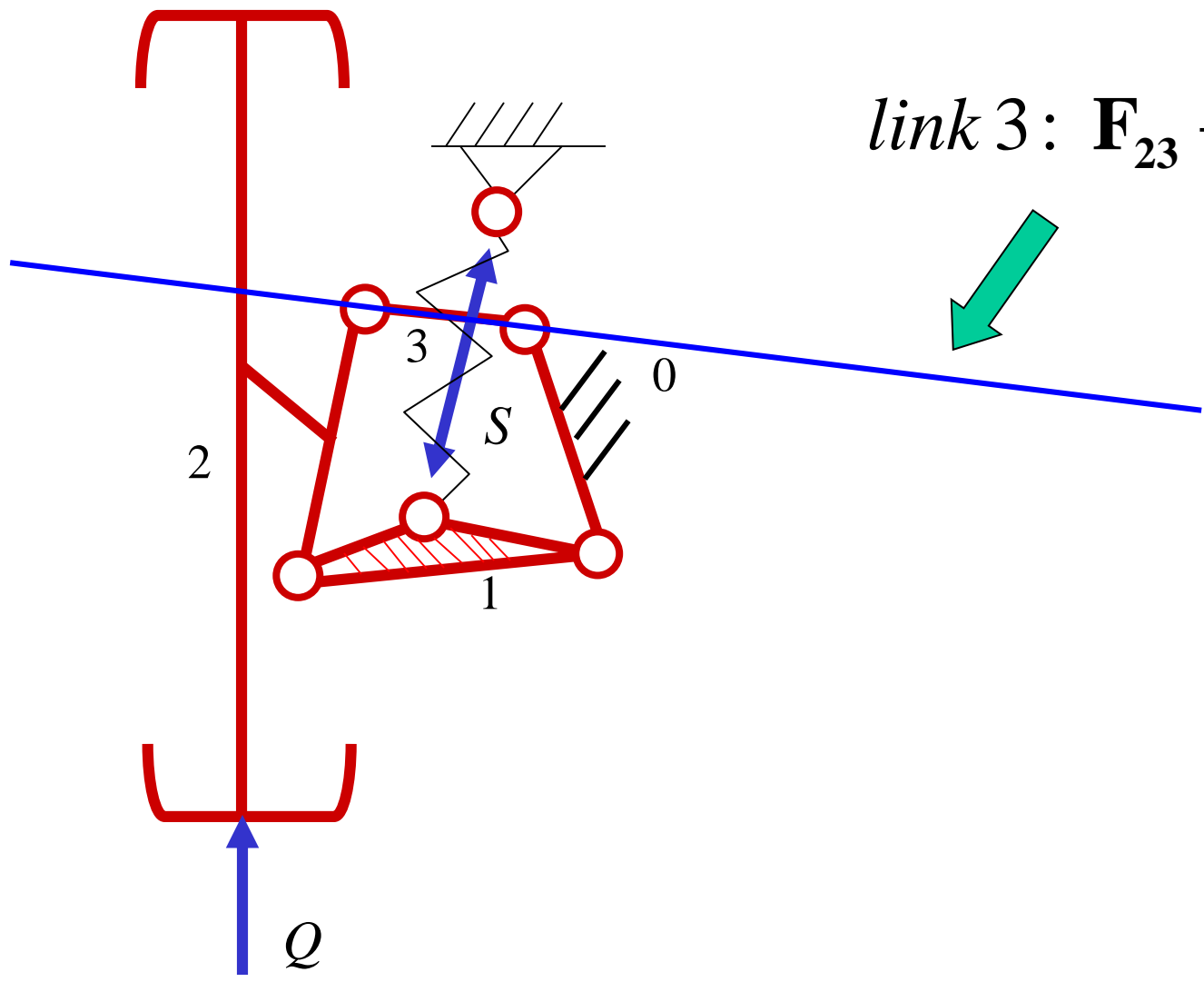




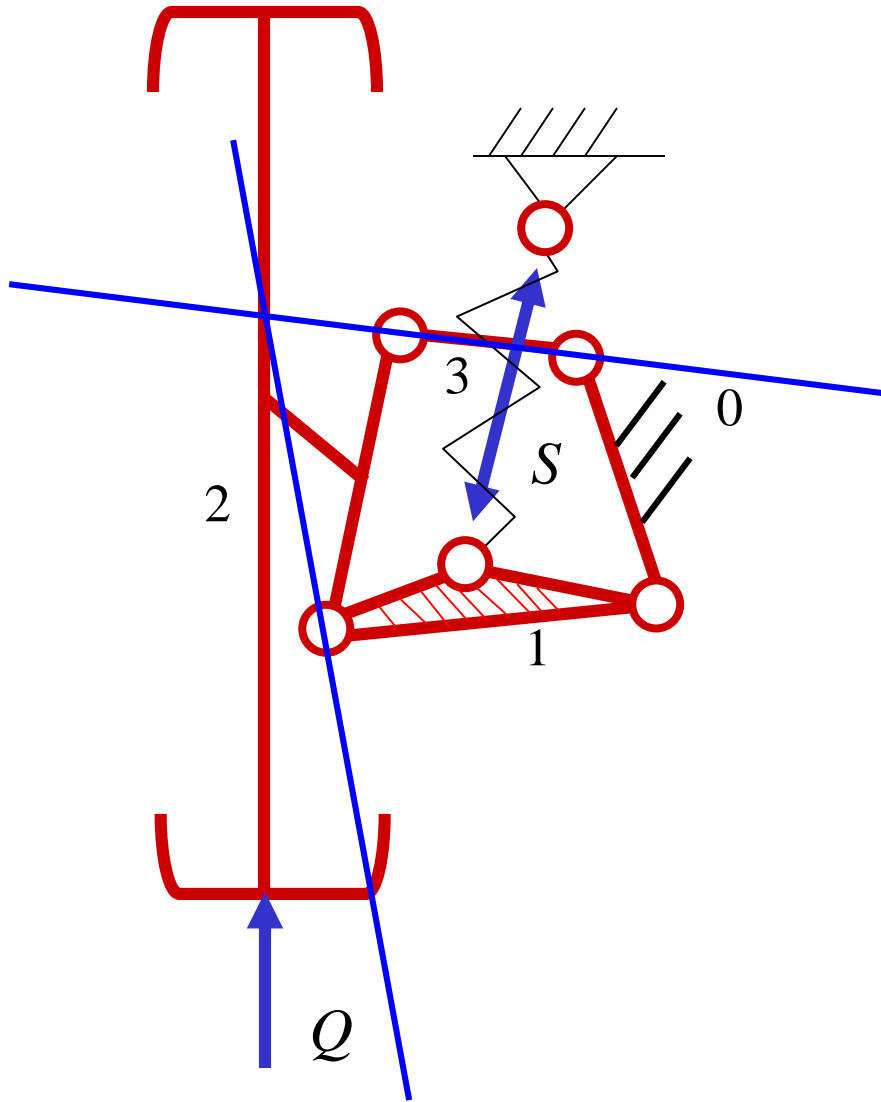
$$\text{link 2: } \mathbf{F}_{32} + \mathbf{Q} + \mathbf{F}_{12} = 0$$

points of application only

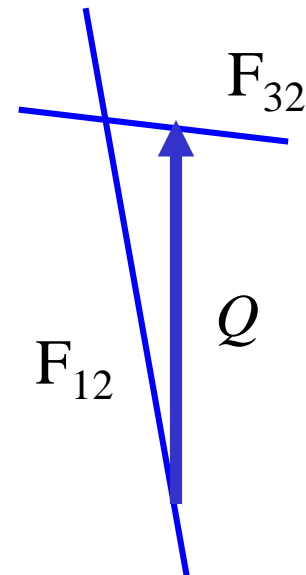
?



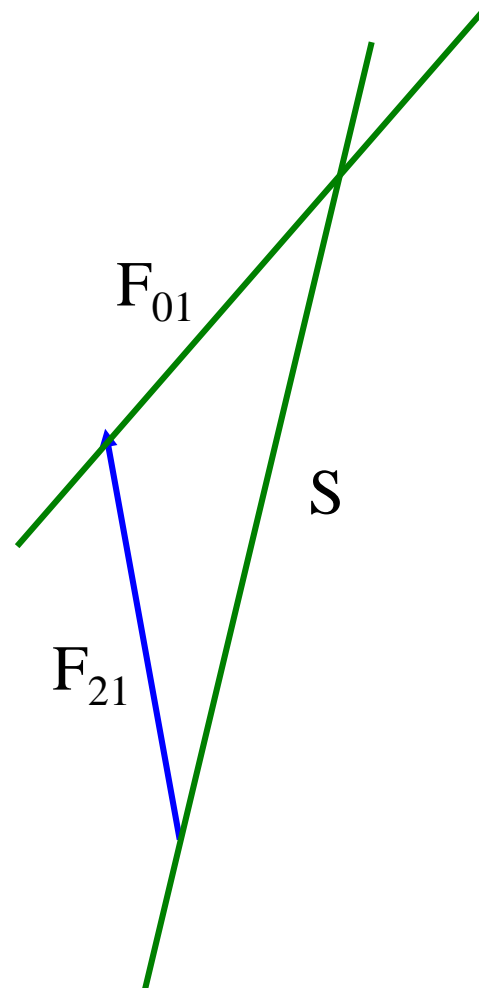
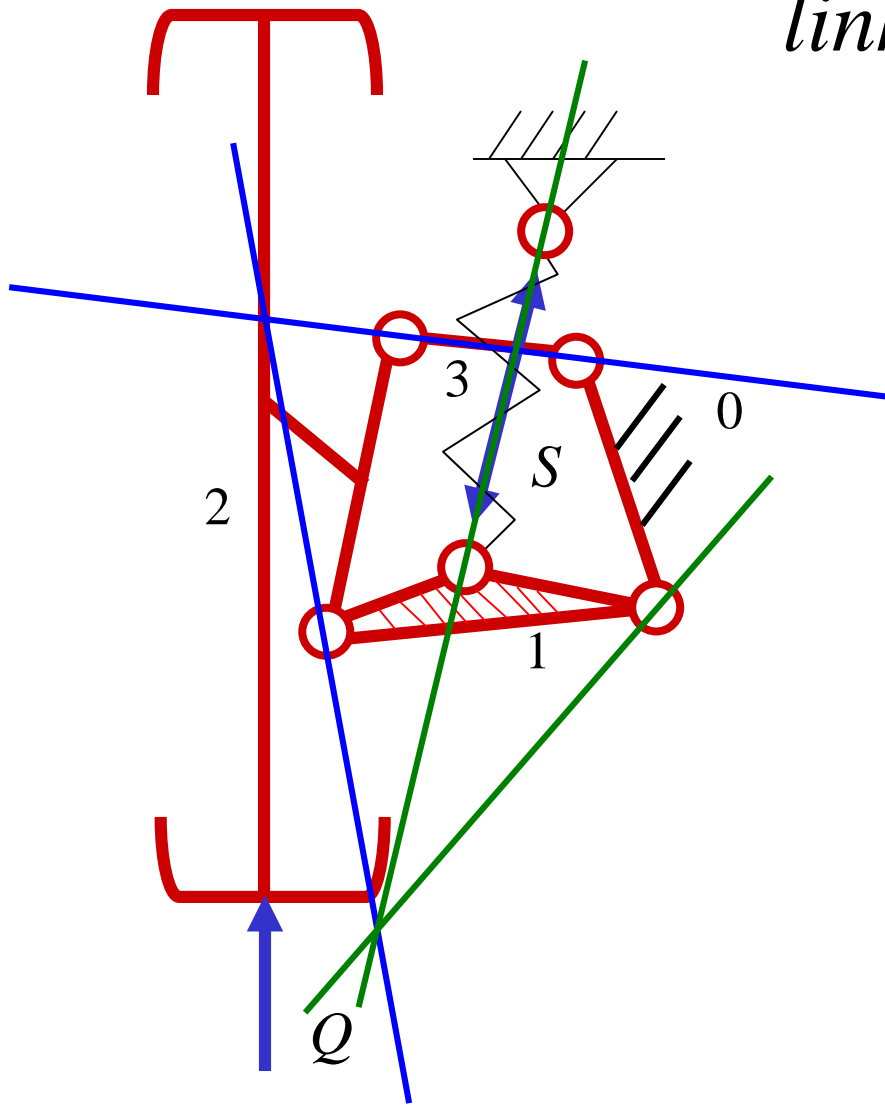
link 3: $\mathbf{F}_{23} + \mathbf{F}_{03} = 0$



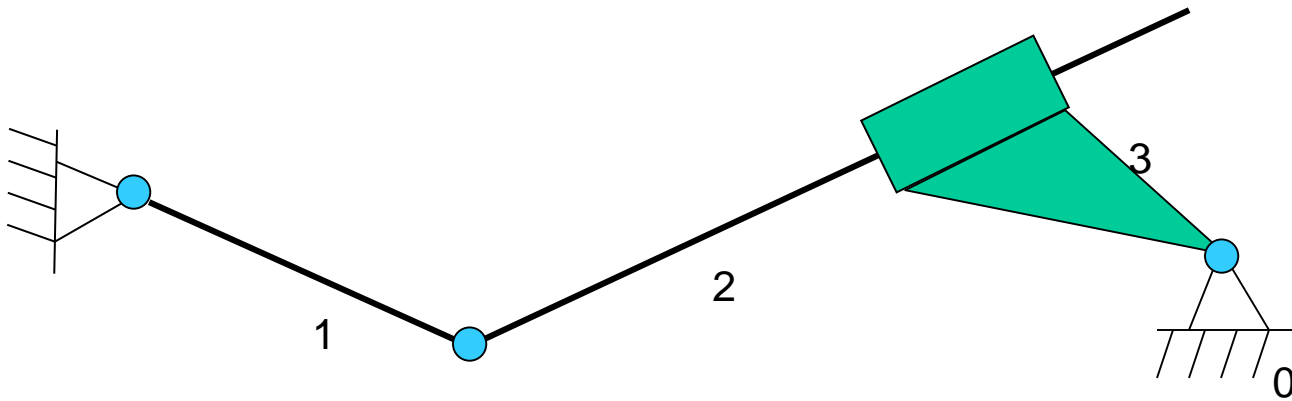
$$\mathbf{F}_{32} + \mathbf{Q} + \mathbf{F}_{12} = 0$$



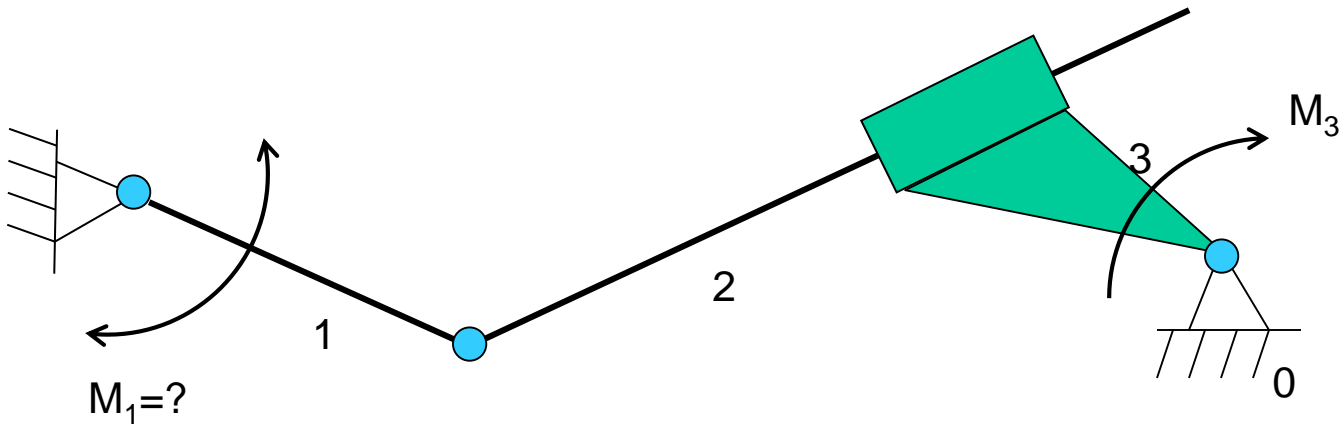
$$\text{link 1: } \mathbf{F}_{21} + \mathbf{F}_{01} + \mathbf{S} = 0$$



RRTR example (1)

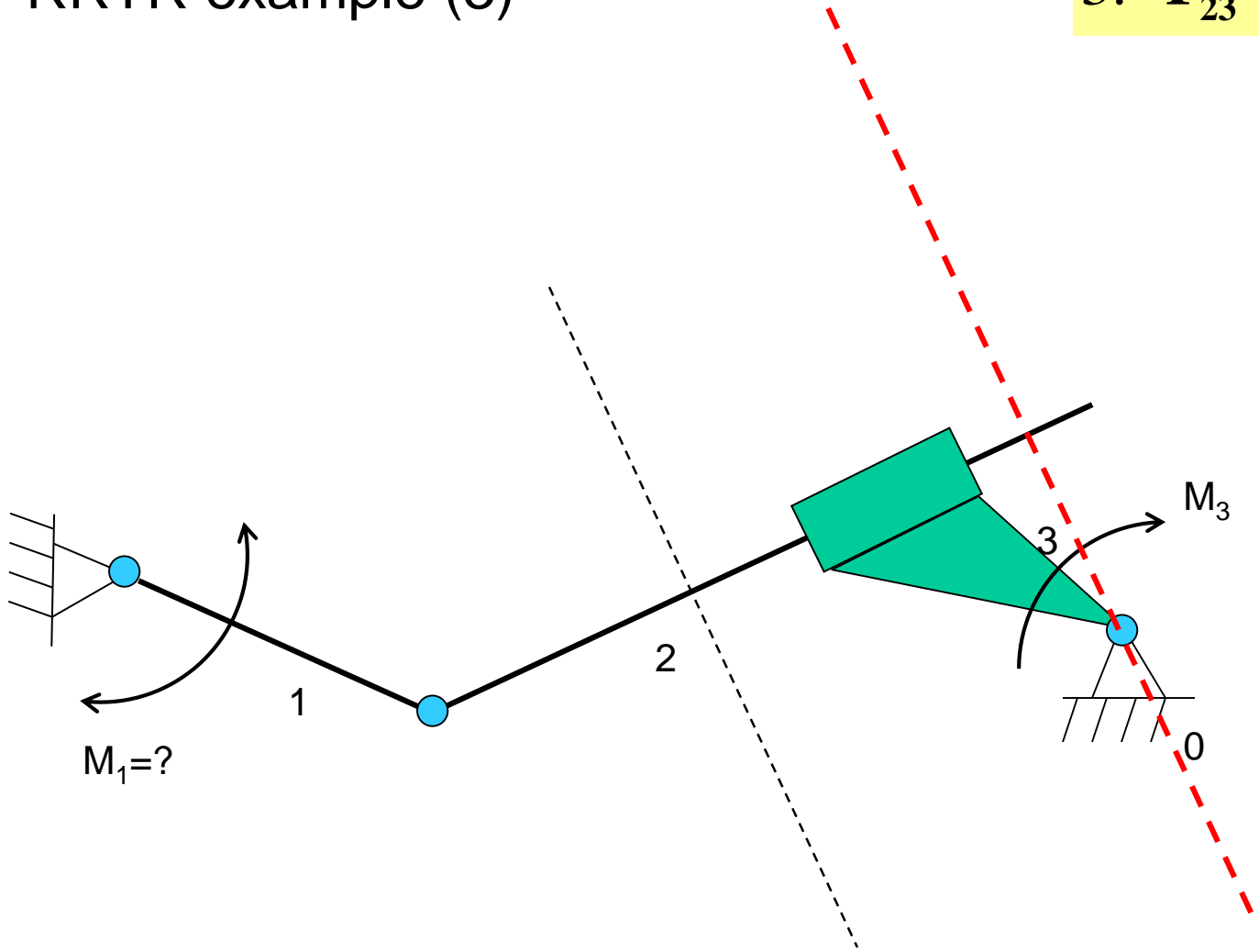


RRTR example (2)



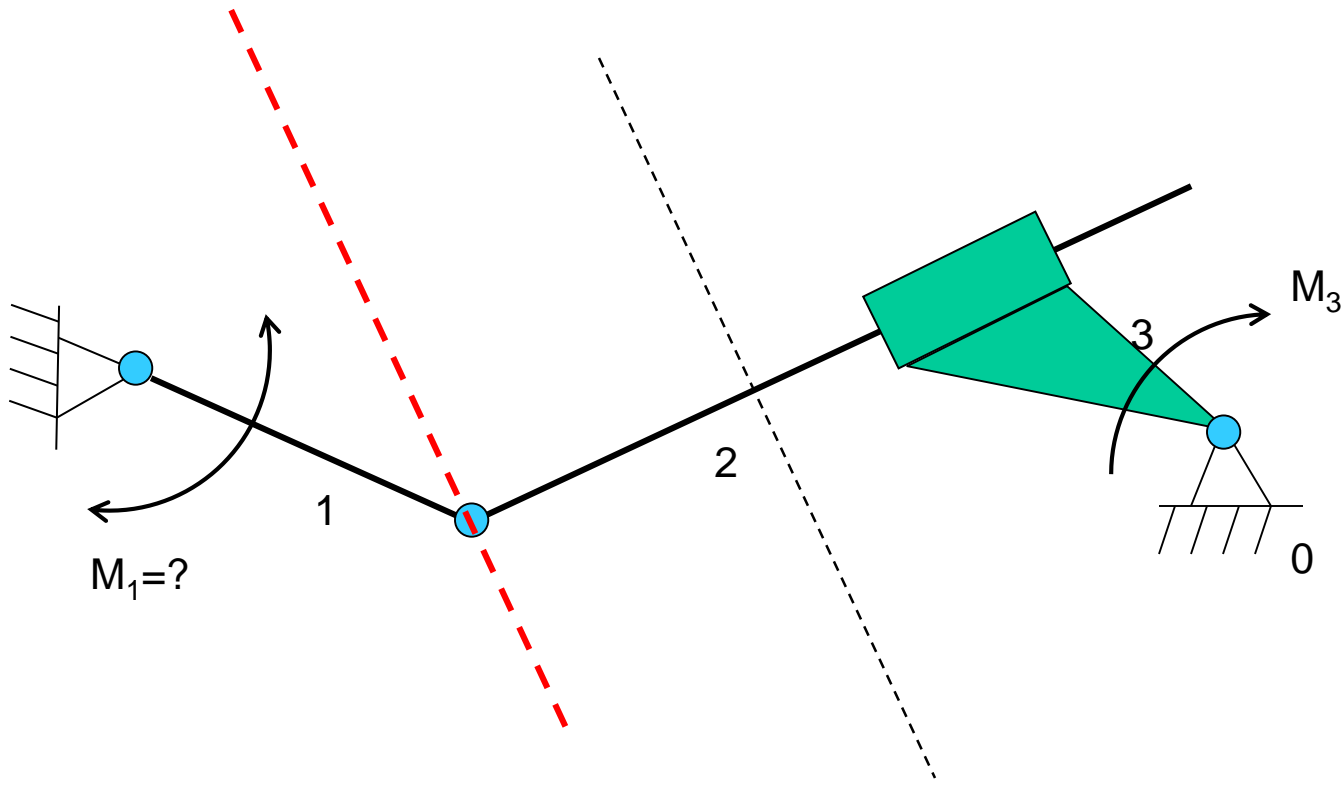
RRTR example (3)

$$3. \mathbf{F}_{23} + \mathbf{F}_{03} = 0$$



RRTR example (3)

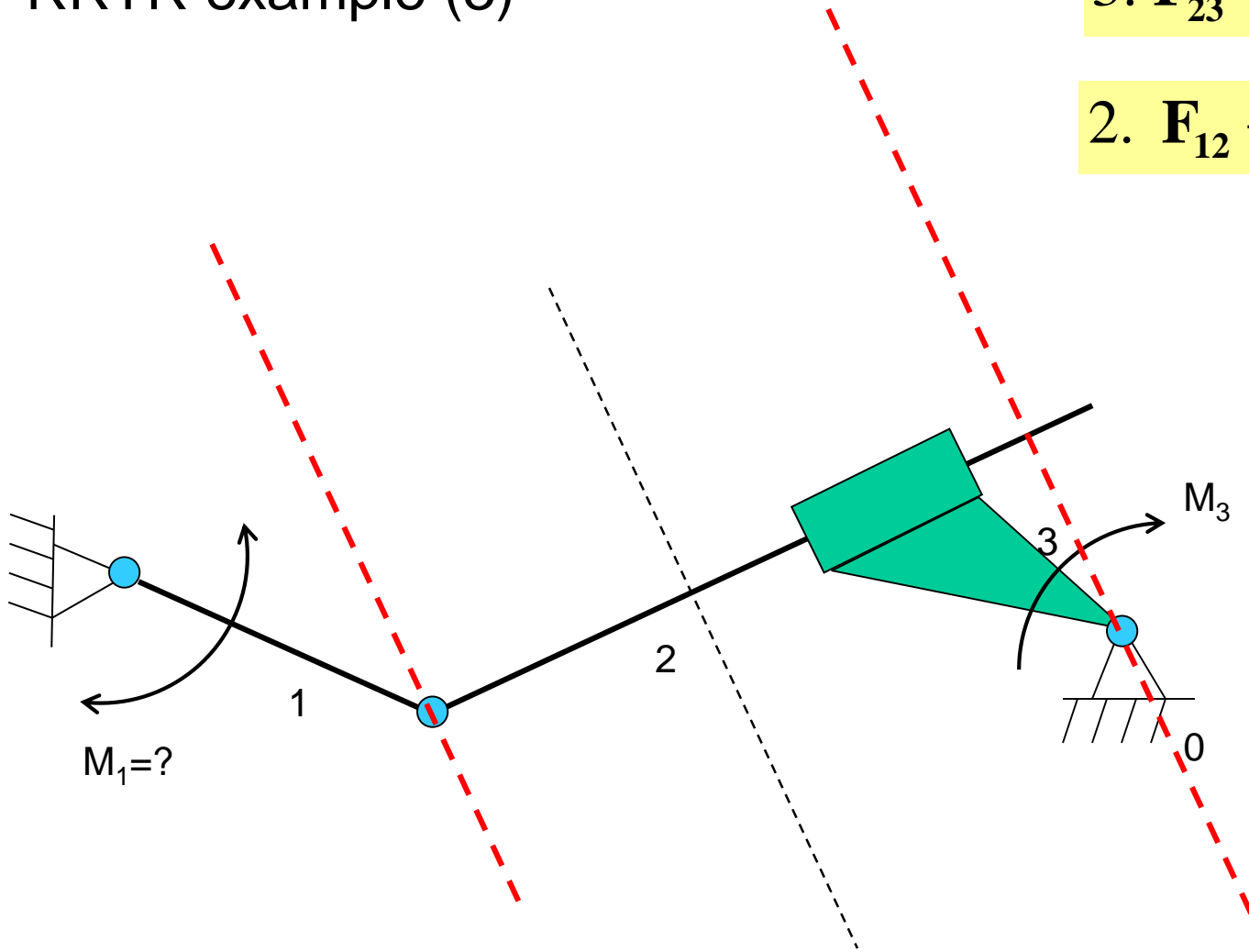
$$2. \mathbf{F}_{12} + \mathbf{F}_{32} = 0$$



RRTR example (3)

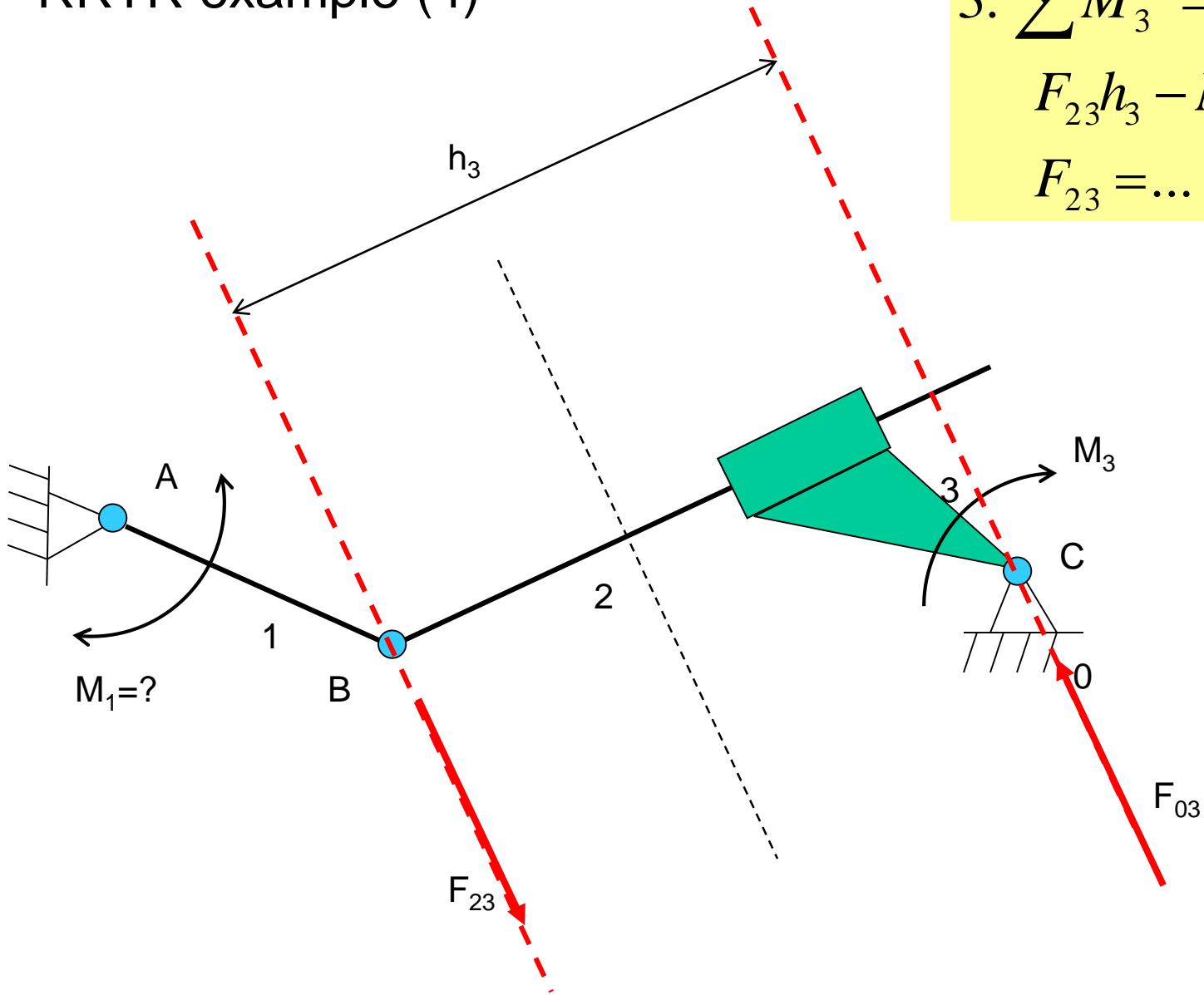
$$3. \mathbf{F}_{23} + \mathbf{F}_{03} = 0$$

$$2. \mathbf{F}_{12} + \mathbf{F}_{32} = 0$$



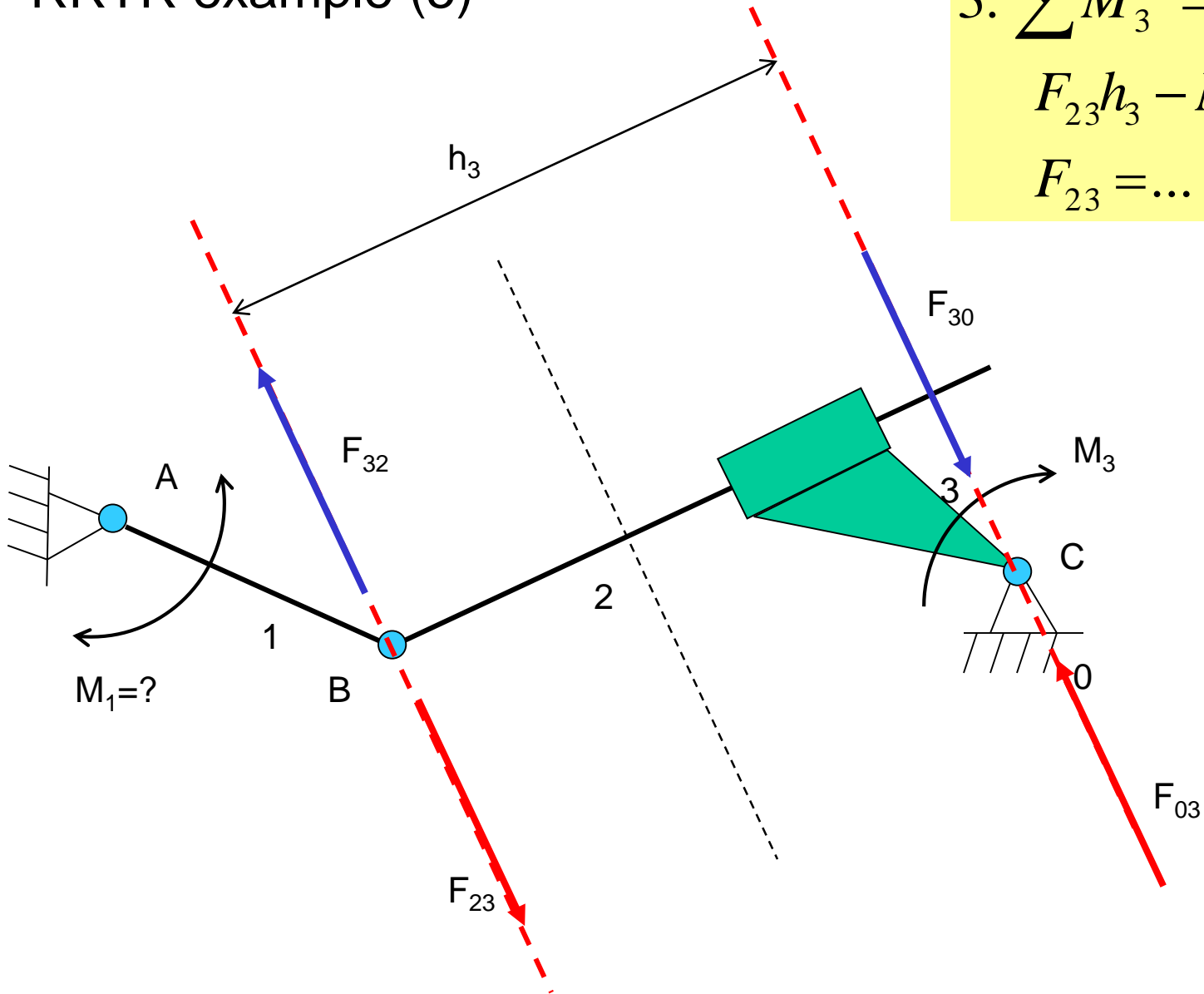
RRTR example (4)

$$3. \sum M_3^C = 0$$
$$F_{23}h_3 - M_3 = 0$$
$$F_{23} = \dots$$

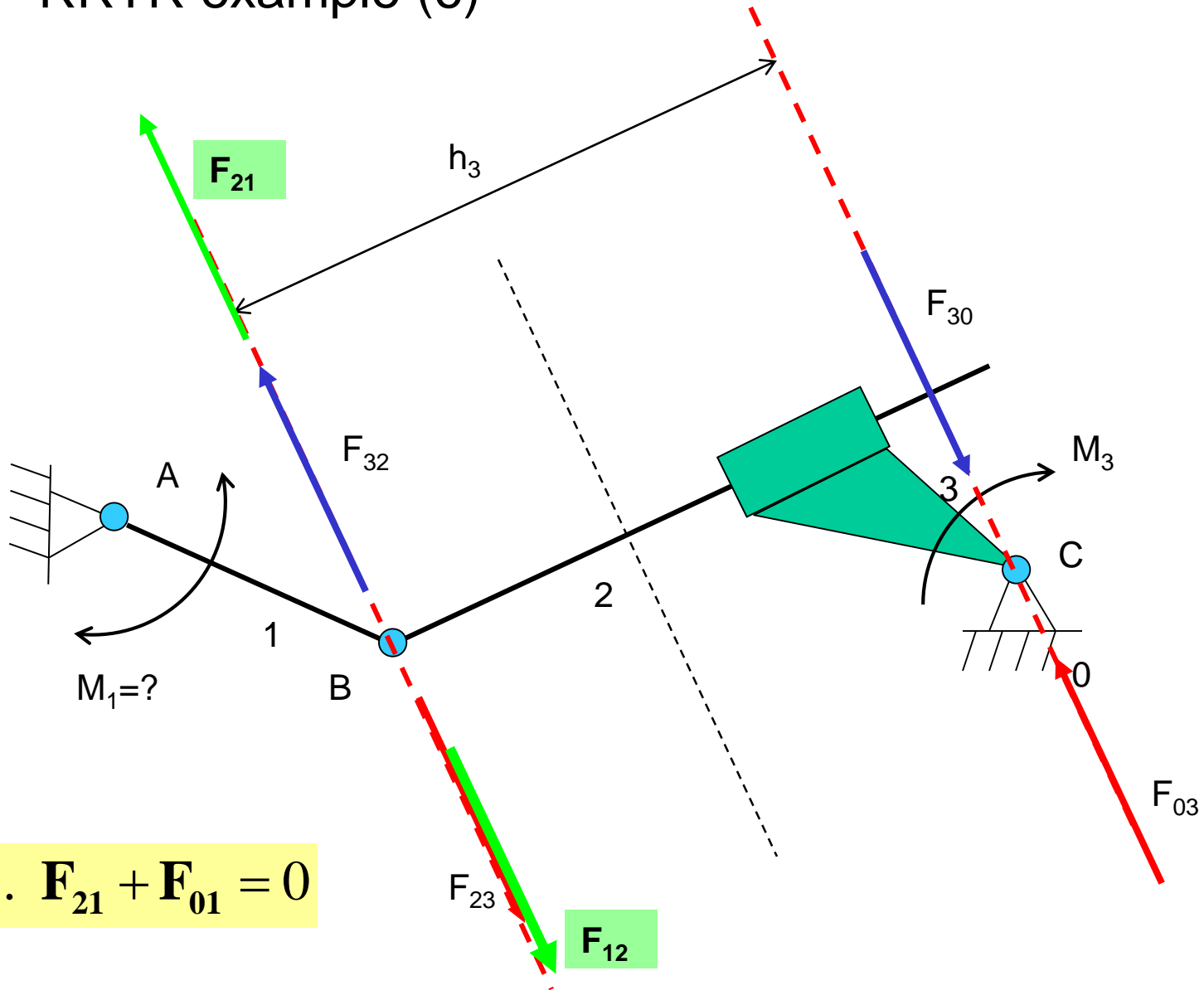


RRTR example (5)

$$3. \sum M_3^C = 0$$
$$F_{23}h_3 - M_3 = 0$$
$$F_{23} = \dots$$



RRTR example (6)

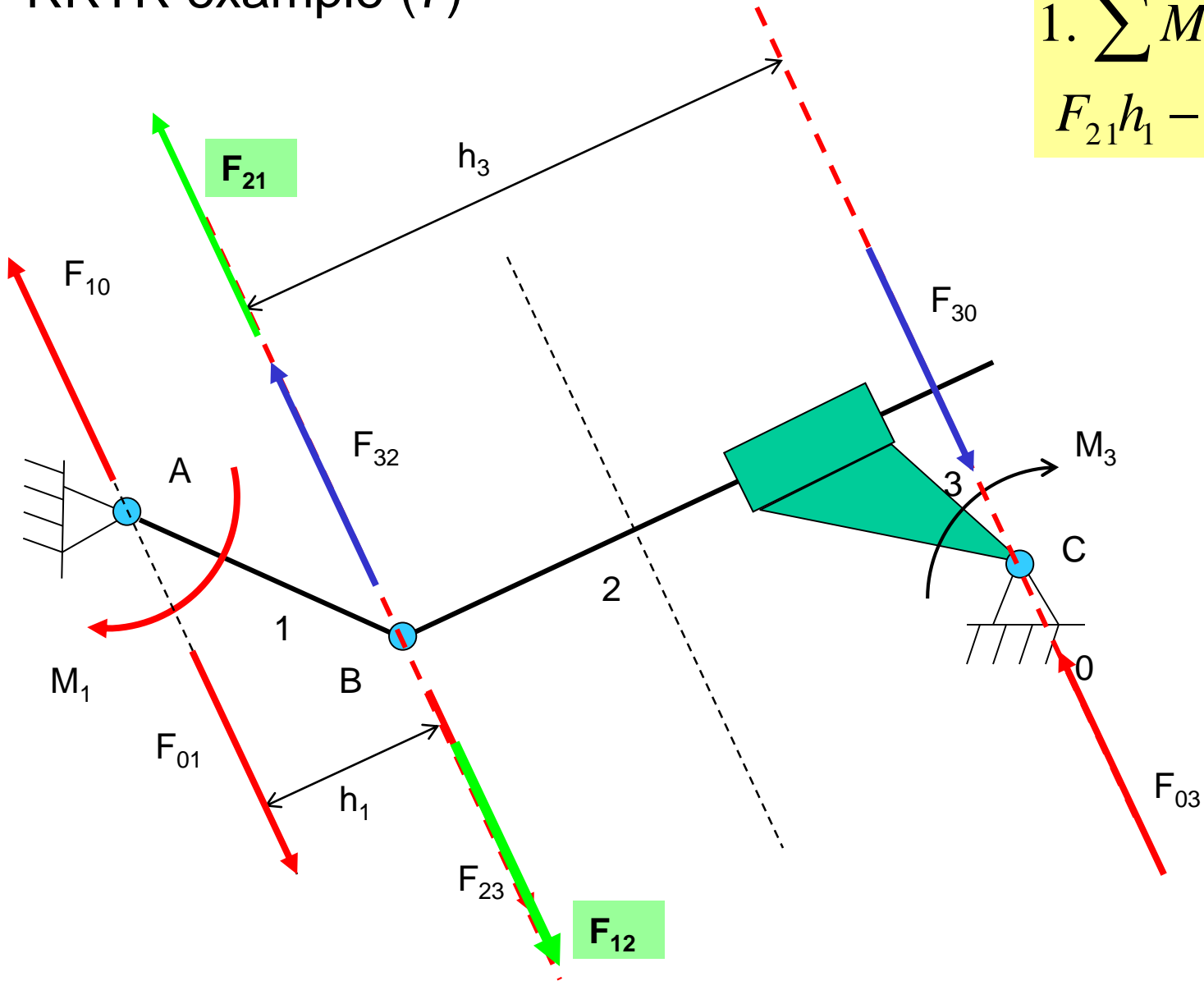


1. $F_{21} + F_{01} = 0$

RRTR example (7)

$$1. \sum M_1^A = 0$$

$$F_{21}h_1 - M_1 = 0$$



IN KINETOSTATICS
THE MOTION OF MASS BODIES (LINKS)
IS REPRESENTED BY INERTIA FORCES

Inertia forces

$$\mathbf{F} = m\mathbf{a}$$

$$\mathbf{F} - m\mathbf{a} = 0 \leftrightarrow \mathbf{F} + \mathbf{F}_b = 0$$



$$\mathbf{F}_b = -m\mathbf{a}$$

$$\mathbf{M} = I_S \boldsymbol{\varepsilon}$$

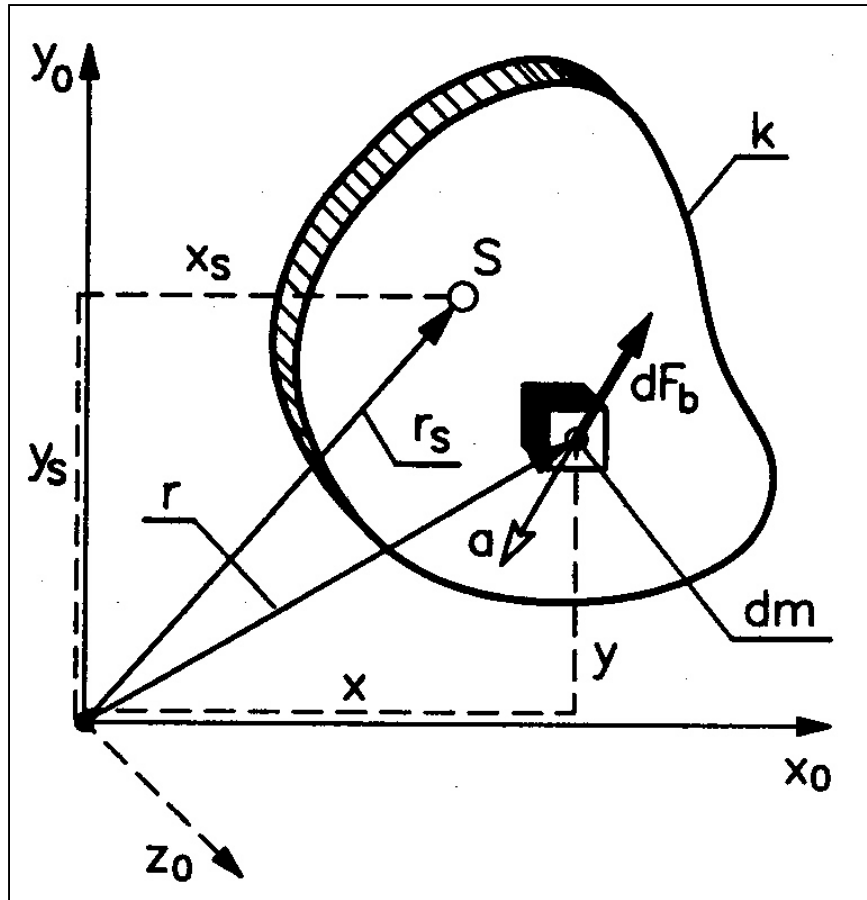
$$\mathbf{M} - I_S \boldsymbol{\varepsilon} = 0 \leftrightarrow \mathbf{M} + \mathbf{M}_b = 0$$



$$\mathbf{M}_b = -I_S \boldsymbol{\varepsilon}$$

Newton, Euler, D'Alembert

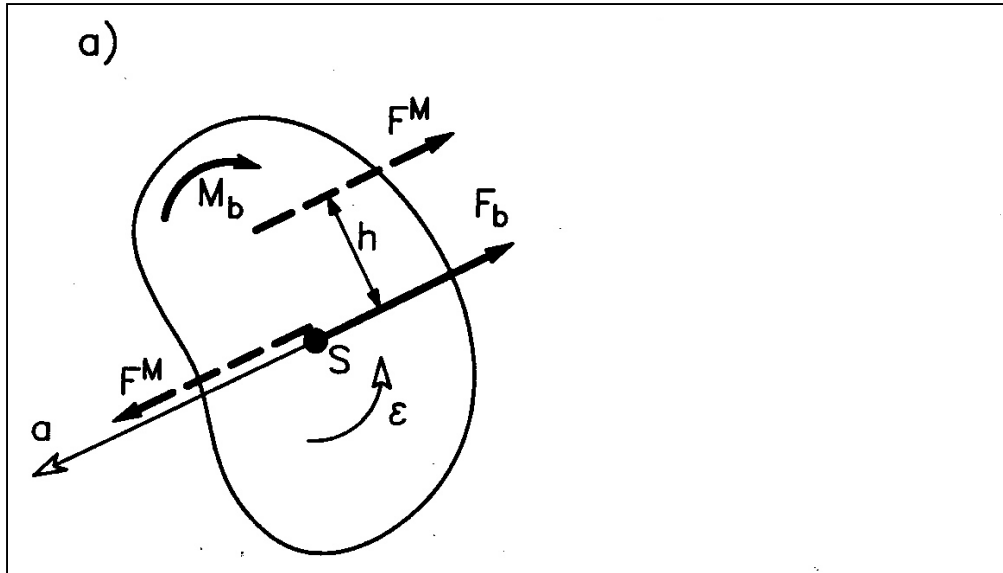
Mass moment of inertia



$$I_Z = \int_m (x^2 + y^2) dm$$

$$I_Z = I_S + m(x_s^2 + y_s^2)$$

FORCE (F_b) and MOMENT (M_b) OF INERTIA



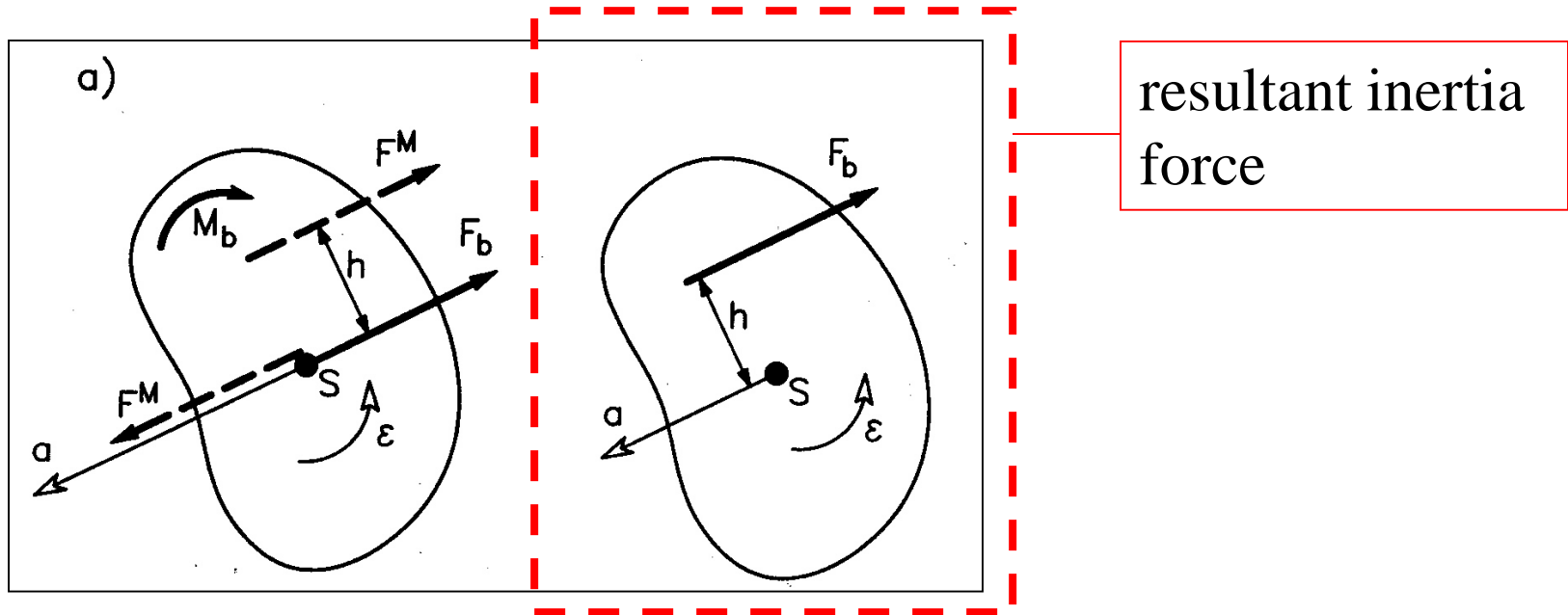
Given:
 m, I_S, a, ε

$$\mathbf{F}_b = -m\mathbf{a}$$

$$\mathbf{M}_b = -I_S \varepsilon$$

$$h = \frac{M_b}{F^M} = \frac{M_b}{F_b} = \frac{I_S \varepsilon}{ma}$$

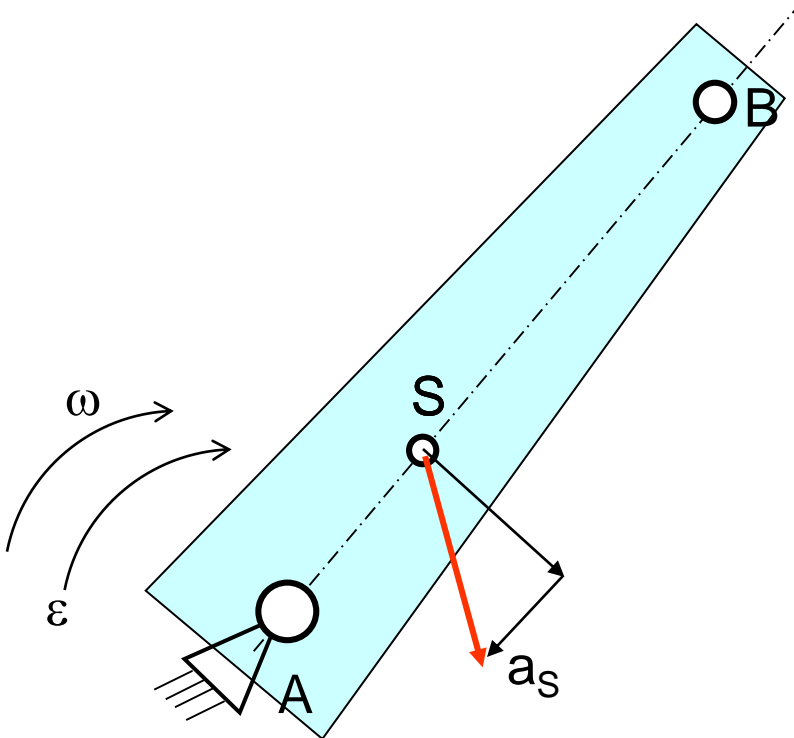
FORCE (F_b) and MOMENT (M_b) OF INERTIA



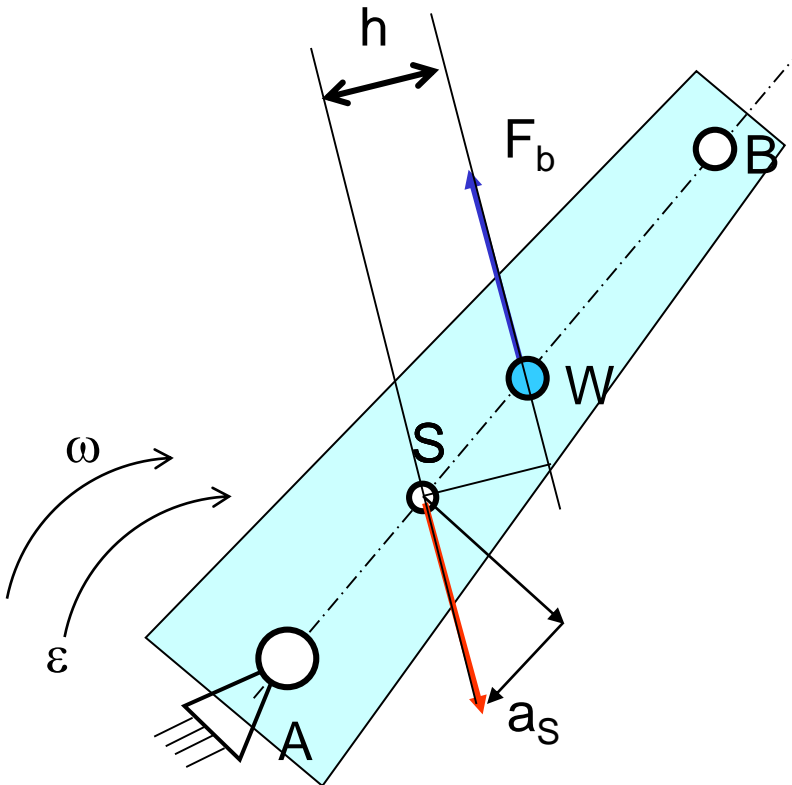
Center of percussion

$$\mathbf{a}_S = \mathbf{a}_S^n + \mathbf{a}_S^t$$

$$\mathbf{a}_S^n = \omega^2 AS; \quad \mathbf{a}_S^t = \varepsilon AS$$



Center of percussion

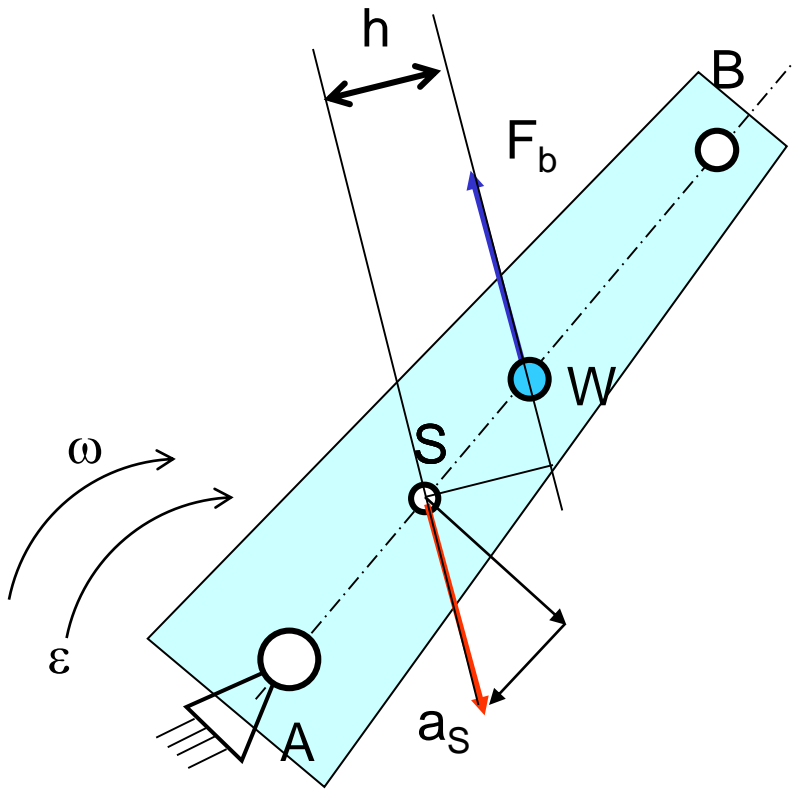


$$\mathbf{F}_b = -\mathbf{a}_S m$$

$$\mathbf{M}_b = -\boldsymbol{\varepsilon} I_S$$

$$h = \frac{M_b}{F_b} = \frac{I_S \boldsymbol{\varepsilon}}{m a_S}$$

Center of percussion



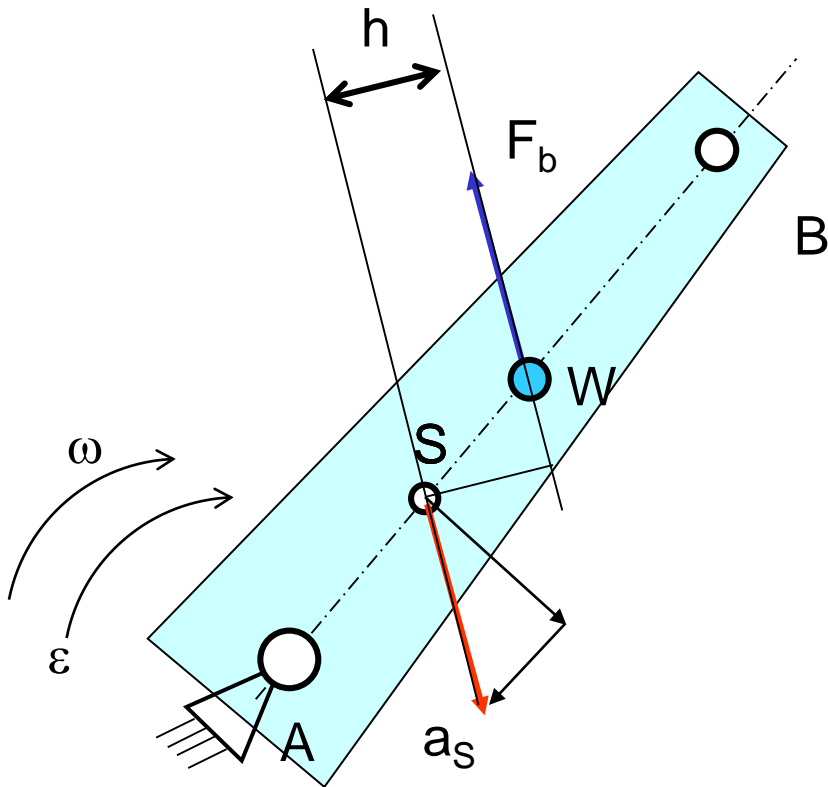
$$\frac{h}{SW} = \frac{a_s^t}{a_s} = \cos(?)$$

$$SW = h \frac{a_s}{a_s^t} = \left(\frac{I_S \epsilon}{m a_s} \right) \left(\frac{a_s}{a_s^t} \right) =$$

$$= \left(\frac{I_S \frac{a_s^t}{AS}}{m a_s} \right) \left(\frac{a_s}{a_s^t} \right) =$$

$$= \frac{I_S a_s^t}{m a_s AS} \frac{a_s}{a_s^t} = \frac{I_S}{m AS}$$

Center of percussion



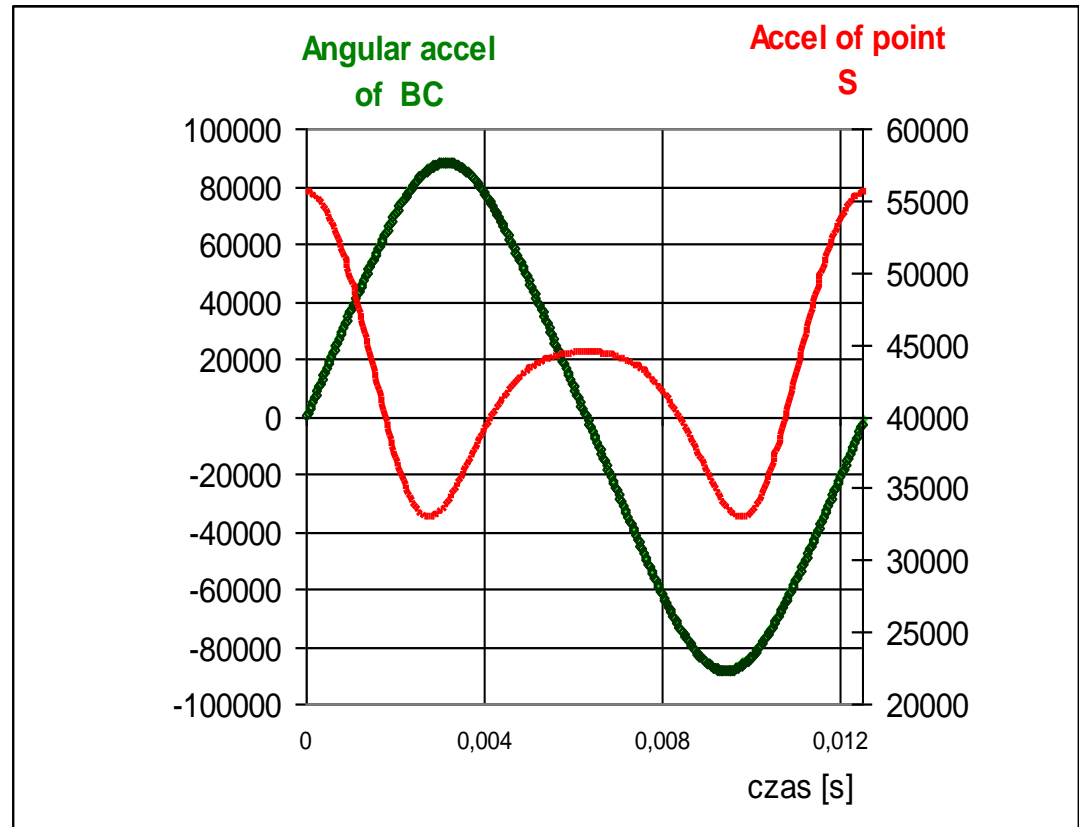
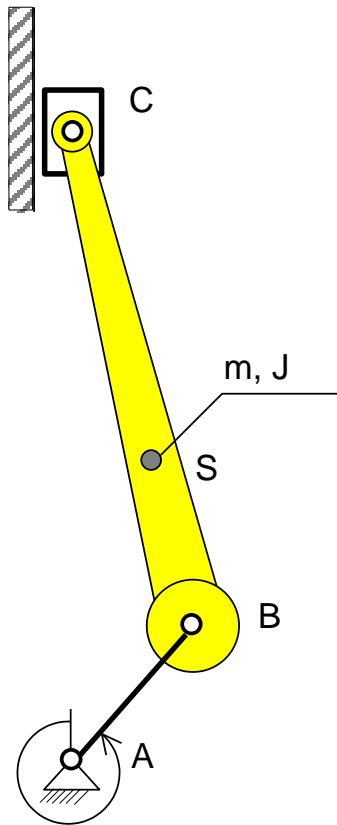
$$SW = \frac{I_S}{mAS} = \frac{mi_S^2}{mAS} = \frac{i_S^2}{AS}$$

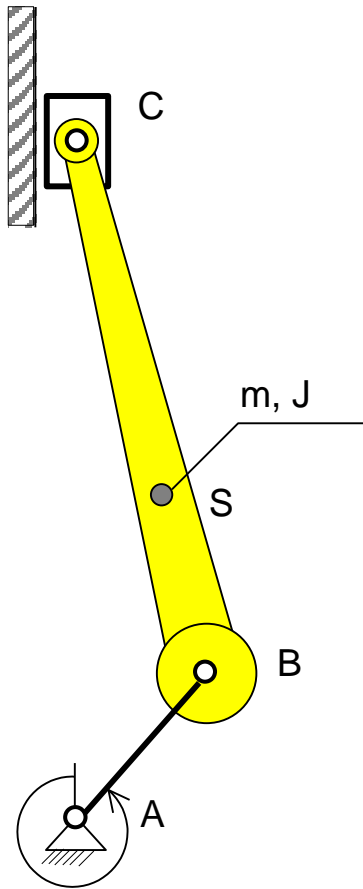
i_S – radius of inertia

$$BC = 0,2 \text{ m}$$

$$\omega_1 = 500 \text{ s}^{-1} (\omega_1 = \text{const})$$

$$(n_1 = 5000 \text{ rev/min})$$

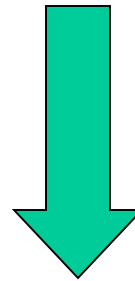




$$a_{S\max} = 55000 \text{ ms}^{-2}$$

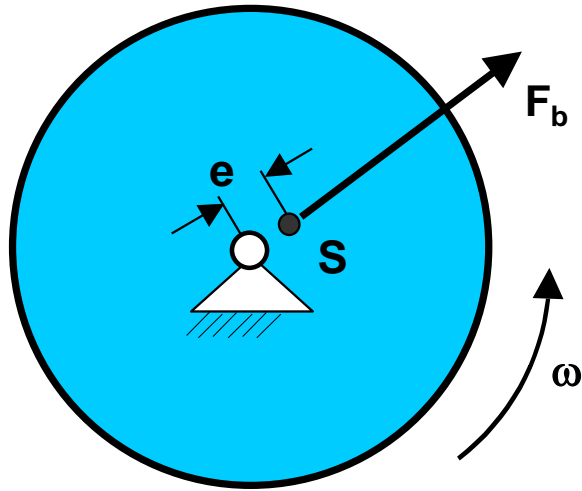
$$\varepsilon_{Bc\max} = 90\,000 \text{ s}^{-2}$$

$$m=0,2 \text{ kg}, \quad J=0,01 \text{ kgm}^2$$



$$F_{b\max} = -ma = 11000 \text{ N} !!!$$

$$M_{b\max} = -J\varepsilon = 900 \text{ Nm} !!!$$



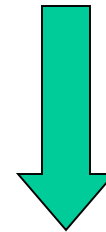
$$e = 0,001m$$

$$\omega = 500[1/s]$$

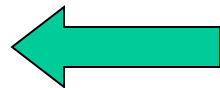
$$a_s = e\omega^2 = 250[m/s^2]$$

$$m = 1kg$$

S – center of gravity

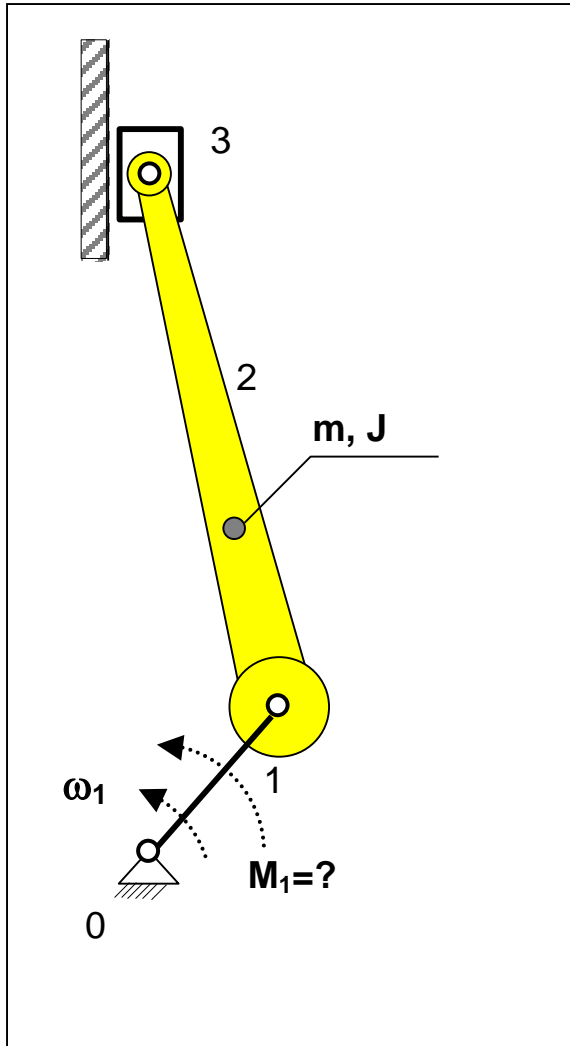


$$F_b = -ma = 250N$$



$$\frac{F_b}{F_{STATIC}} = 25$$

Example of analysis (kinetostatics)

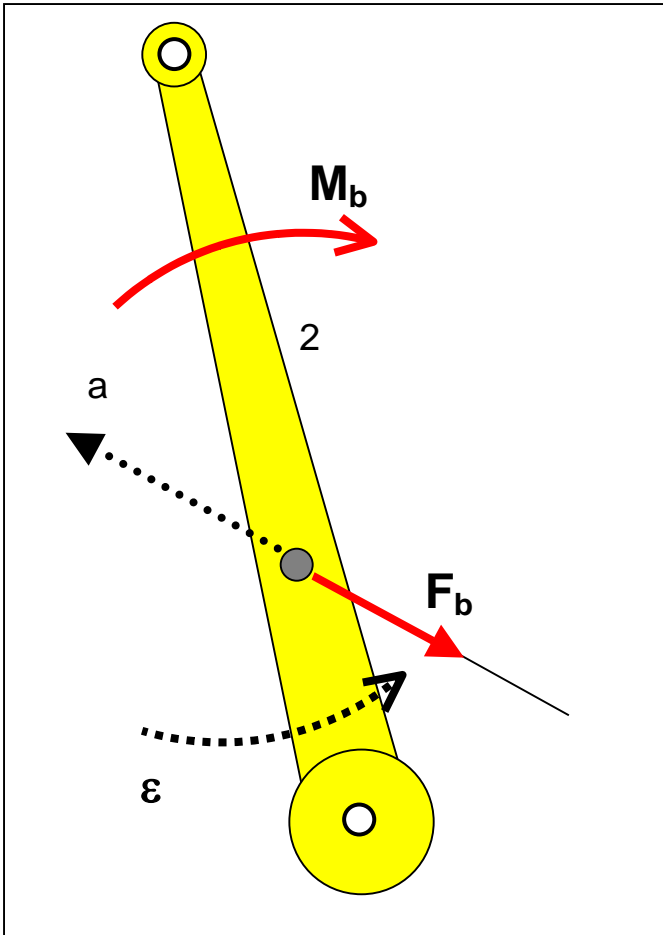


Data:

m, J – mass, mass moment of inertia

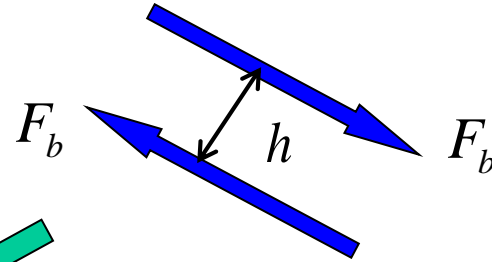
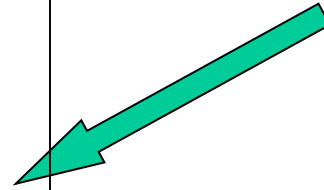
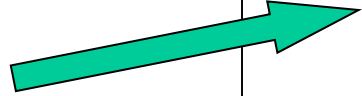
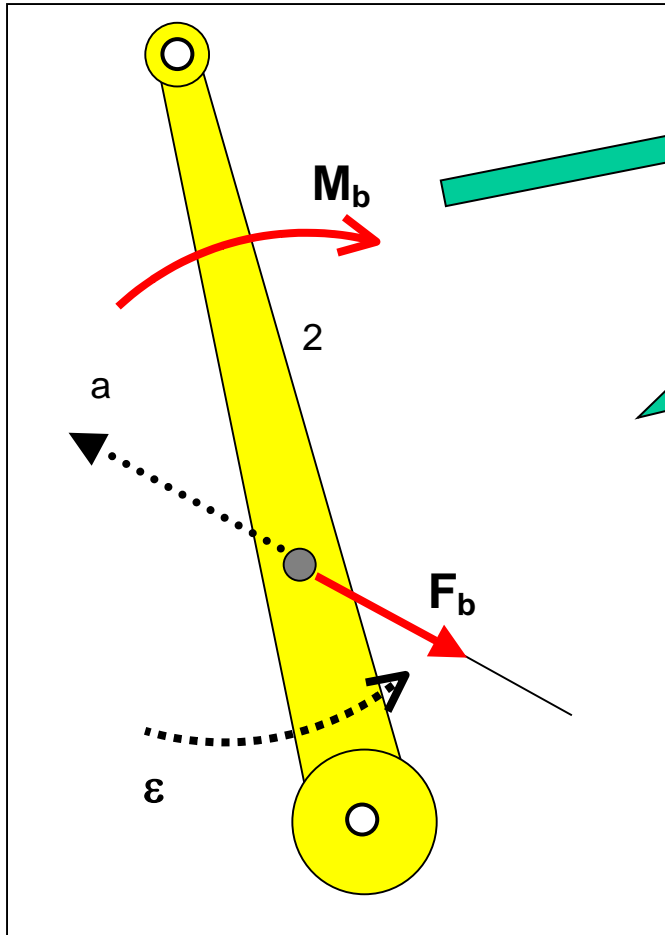
ω_1 – angular velocity of link 1

$M_1 = ?$ and joint forces

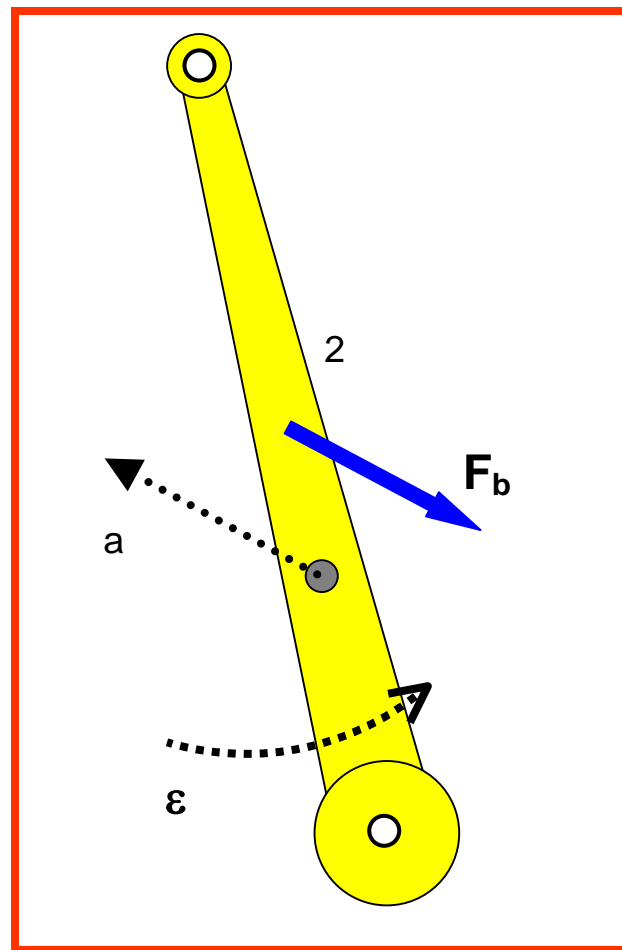
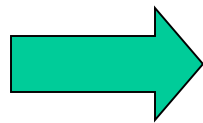
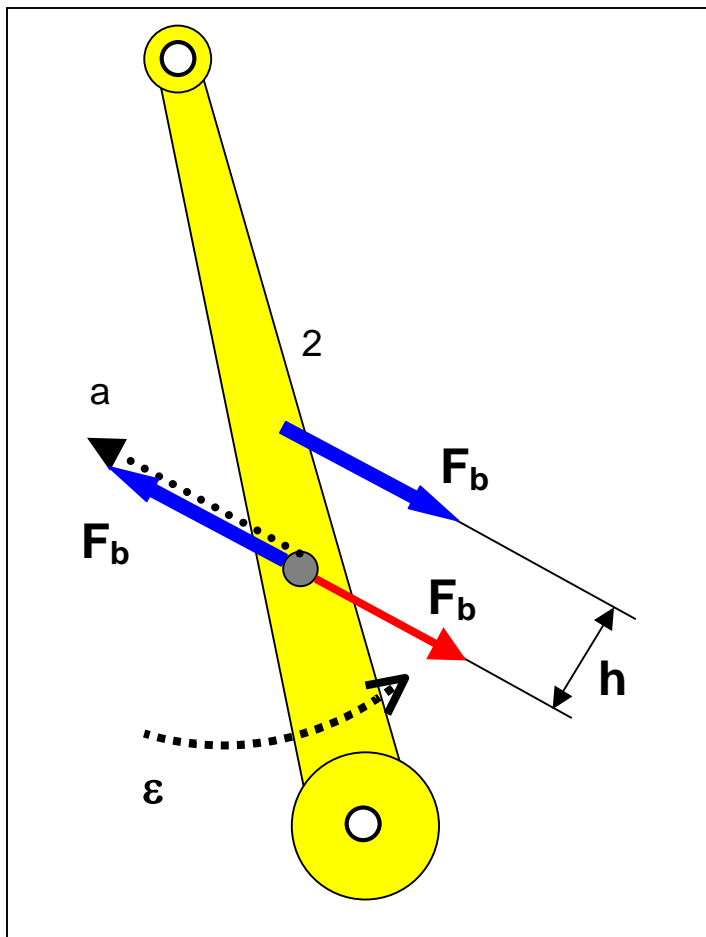


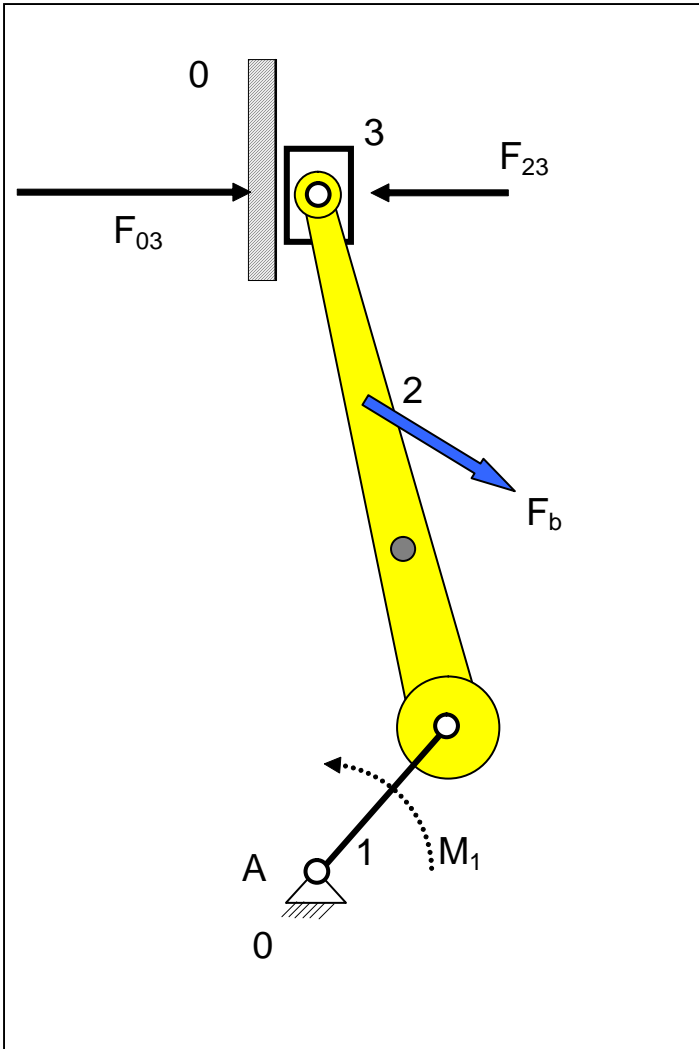
$$F_b = -ma$$

$$M_b = -J\varepsilon$$

M_b Couple of forces F_b 

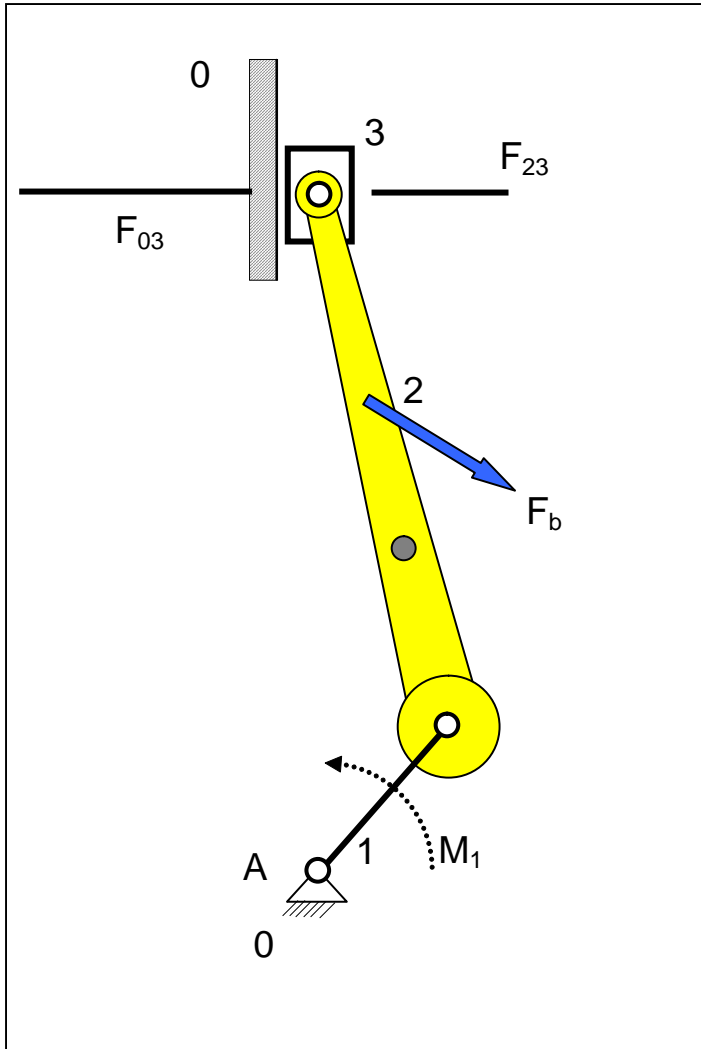
$$h = \frac{M_b}{F_b} = \frac{J\varepsilon}{ma}$$





Link 3:

$$\mathbf{F}_{23} + \mathbf{F}_{03} = 0$$

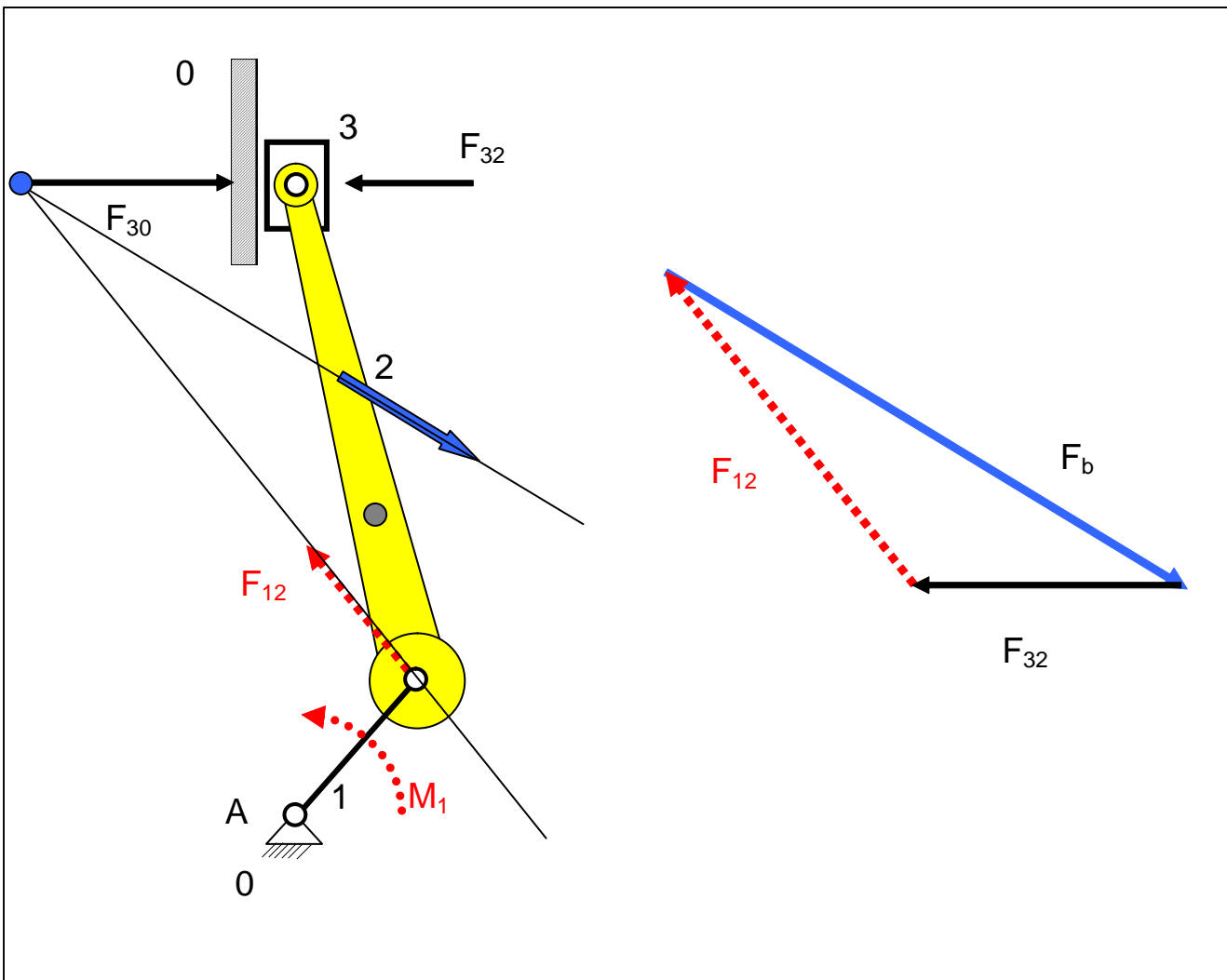


Link 3:

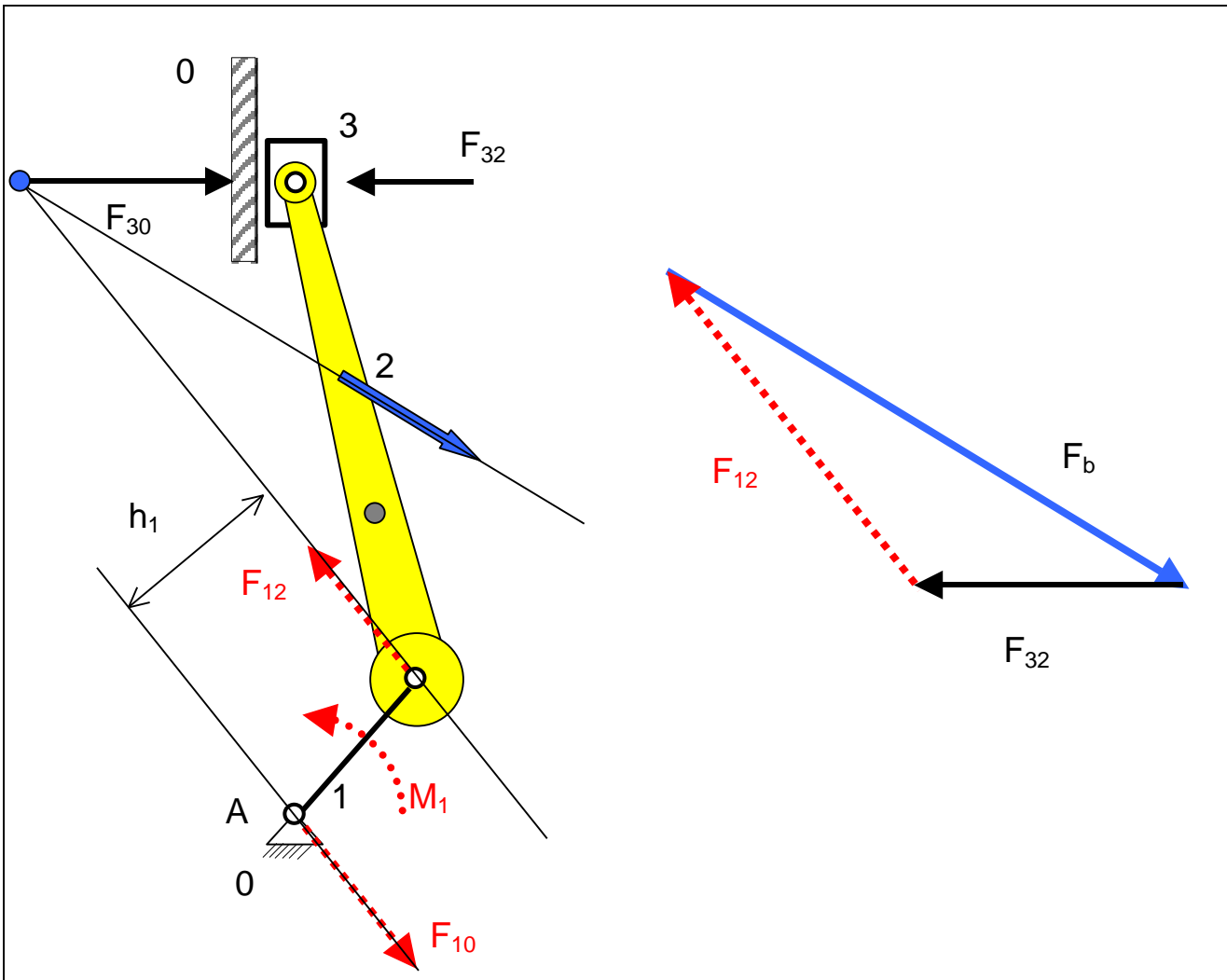
$$\mathbf{F}_{23} + \mathbf{F}_{03} = 0$$

Link 2:

$$\mathbf{F}_{32} + \mathbf{F}_b + \mathbf{F}_{12} = 0$$

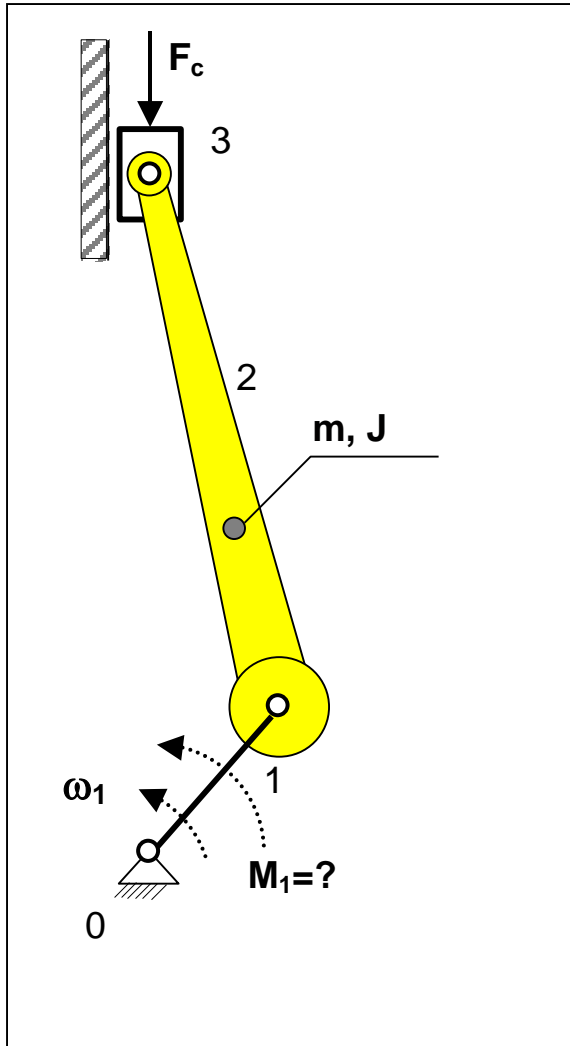


$$\mathbf{F}_{32} + \mathbf{F}_b + \mathbf{F}_{12} = \mathbf{0}$$



$$\mathbf{F}_{21} + \mathbf{F}_{01} = 0 \quad \sum M_1^A = 0 \rightarrow M_1 - F_{21} h_1 = 0$$

Example of analysis (kinetostatics)



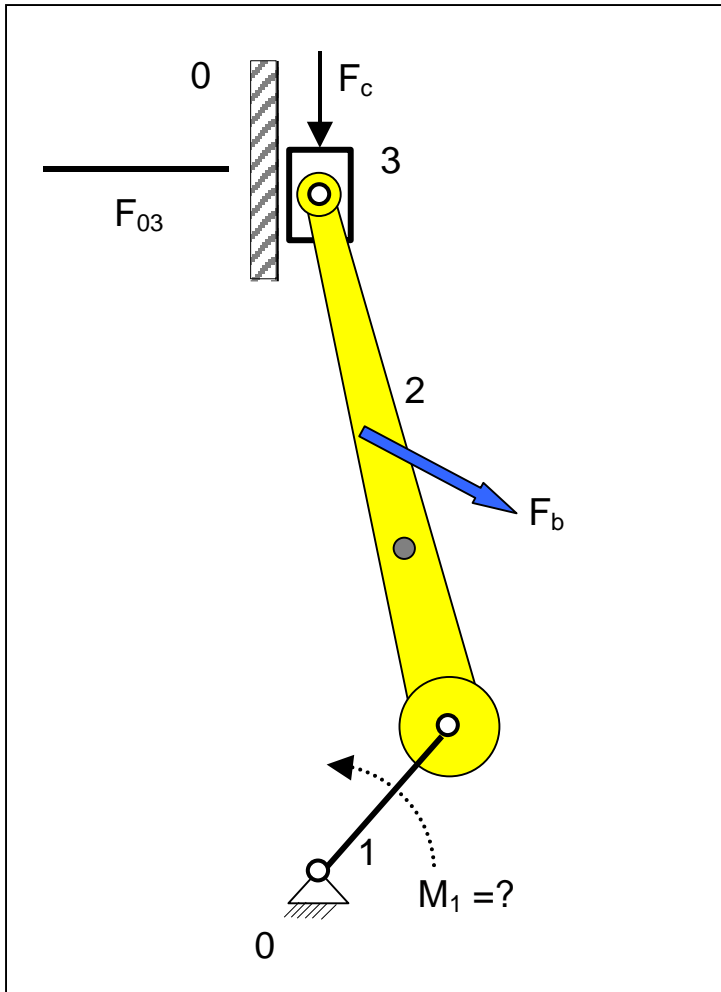
Data:

F_c – input force (driver)

m, J – mass, mass mom. of inertia

ω_1 – angular vel of crank 1

$M_1 = ?$ and joint forces = ?



Slider 3:

$$\mathbf{F}_C + \mathbf{F}_{03} + \mathbf{F}_{23} = 0$$

Coupler 2:

$$\mathbf{F}_{32} + \mathbf{F}_b + \mathbf{F}_{12} = 0$$

???

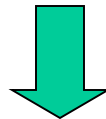
Statically determined group (of links)

forces may be fully determined using the six (three) equations of 3D (2D) statics without using deflection and stiffness criteria.

kinematic system in equilibrium

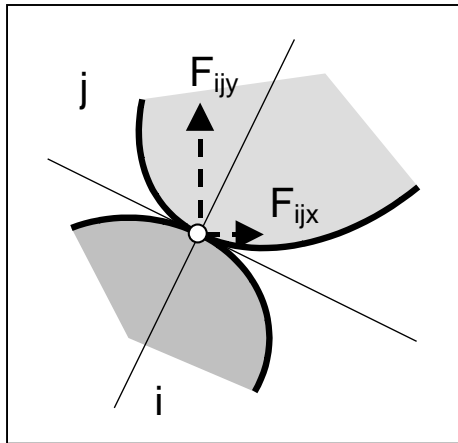


every link in equilibrium

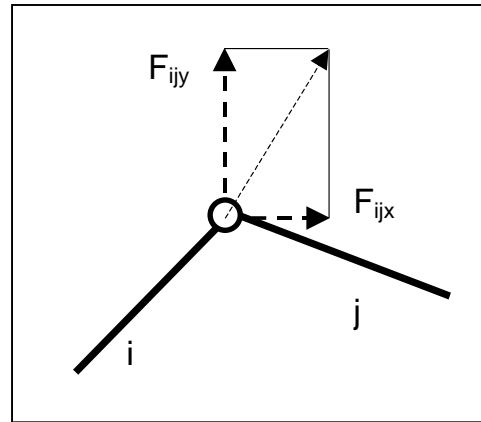


any group of links in equilibrium

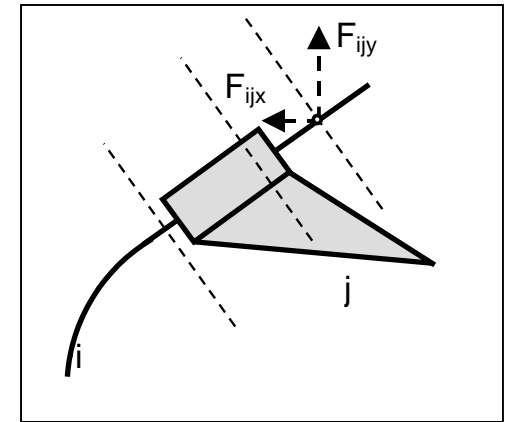
In kinetostatics unknowns are joint forces



1 unknown



2 unknowns



2 unknowns

Number of equations

3k

=

Number of unknowns

=

$2p_1 + p_2$

Examples of static. determined groups

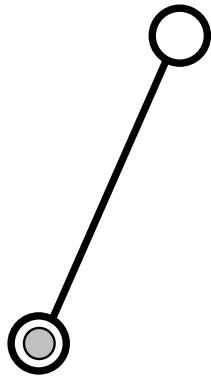
$DOF = 3(n - 1) - 2p_1 - 1p_2$ Mobility

$3k - 2p_1 - 1p_2 = 0$ Condition of static determination

k	1	2	...	4	
p₁	1	3	...	6	
p₂	1	0	...	0	
(k-p₁-p₂)	(1-1-1)	(2-3-0)		(4-6-0)	

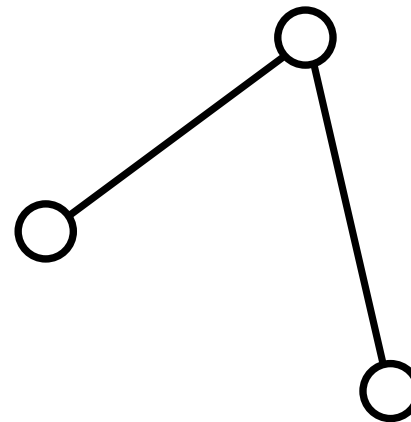
group (1-1-1)

I kl - R lub T



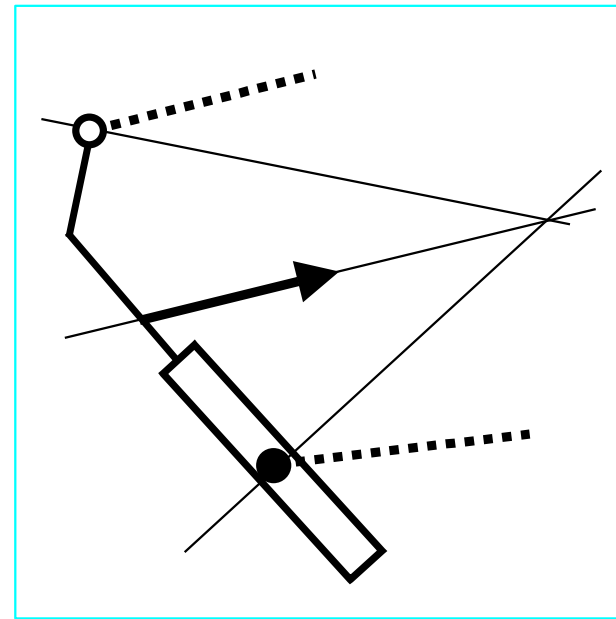
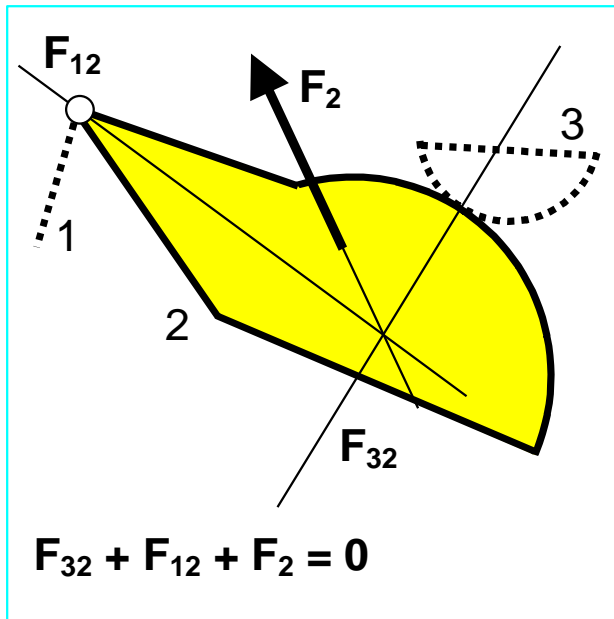
II class - K or J or Z

group (2-3-0)



II class - R or T

1-1-1 stat. determined group (one link)



2-link groups ($k-p_1-p_2 = 230$)

