Statically determined group (of links)

forces may be fully determined using the six (three) equations of 3D (2D) statics without using deflection and stiffness criteria.

kinematic system in equilibrium

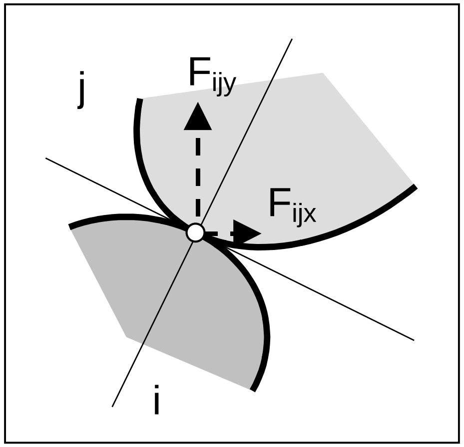
↓

every link in equilibrium

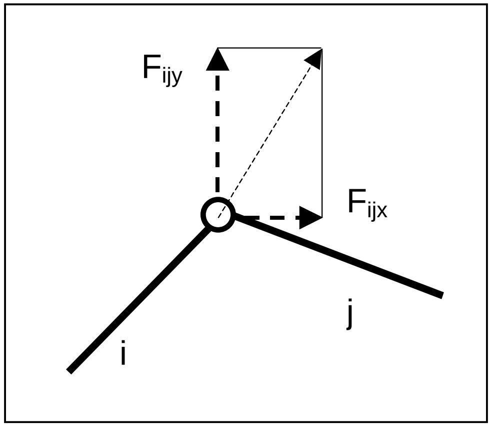
↓

any group of links in equlibrium

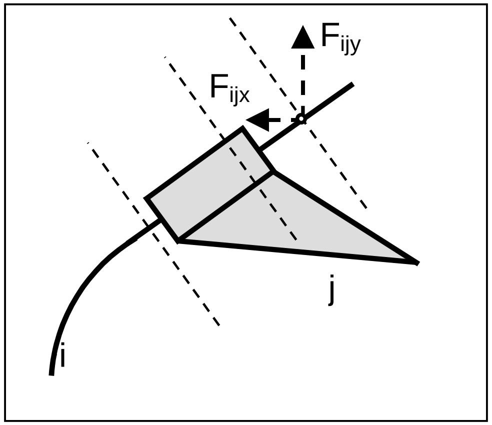
In kinetostatics unknowns are joint forces



1 unknown



2 unknowns



2 unknowns

Number of equations	=	*Number of unknowns*
3k	**=**	$\mathbf{2p_1 + p_2}$

Examples of static. determined groups

$$DOF = 3(n-1) - 2p_1 - 1p_2$$

Mobility

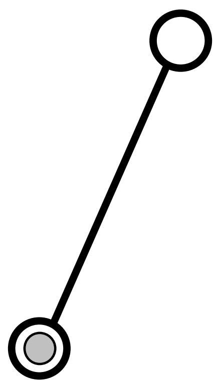
$$3k - 2p_1 - 1p_2 = 0$$

Condition of static determination

k	**1**	**2**	**…**	**4**	
p_1	**1**	**3**	**…**	**6**	
p_2	**1**	**0**	**…**	**0**	
(k-p_1–p_2)	**(1-1-1)**	**(2-3-0)**		**(460)**	

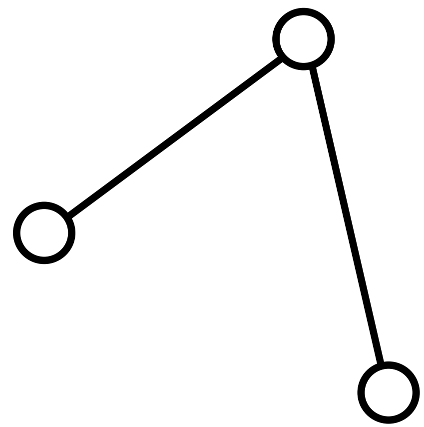
group (1-1-1)

I kl - R lub T



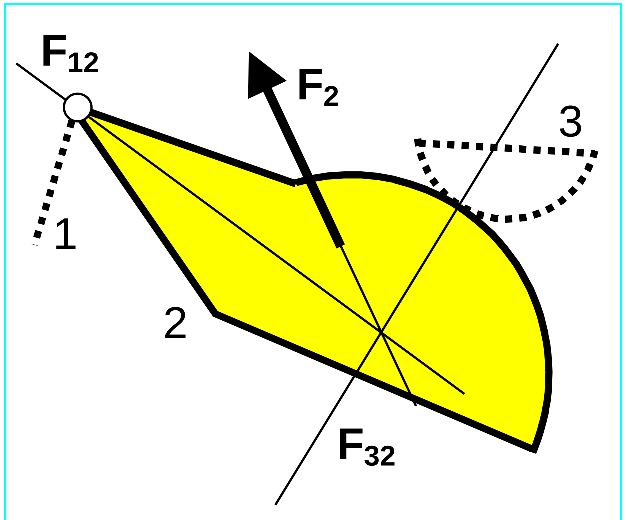
II class - K or J or Z

group (2-3-0)

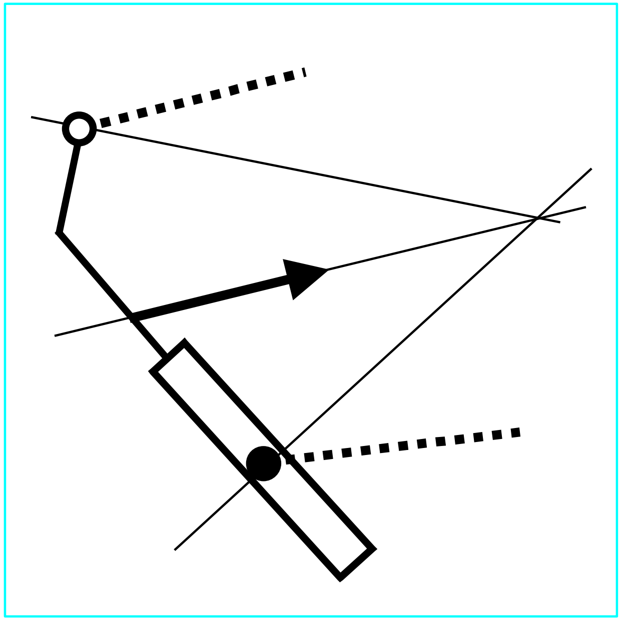


II class - R or T

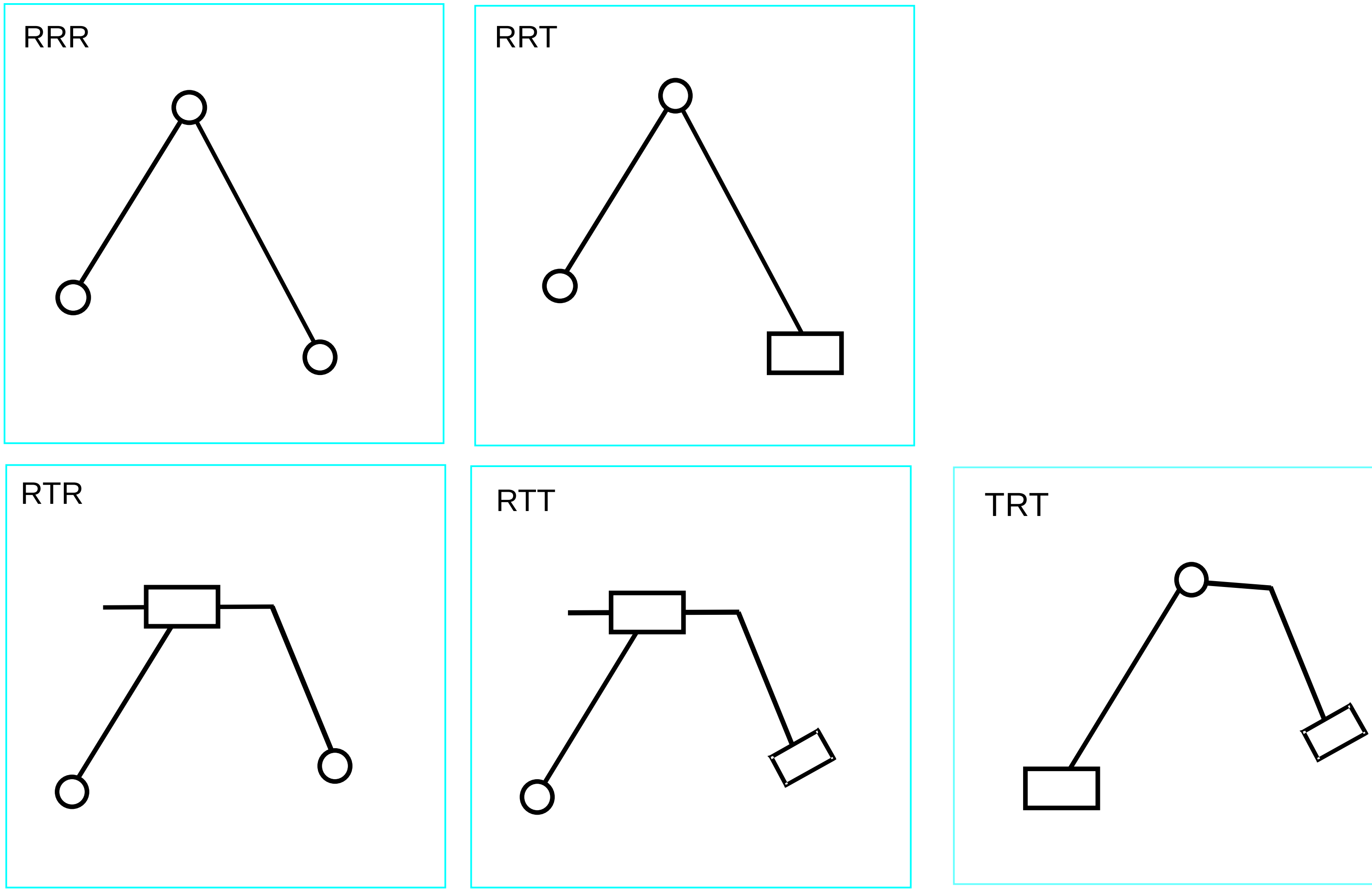
1-1-1 stat. determined group (one link)



$$F_{32} + F_{12} + F_2 = 0$$



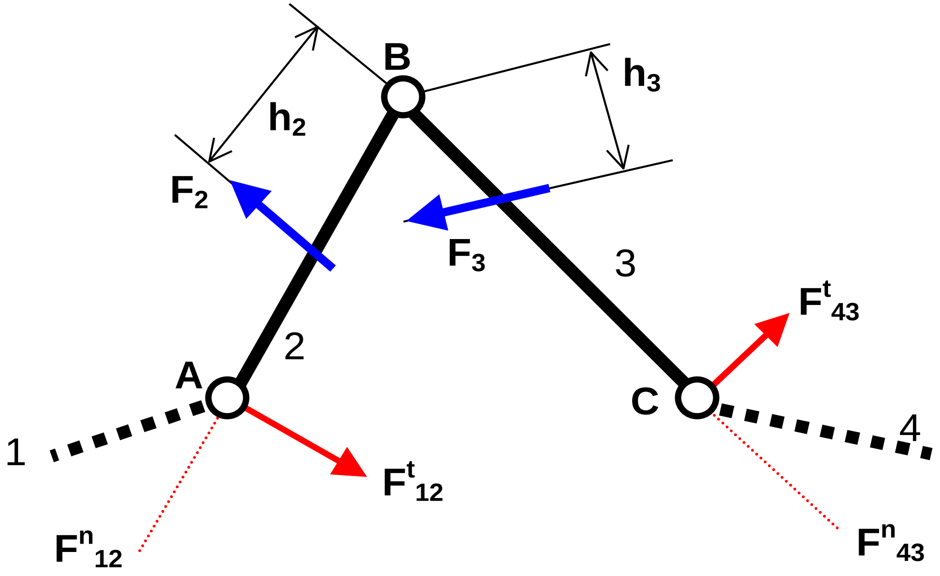
2-link groups (k-p_1-p_2 = 230)



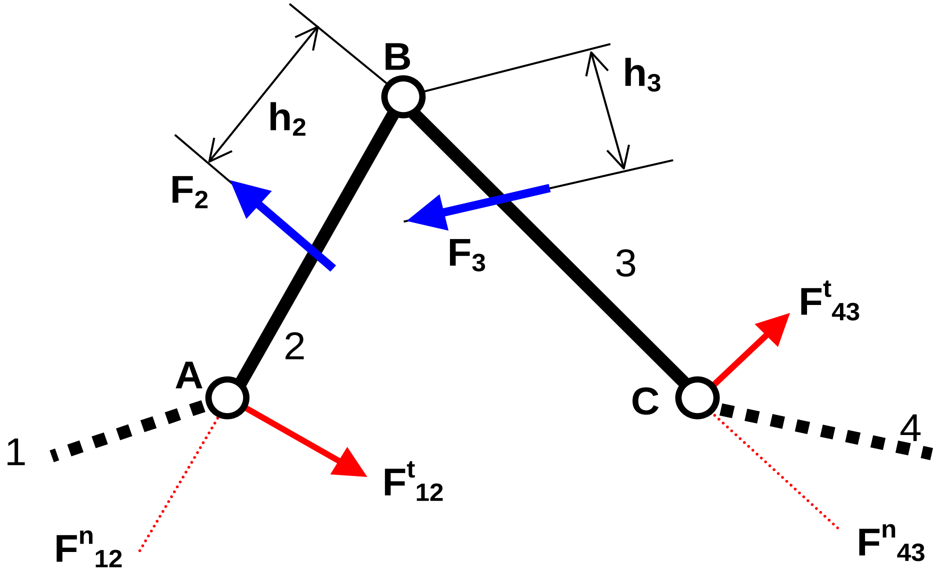
EXAMPLE 1

Group RRR

Group RRR – forces



Group RRR – forces



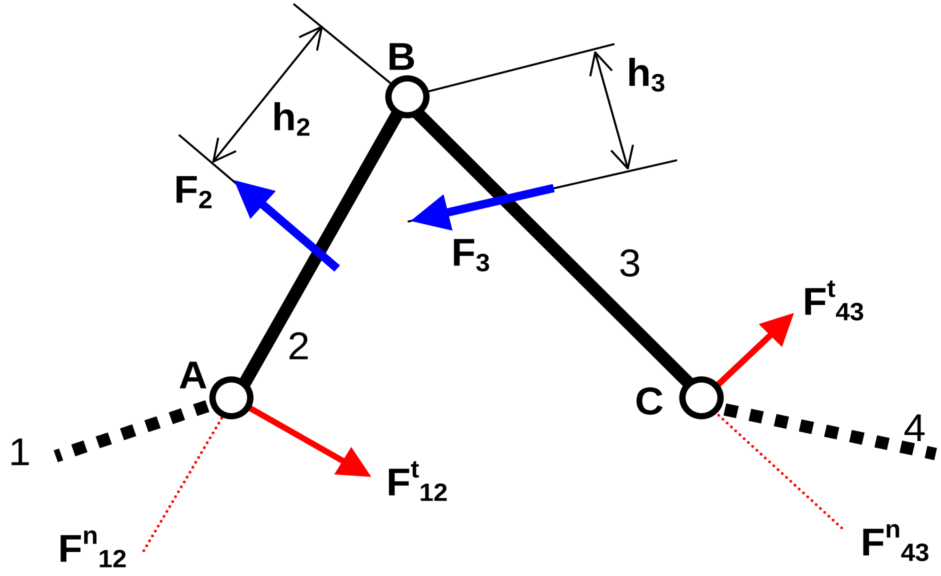
Equilibrium equation (forces)
of group (links 2 and 3):

$$\mathbf{F}_{12}^{t} + \mathbf{F}_{12}^{n} + \mathbf{F}_{2} + \mathbf{F}_{3} + \mathbf{F}_{43}^{t} + \mathbf{F}_{43}^{n} = 0$$

$$F_{12}^{t} = ?$$

$$F_{43}^{t} = ?$$

Group RRR – forces



Equilibrium equation (torques) of link 2 relative to point B:

$$\sum M_2^B = 0 \rightarrow -F_2 h_2 + F_{12}^t l_{AB} = 0$$

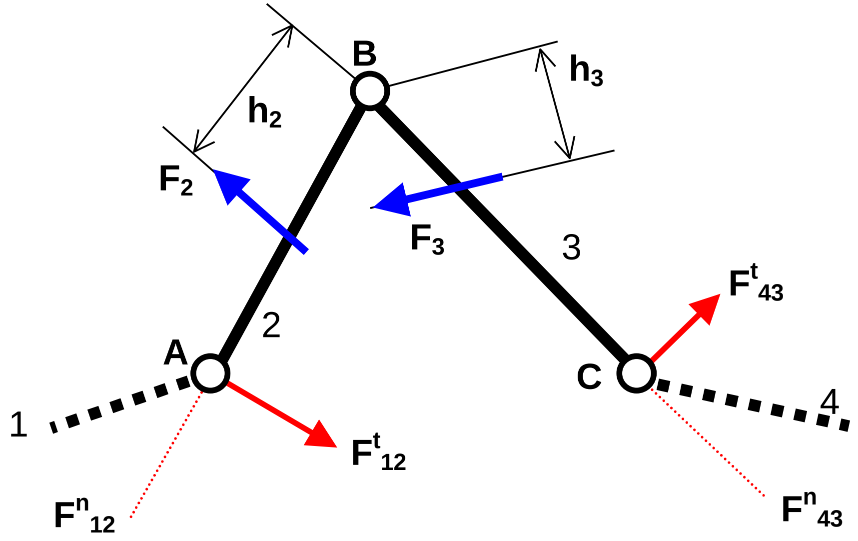
$$\rightarrow F_{12}^t = \frac{F_2 h_2}{l_{AB}}$$

Equilibrium equation (torques) of link 3 relative to point B:

$$\sum M_3^B = 0 \rightarrow -F_3 h_3 + F_{43}^t l_{BC} = 0$$

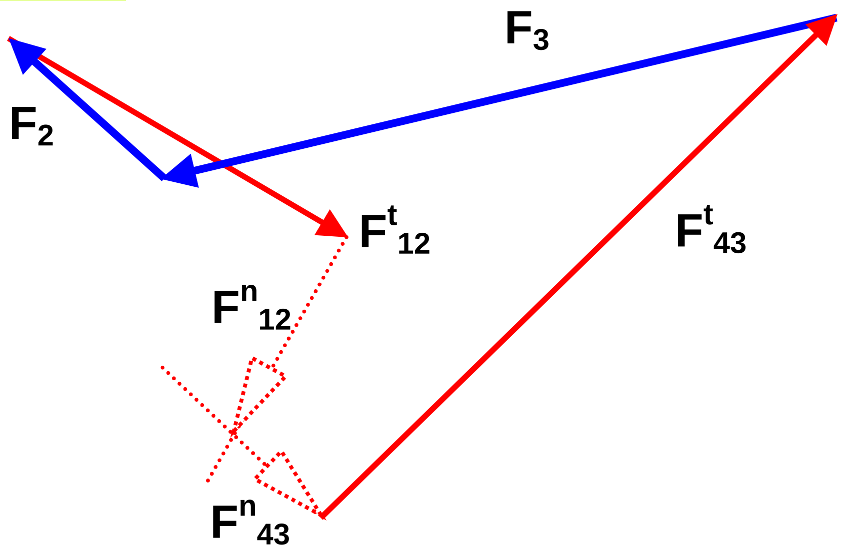
$$\rightarrow F_{43}^t = \frac{F_3 h_3}{l_{BC}}$$

Group RRR – forces



Equilibrium equation (forces)
of group (links 2 and 3):

$$\mathbf{F}_{12}^{t} + \mathbf{F}_{12}^{n} + \mathbf{F}_{2} + \mathbf{F}_{3} + \mathbf{F}_{43}^{t} + \mathbf{F}_{43}^{n} = 0$$



Equilibrium equation (forces)
of link 2:

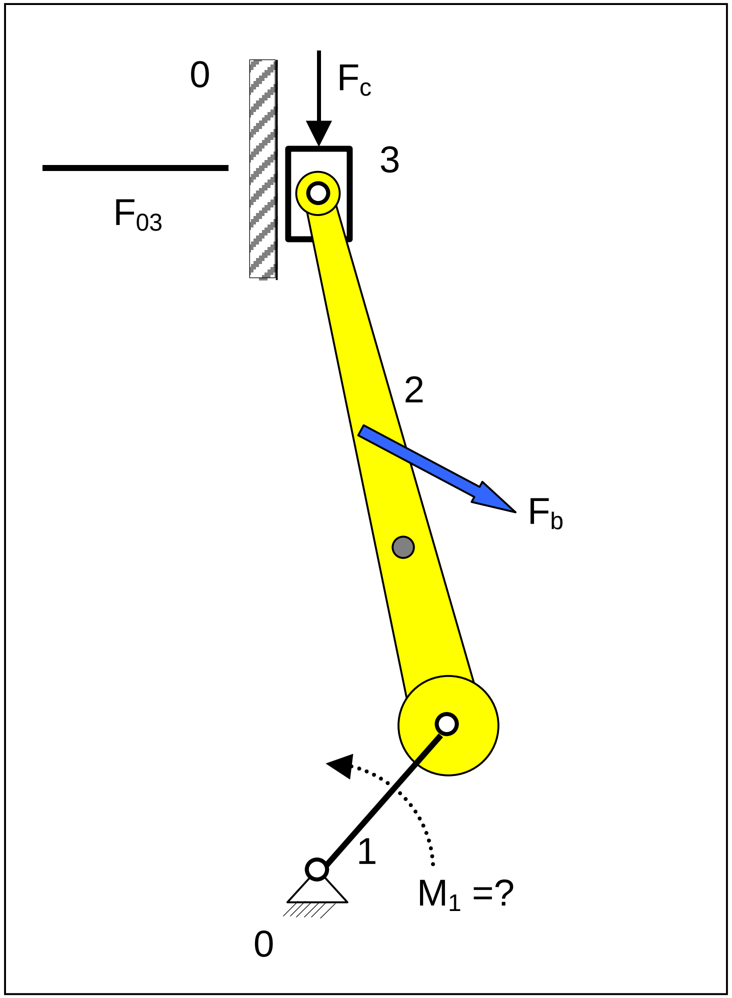
$$\mathbf{F}_{12} + \mathbf{F}_{2} + \mathbf{F}_{32} = 0$$

Equilibrium equation (forces)
of link 3:

$$\mathbf{F}_{23} + \mathbf{F}_{3} + \mathbf{F}_{43} = 0$$

EXAMPLE 2

Group RRT



Equilibrium equation (forces) of link 3:

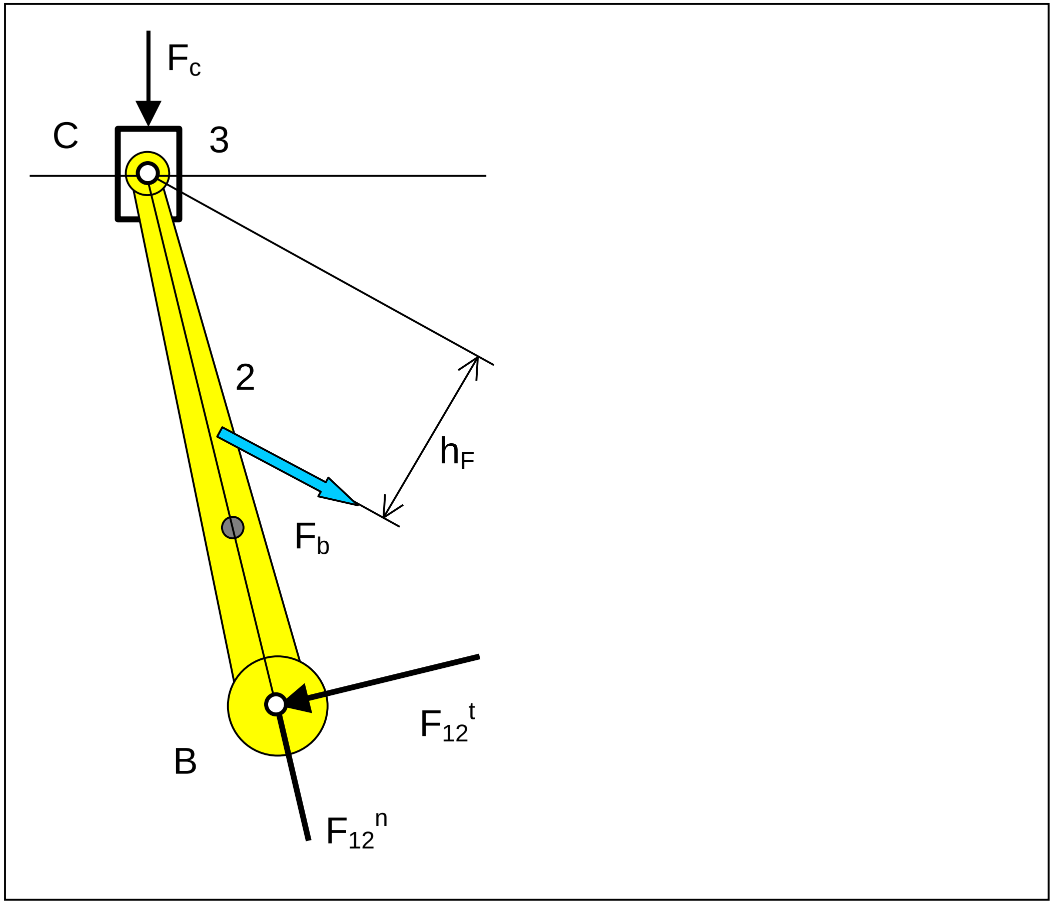
$$\mathbf{F_C} + \mathbf{F_{03}} + \mathbf{F_{23}} = 0$$

Equilibrium equation (forces) of link 2:

$$\mathbf{F_{32}} + \mathbf{F_b} + \mathbf{F_{12}} = 0$$

Links 2-3

group RRT



Equilibrium equation (forces) of group (links 2 and 3):

$$F_c + F_{03} + F_b + F_{12}^t + F_{12}^n = 0$$

because

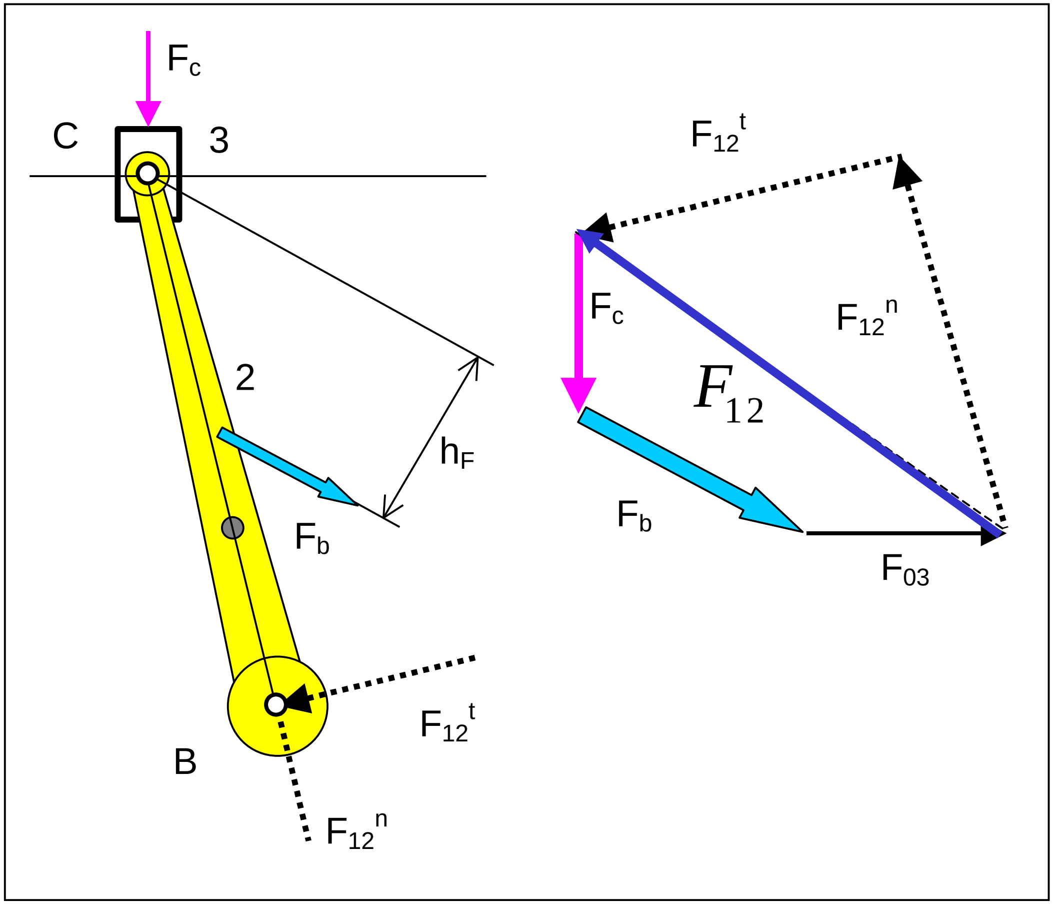
$$\mathbf{F}_{32} + \mathbf{F}_{23} = 0$$

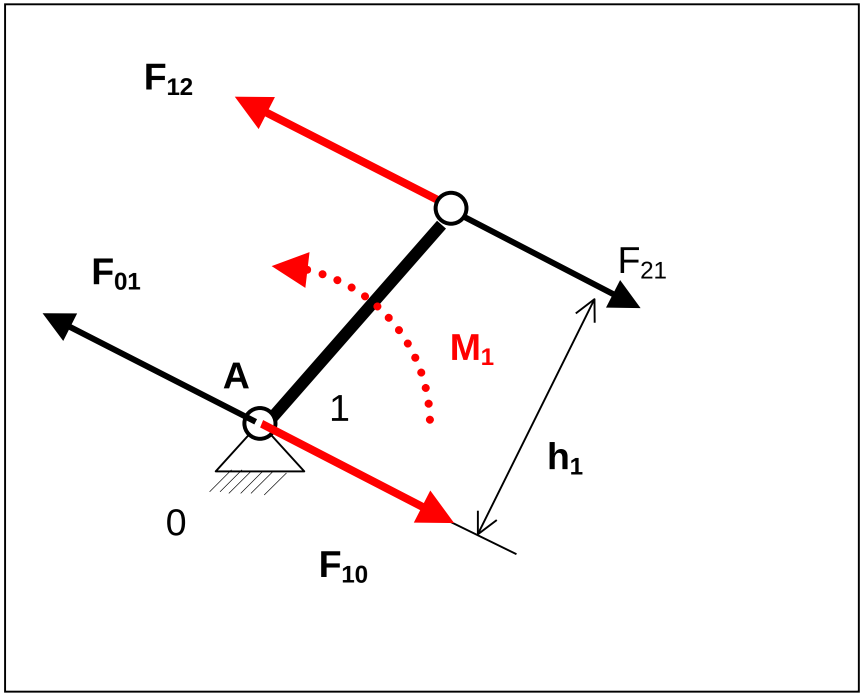
Equilibrium equation (torques) of link 2 relative to point C:

$$\sum M_2^C = 0 \rightarrow F_b h_F - F_{12}^t l_{BC} = 0 \rightarrow F_{12}^t = \frac{F_b h_F}{l_{BC}}$$

Equilibrium equation (forces) of group (links 2 and 3):

$$\mathbf{F_c} + \mathbf{F_{03}} + \mathbf{F_b} + \mathbf{F_{12}^t} + \mathbf{F_{12}^n} = 0$$





Equilibrium equation (forces)
of link 1:

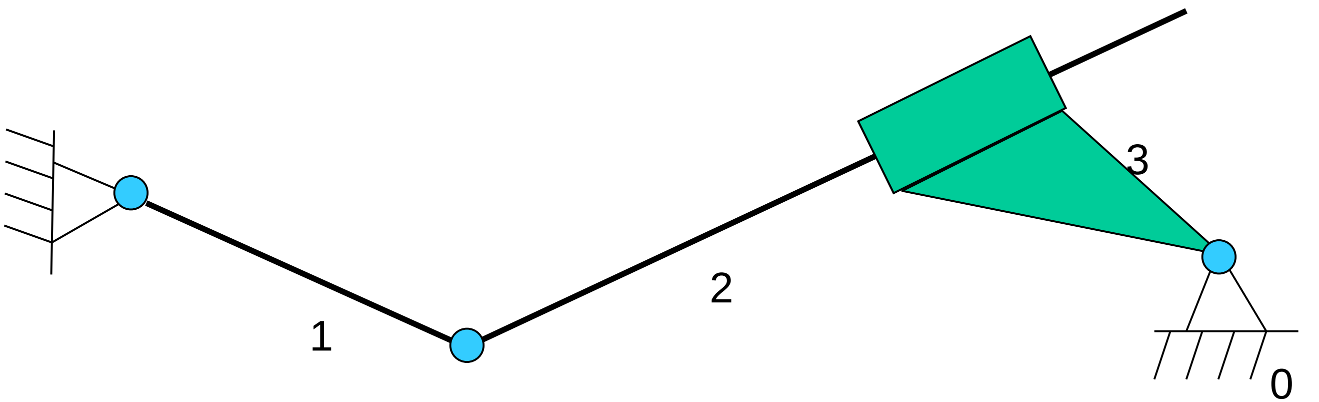
$$\mathbf{F}_{21} + \mathbf{F}_{01} = 0$$

Equilibrium equation (torques)
of link 1 relative to point A:

$$\sum M_1^A = 0 \rightarrow M_1 - F_{21}h_1 = 0$$

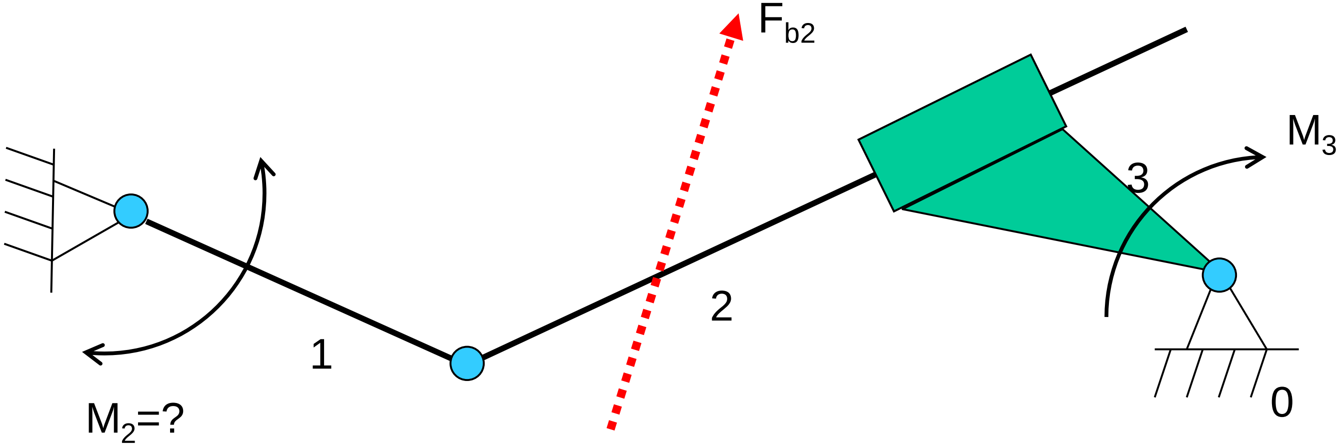
EXAMPLE 3

Group RTR

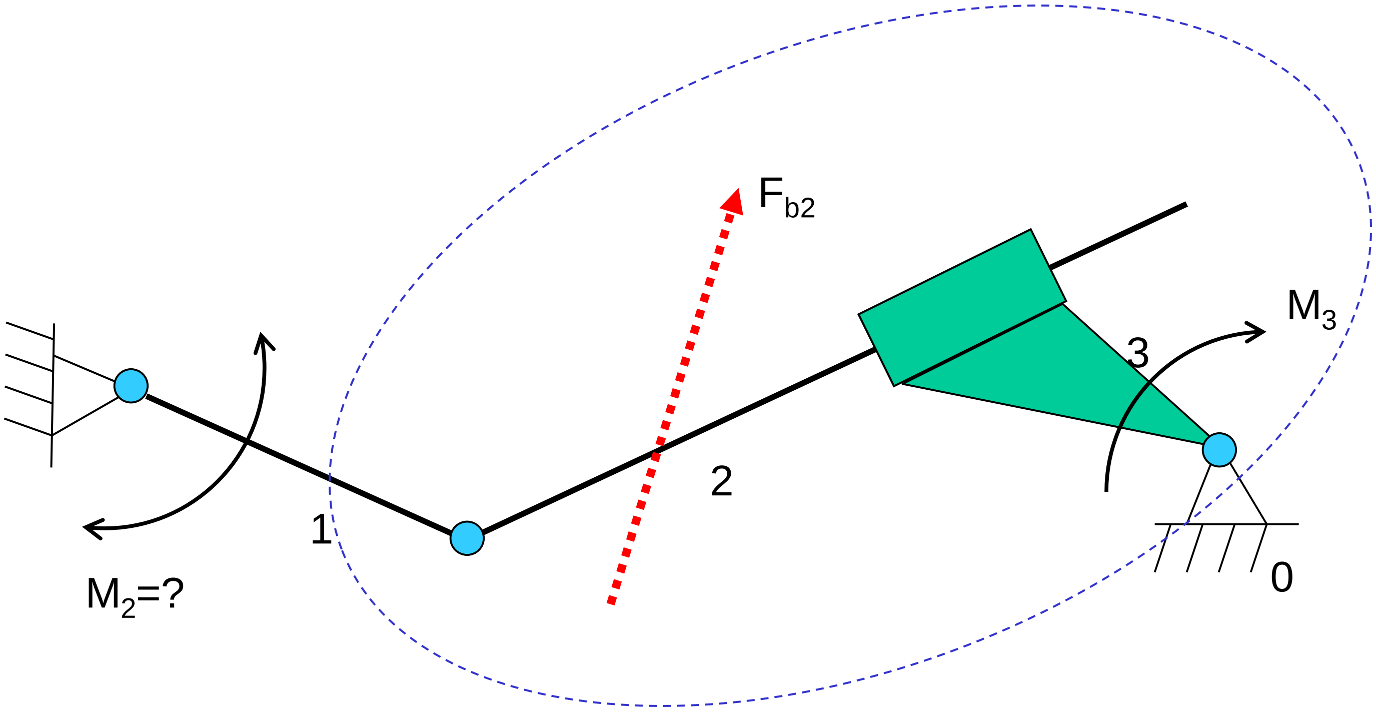

3
2
1
0

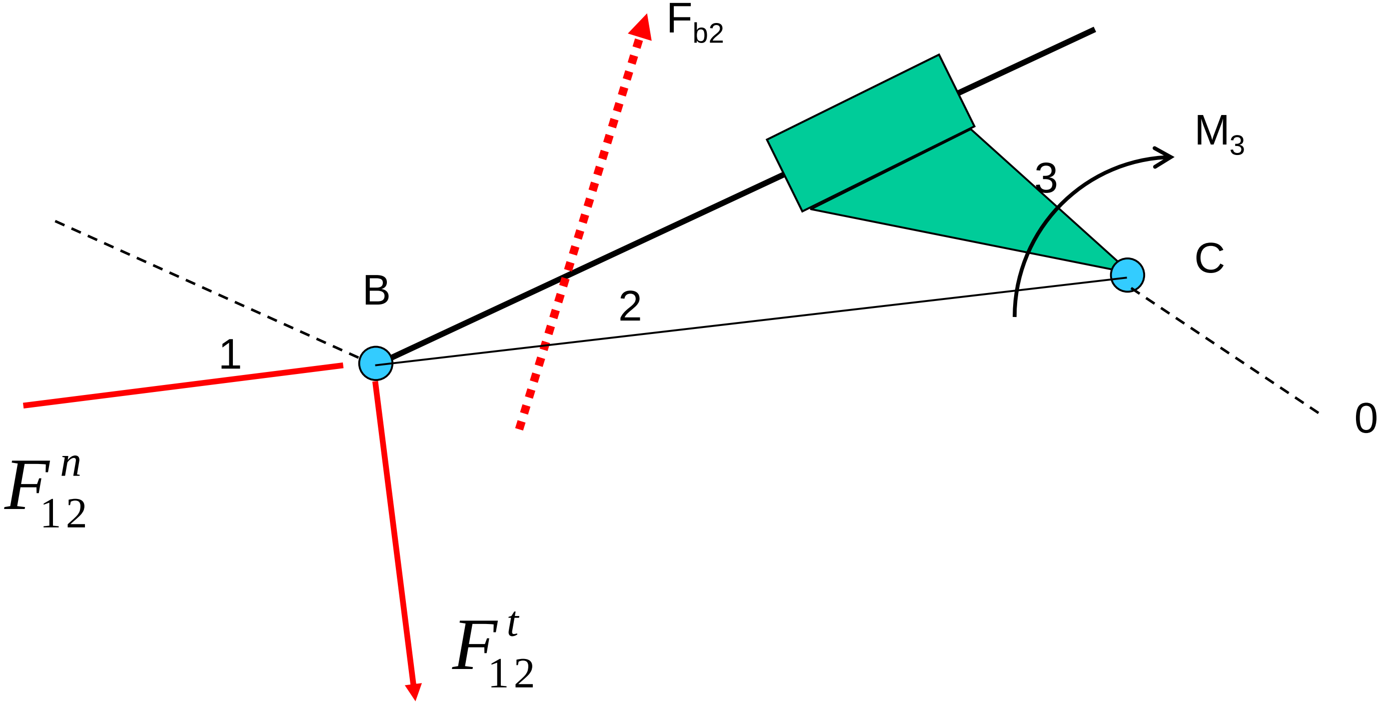
Data: $\mathbf{M_3}$, $\mathbf{F_{b2}}$

Unknown: $\mathbf{M_2}$



stat determined group (2-3)



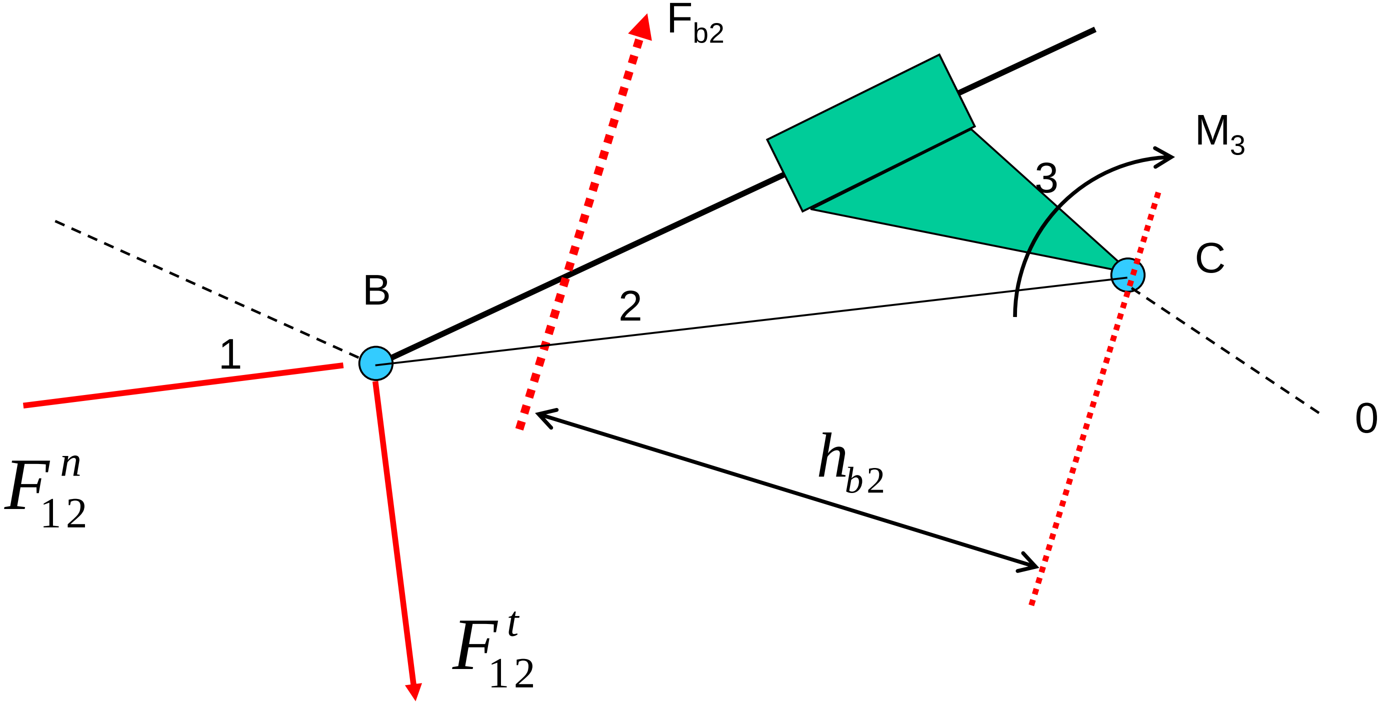


F_{b2}
M_3
3
C
B
2
1
0
F_{12}^n
F_{12}^t

Equilibrium equation (torques)
of group (links 2 and 3) relative to point C:

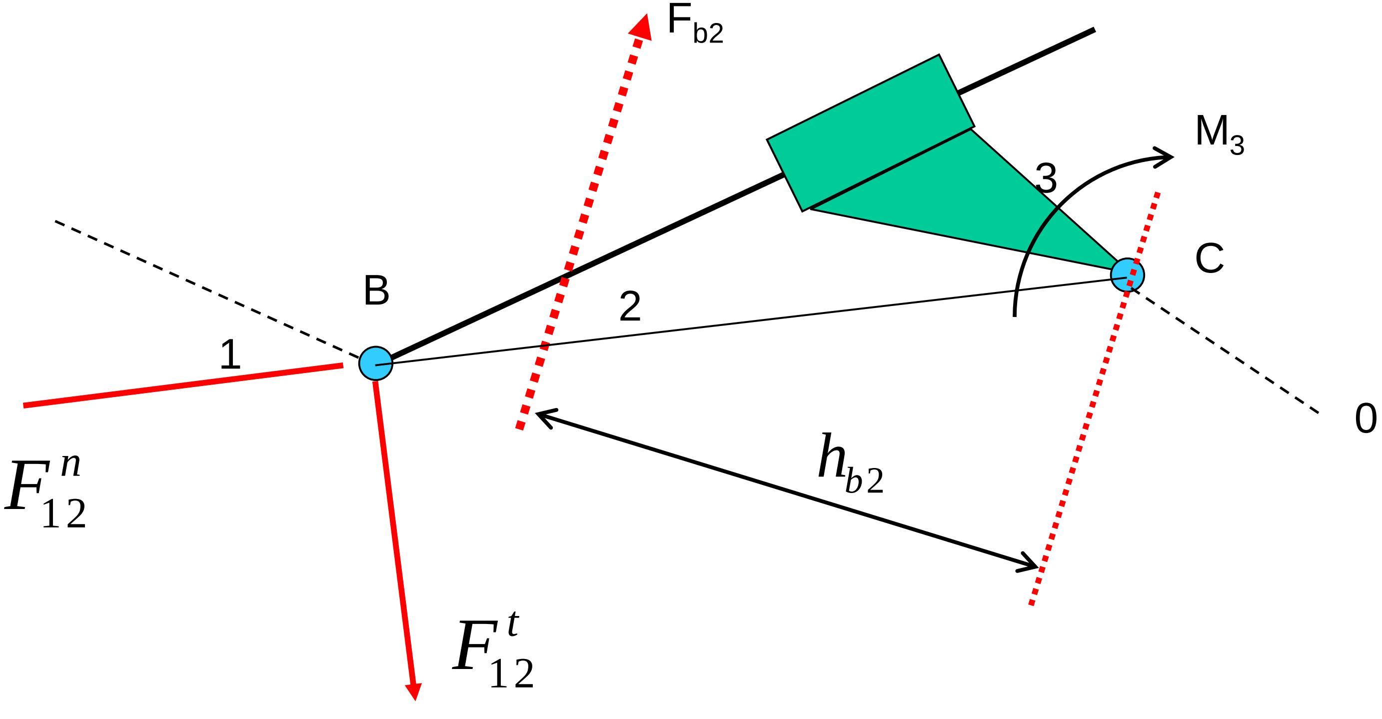
$$F_{12}^{t} h_{CB} - F_{b2} h_{b2} - M_3 = 0$$

$$F_{12}^{t} = \frac{F_{b2} h_{b2} + M_3}{h_{CB}}$$

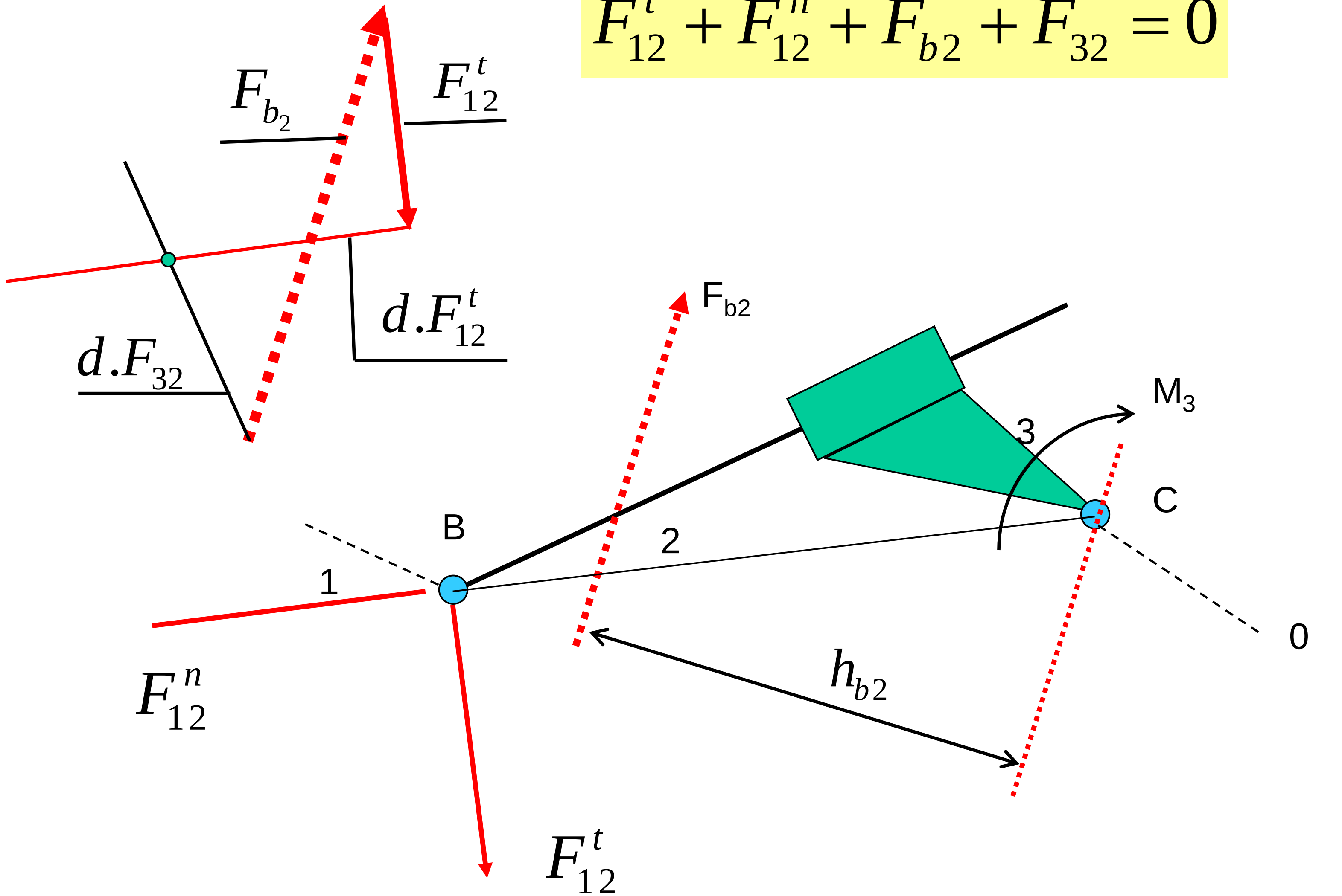


Equilibrium equation (forces) of link 2:

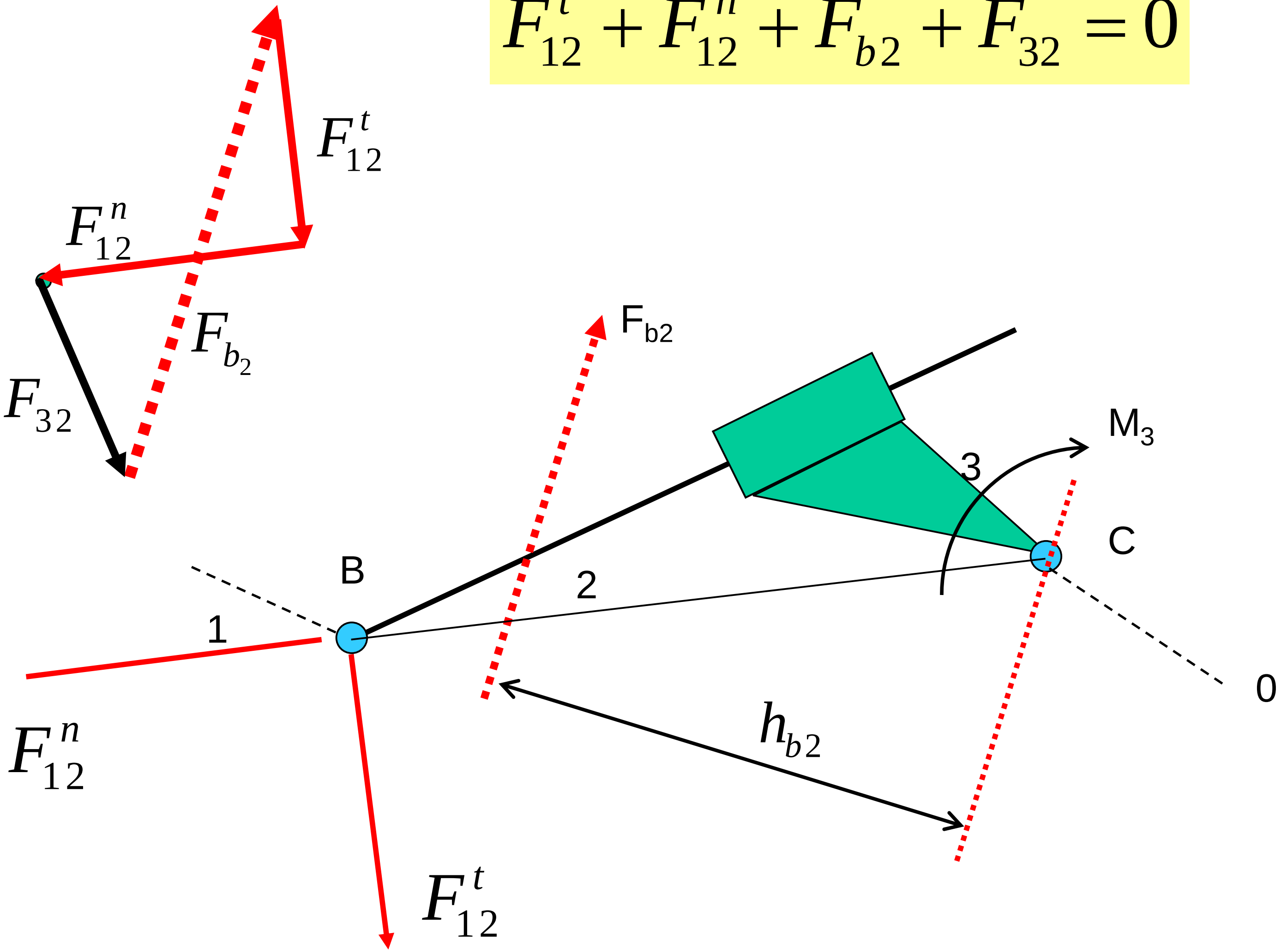
$$F_{12}^{t} + F_{12}^{n} + F_{b2} + F_{32} = 0$$



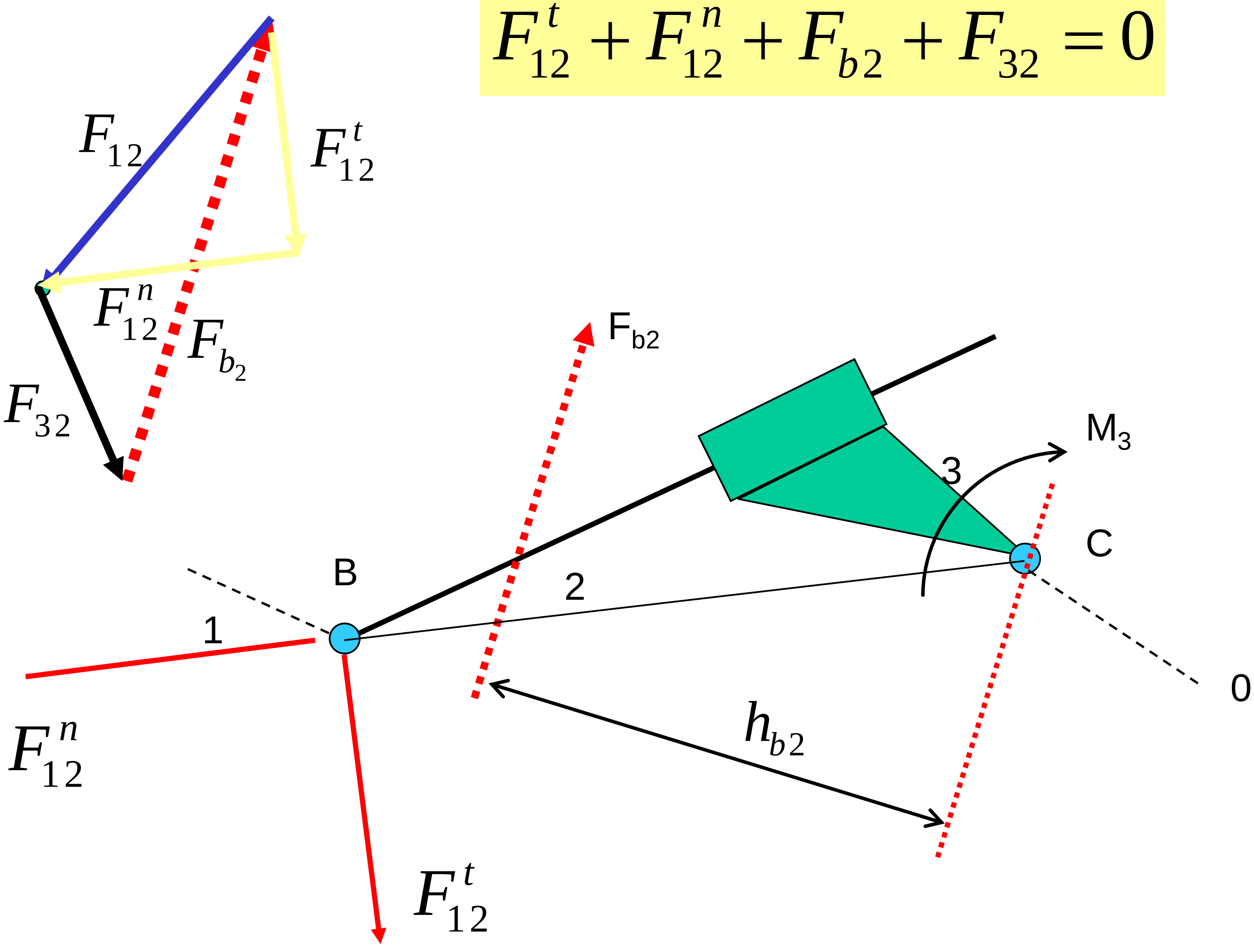
$$F_{12}^{t} + F_{12}^{n} + F_{b2} + F_{32} = 0$$



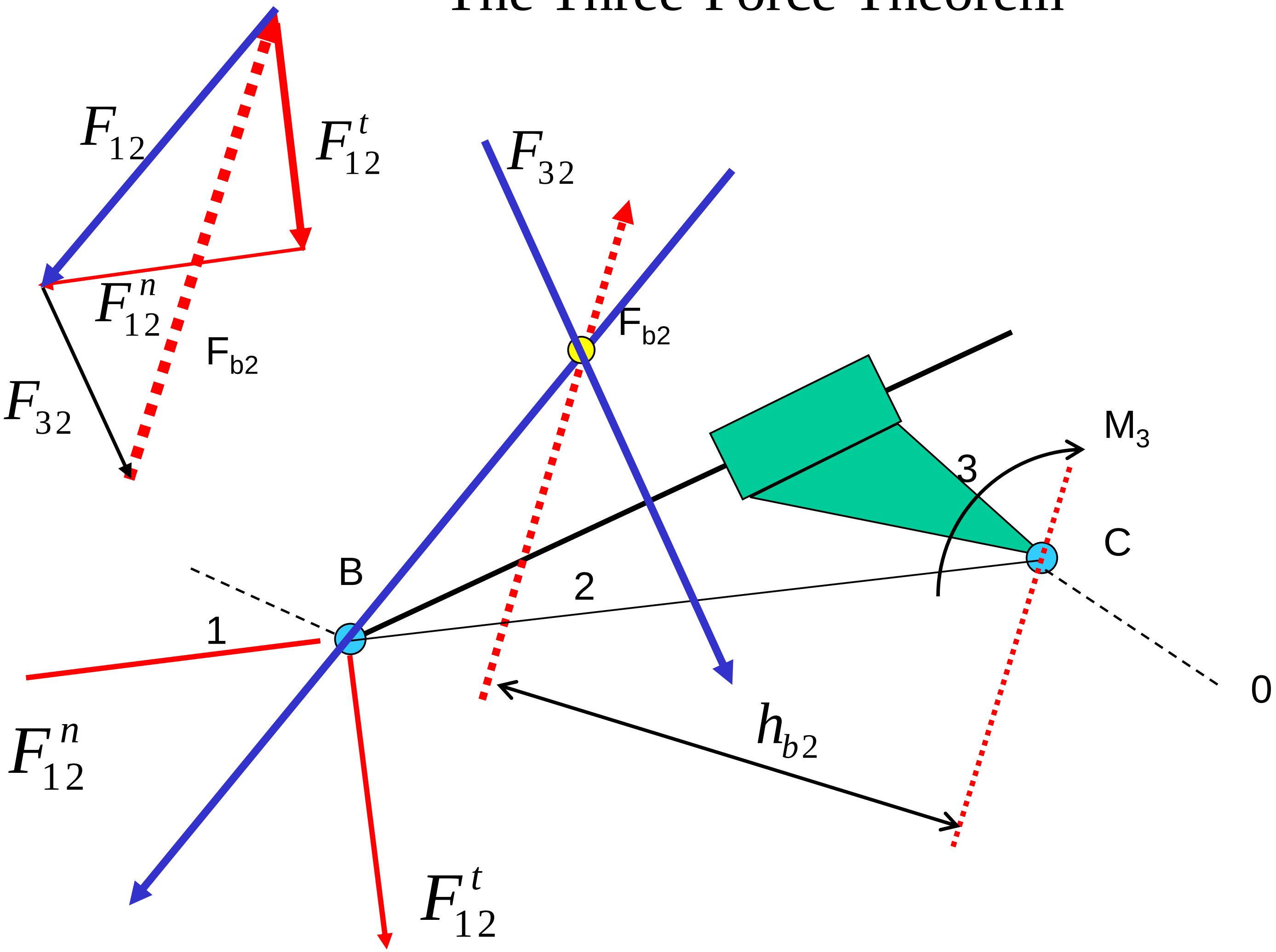
$$F_{12}^{t} + F_{12}^{n} + F_{b2} + F_{32} = 0$$

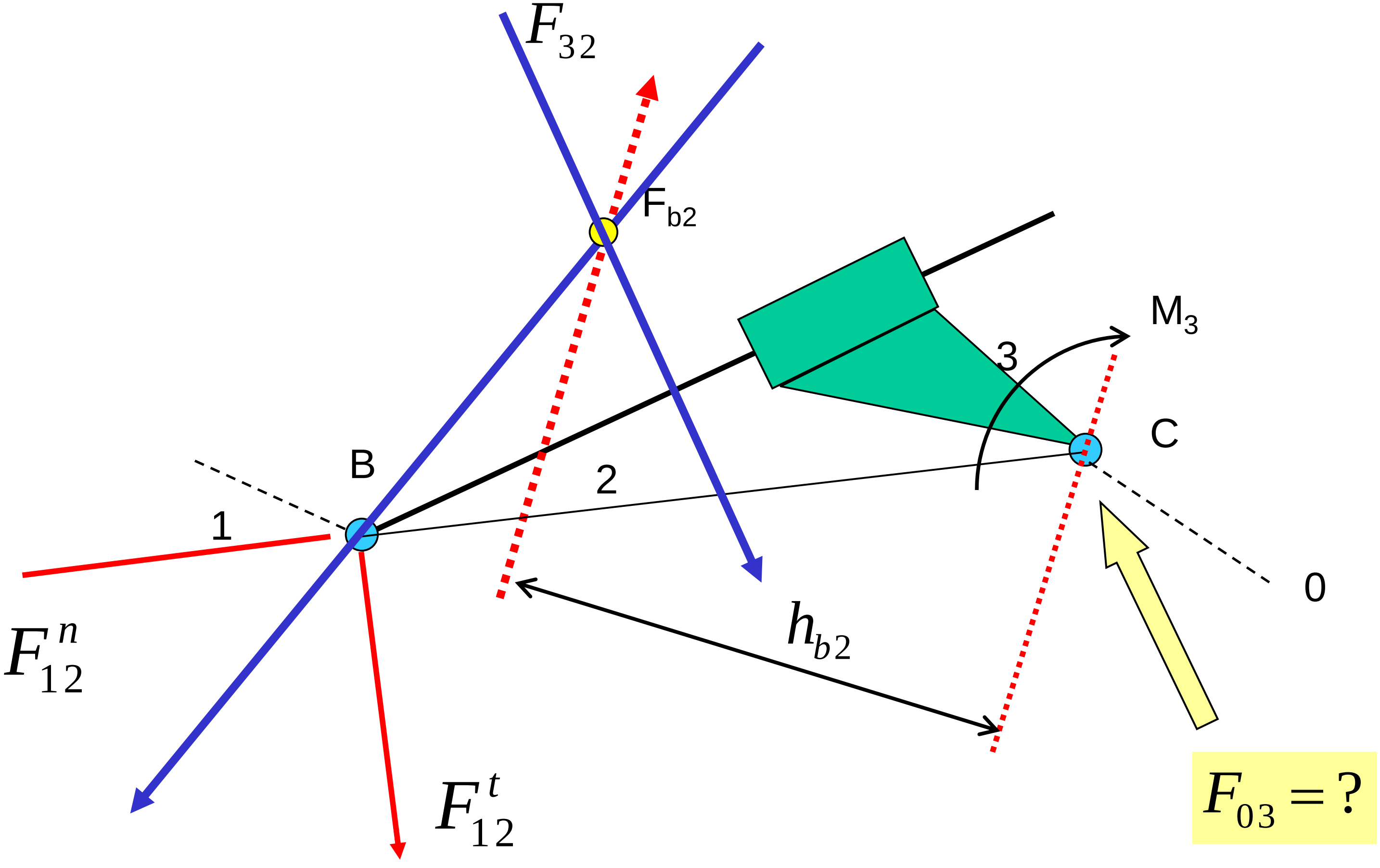


$$F_{12}^{t} + F_{12}^{n} + F_{b2} + F_{32} = 0$$



The Three-Force Theorem

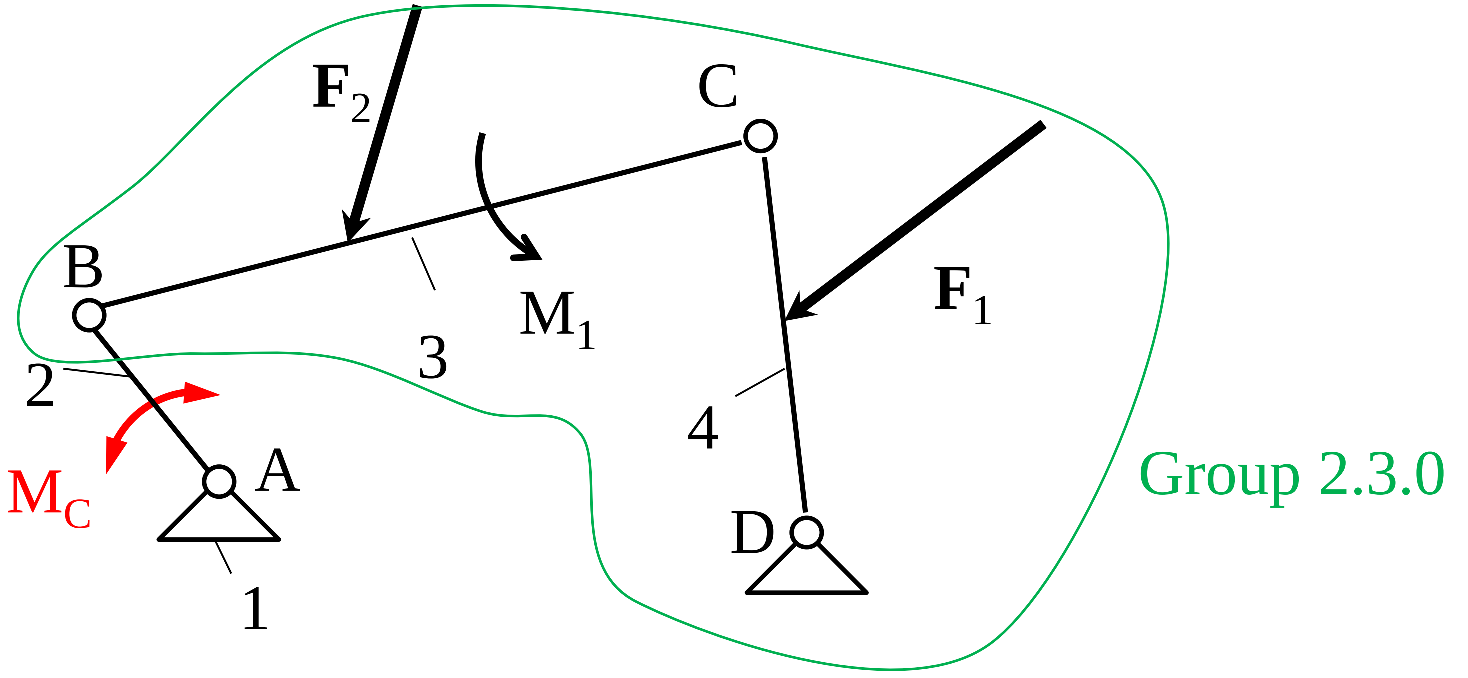




F_{32}
F_{b2}
M_3
3
C
B
2
1
0
h_{b2}
F_{12}^{n}
F_{12}^{t}
$F_{03} = ?$

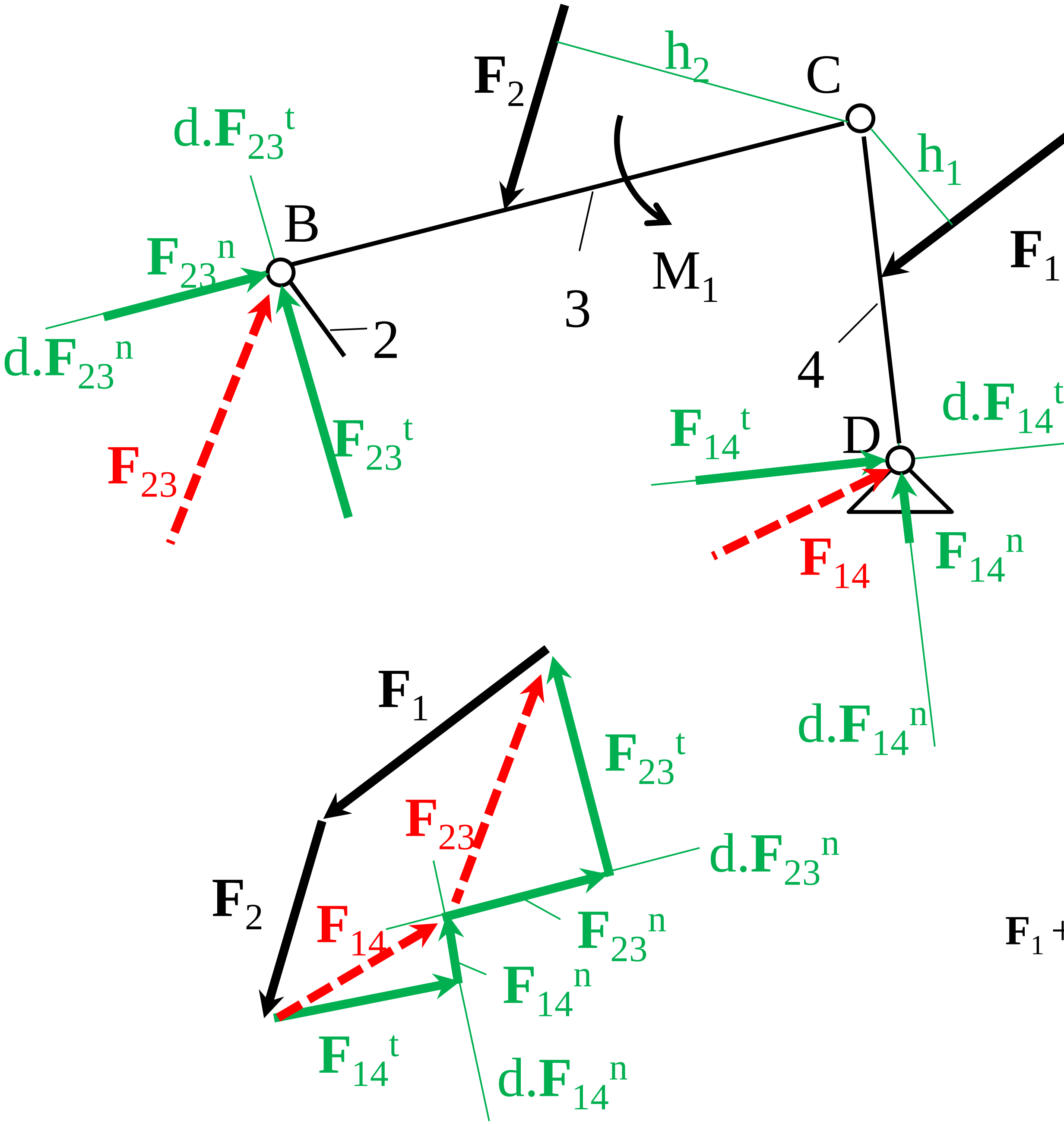
EXAMPLE 4

Group RRR



Data: $\mathbf{F_1}$, $\mathbf{F_2}$, $\mathbf{M_1}$

Unknown: $\mathbf{M_c}$, $\mathbf{F_{12}}$, $\mathbf{F_{23}}$, $\mathbf{F_{34}}$, $\mathbf{F_{14}}$



Equilibrium equation (torques) of link 3 relative to point C:

$$\mathbf{M}_1 + \mathbf{F}_2\, h_2 \; - \; \mathbf{F}_{23}^{t}\, BC = 0$$

$$\mathbf{F}_{23}^{t} = (\mathbf{M}_2 + \mathbf{F}_2\, h_2)/BC$$

Equilibrium equation (torques) of link 4 relative to point C:

$$-\mathbf{F}_1\, h_1 + \mathbf{F}_{14}^{t}\, DC = 0$$

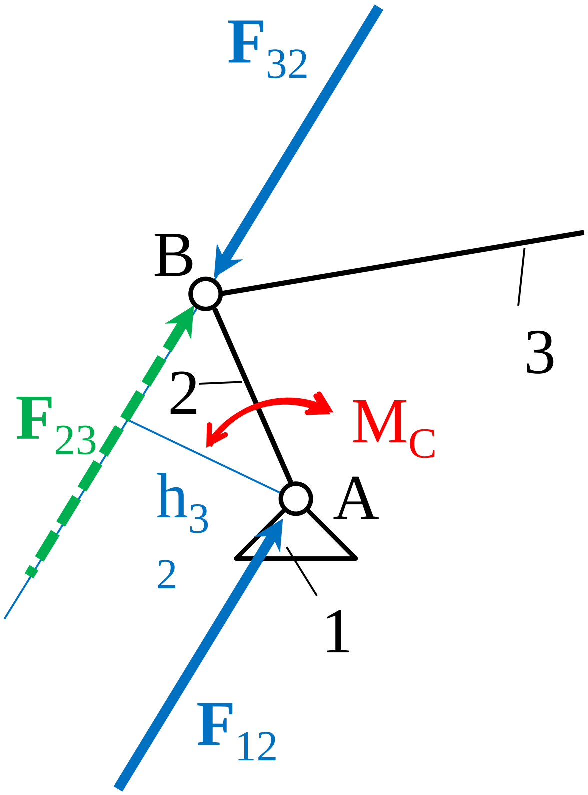
$$\mathbf{F}_{14}^{t} = \mathbf{F}_1\, h_1/DC$$

Equilibrium equation (forces) of group (links 3 and 4):

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_{23}^{t} + \mathbf{F}_{23}^{n} + \mathbf{F}_{14}^{t} + \mathbf{F}_{14}^{n} = 0$$

$$\mathbf{F}_{23} = \mathbf{F}_{23}^{t} + \mathbf{F}_{23}^{n}$$

$$\mathbf{F}_{14} = \mathbf{F}_{14}^{t} + \mathbf{F}_{14}^{n}$$



$$\mathbf{F}_{32} = -\,\mathbf{F}_{23}$$

Equilibrium equation (torques) of link 2 relative to point A:

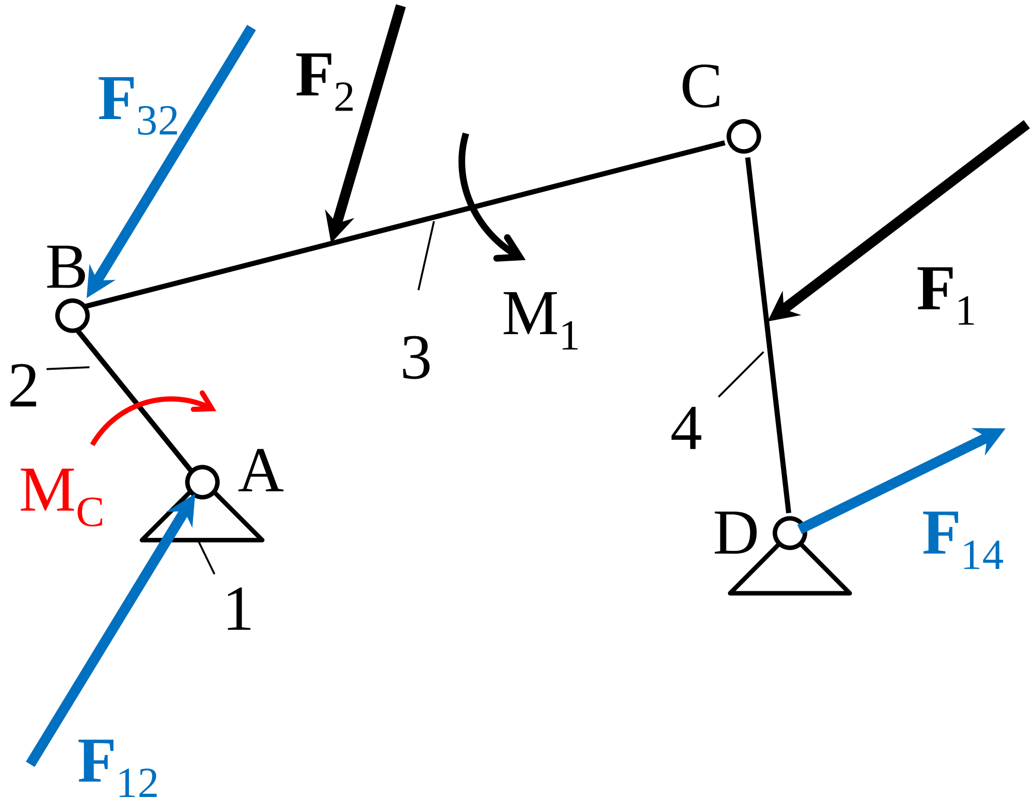
$$\mathbf{F}_{32}\, h_{32} - \mathbf{M}_C = 0$$

$$\mathbf{M}_C = \mathbf{F}_{32}\, h_{32}$$

Equilibrium equation (forces) of link 2:

$$\mathbf{F}_{32} + \mathbf{F}_{12} = 0$$

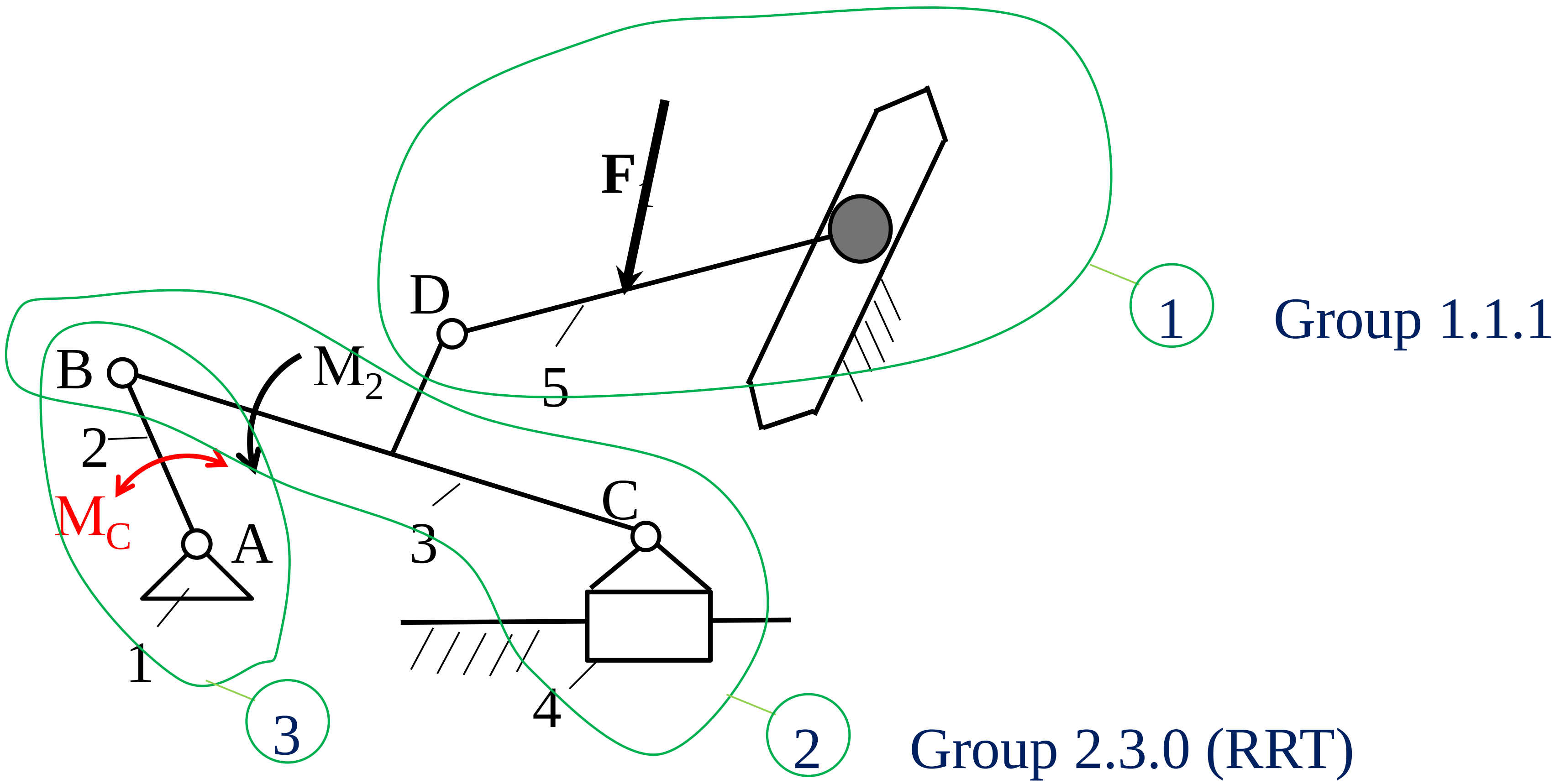
$$\mathbf{F}_{12} = -\mathbf{F}_{32}$$



F32
F2
C
B
F1
M1
3
2
4
MC
A
D
F14
1
F12

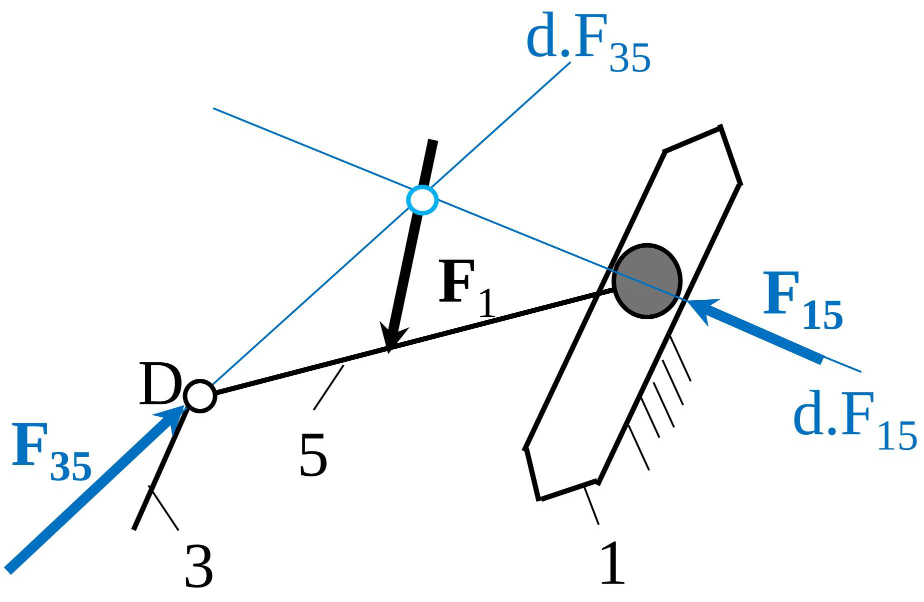
EXAMPLE 5

Group 1.1.1 + RRT



Data: F_1 , M_2

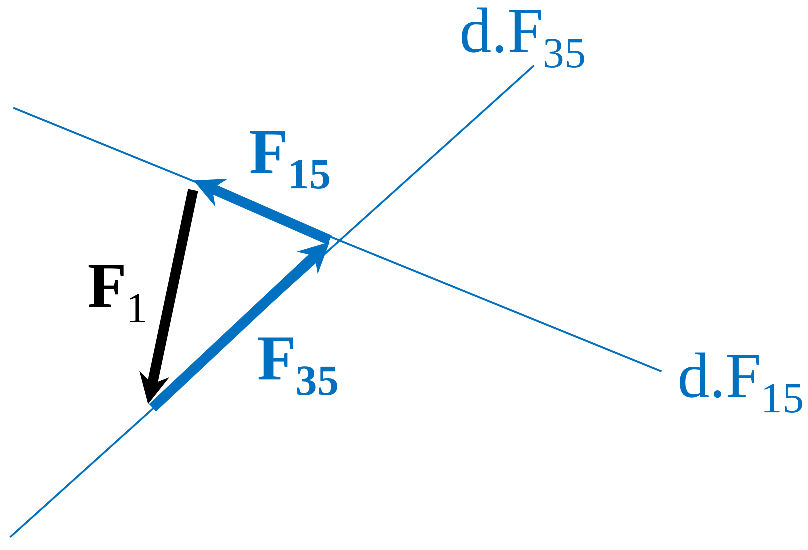
Unknown : M_c , F_{12}, F_{23}, F_{34}, F_{14}, F_{35}, F_{15}

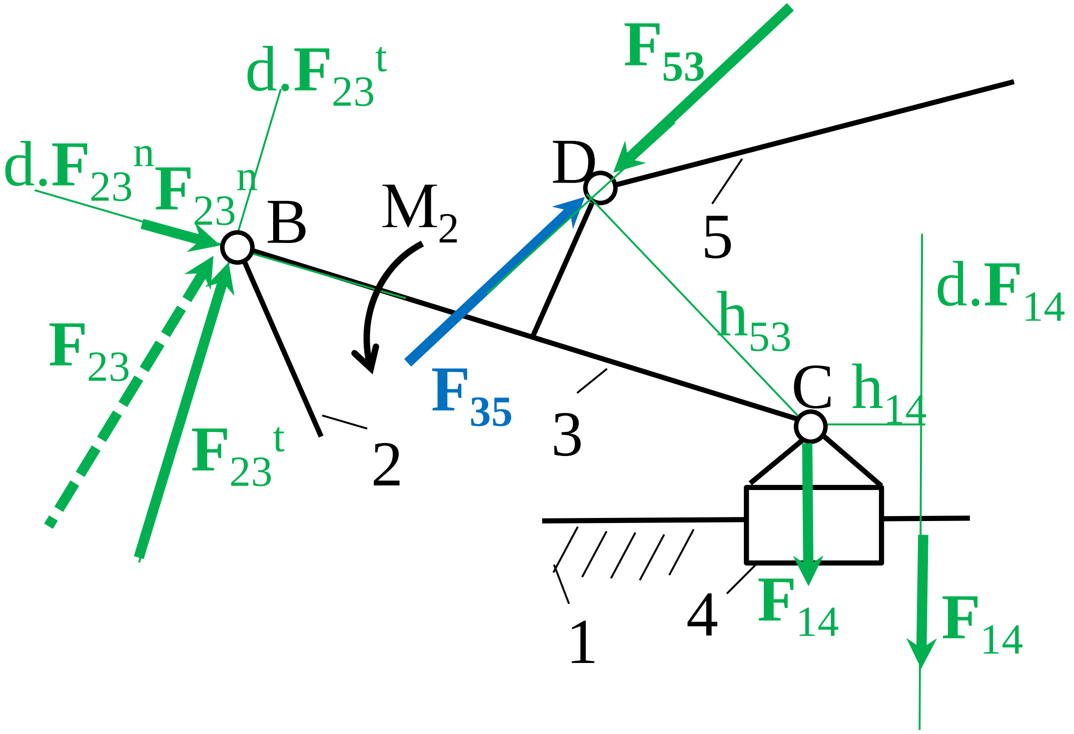


Equilibrium equation (forces) of link 5:

$$\mathbf{F}_1 + \mathbf{F}_{15} + \mathbf{F}_{35} = 0$$

The Three-Force Theorem





$$\mathbf{F}_{53} = - \mathbf{F}_{35}$$

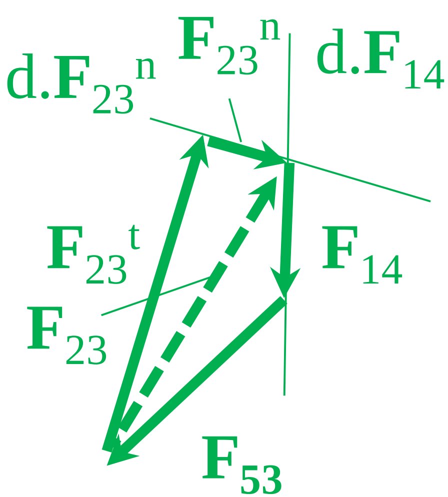
Equilibrium equation (torques) of link 3 relative to point C:

$$\mathbf{M}_2 + \mathbf{F}_{53}\, h_{53} \quad - \quad \mathbf{F}_{23}^{t}\, BC = 0$$

$$\mathbf{F}_{23}^{t} = (\mathbf{M}_2 + \mathbf{F}_{53}\, h_{53})/BC$$

Equilibrium equation (forces) of group (links 3 and 4):

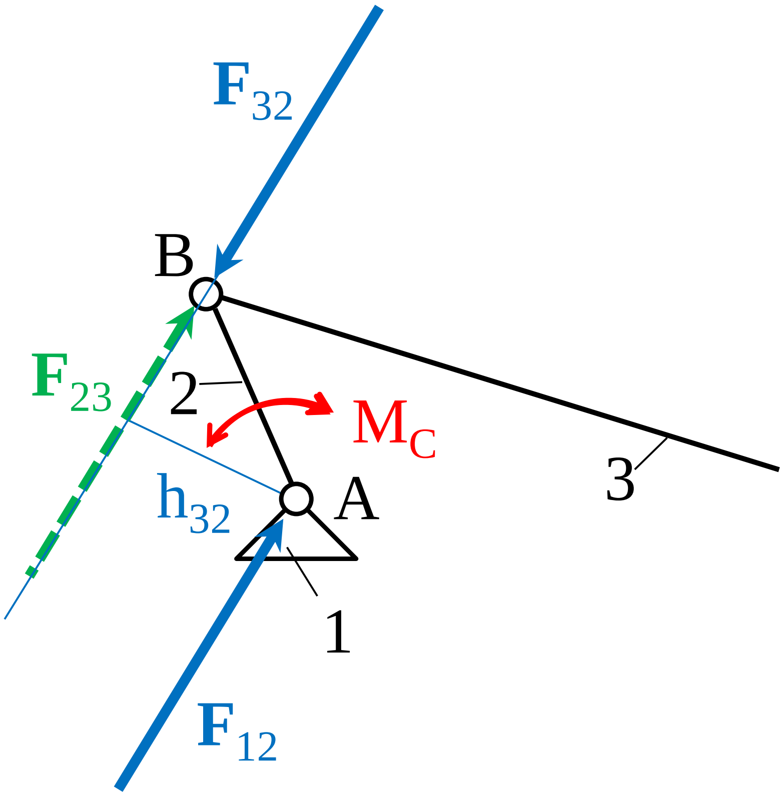
$$\mathbf{F}_{53} + \mathbf{F}_{23}^{t} + \mathbf{F}_{23}^{n} + \mathbf{F}_{14} = 0$$



Equilibrium equation (torques) of link 4 relative to point C:

$$-\mathbf{F}_{14}\, h_{14} = 0 \quad , \quad h_{14} = 0$$

$$\mathbf{F}_{23} = \mathbf{F}_{23}^{t} + \mathbf{F}_{23}^{n}$$



$$\mathbf{F}_{32} = - \mathbf{F}_{23}$$

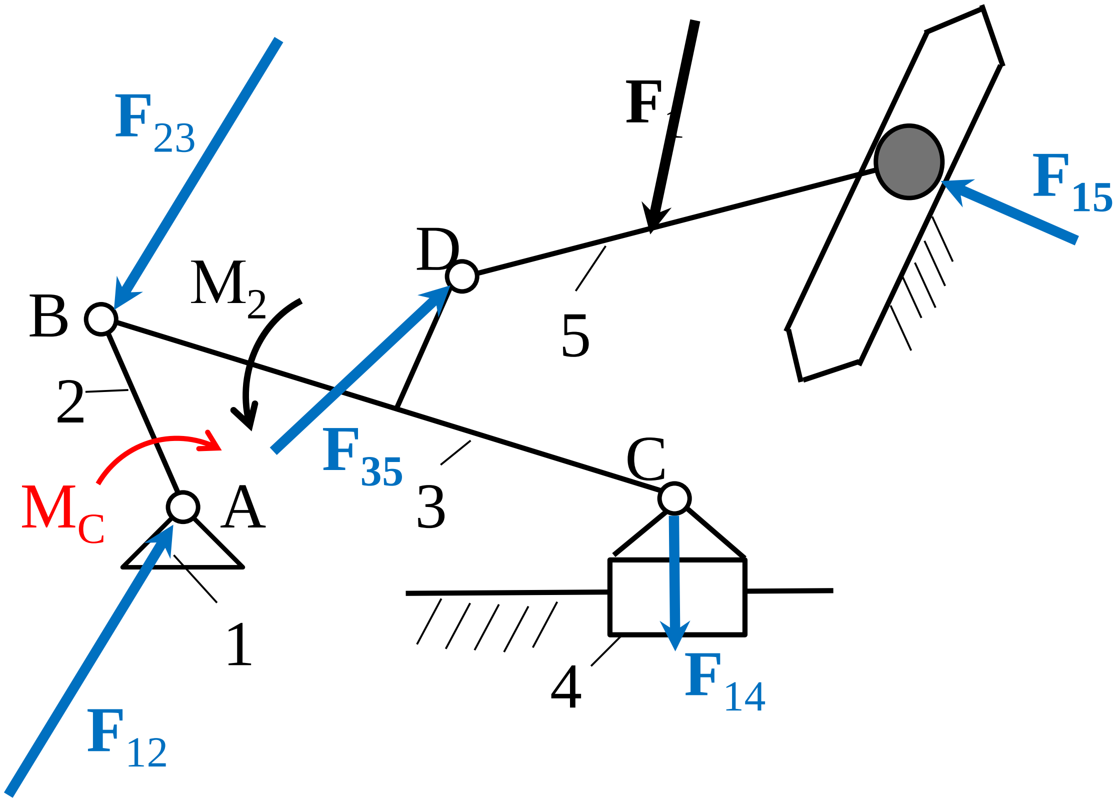
Equilibrium equation (torques) of link 2 relative to point A:

$$\mathbf{F}_{32}\, h_{32} - \mathbf{M}_C = 0$$

$$\mathbf{M}_C = \mathbf{F}_{32}\, h_{32}$$

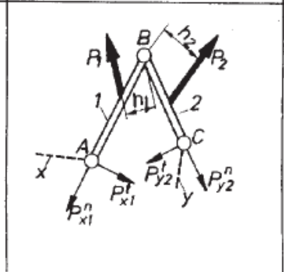
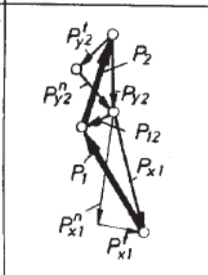
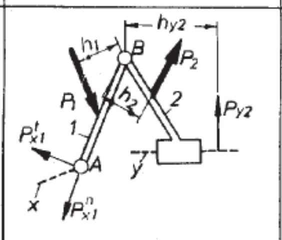
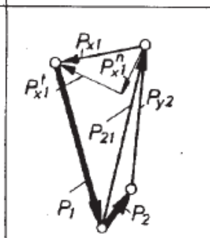
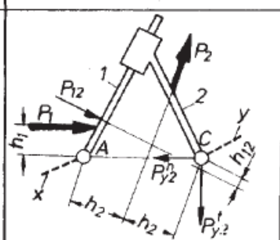
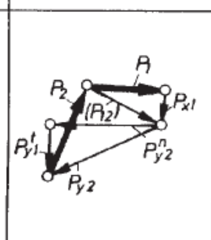
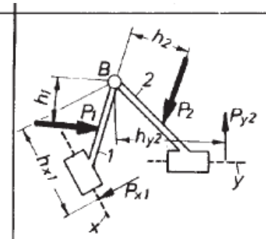
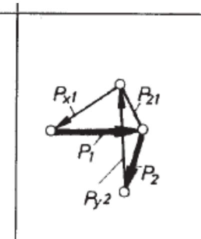
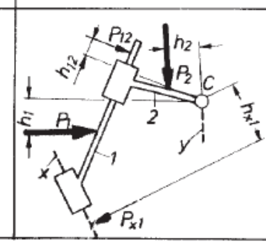
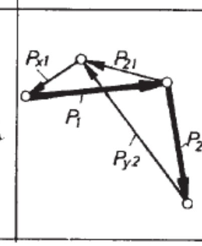
Equilibrium equation (forces) of link 2:

$$\mathbf{F}_{32} + \mathbf{F}_{12} = 0$$
$$\mathbf{F}_{12} = -\mathbf{F}_{32}$$



F23
B
M2
D
F
5
F15
2
MC
A
F35
3
C
1
4
F14
F12

Statically determined groups - examples of solutions

Wersja 2.3.0	Schemat grupy	Plan sił	Równania
A			$P^t_{x1} = \frac{P_1 \cdot h_1}{l_{AB}}$ $P^t_{y2} = \frac{P_2 \cdot h_2}{l_{BC}}$ $\bar{P}^n_{x1} + \bar{P}^t_{x1} + \bar{P}_1 + \bar{P}_2 + \bar{P}^t_{y2} + \bar{P}^n_{y2} = 0$ $\bar{P}_1 + \bar{P}_{x1} + \bar{P}_{21} = 0$
B			$P^t_{x1} = \frac{P_1 \cdot h_1}{l_{AB}}$ $\bar{P}^n_{x1} + \bar{P}^t_{x1} + \bar{P}_1 + \bar{P}_2 + \bar{P}_{y2} = 0$ $h_{y2} = -\frac{P_2 \cdot h_2}{P_{y2}}$ $\bar{P}_2 + \bar{P}_{y2} + \bar{P}_{12} = 0$
C			$P^t_{y2} = \frac{P_2 h_2 - P_1 h_1}{l_{AC}}$ $\bar{P}^n_{y2} + \bar{P}^t_{y2} + \bar{P}_2 + \bar{P}_{12} = 0$ $h_{12} = \frac{P_2 h_2}{P_{12}}$
D			$\bar{P}_{x1} + \bar{P}_1 + \bar{P}_2 + \bar{P}_{y2} = 0$ $h_{x1} = \frac{P_1 \cdot h_1}{P_{x1}}$ $h_{y2} = \frac{P_2 \cdot h_2}{P_{y2}}$ $\bar{P}_1 + \bar{P}_{x1} + \bar{P}_{21} = 0$
E			$\bar{P}_1 + \bar{P}_{21} + \bar{P}_{x1} = 0$ $\bar{P}_2 + \bar{P}_{12} + \bar{P}_{y2} = 0$ $h_{12} = \frac{P_2 \cdot h_2}{P_{12}}$ $h_{x1} = \frac{P_1 \cdot h_1 + P_2 \cdot h_2}{P_{x1}}$