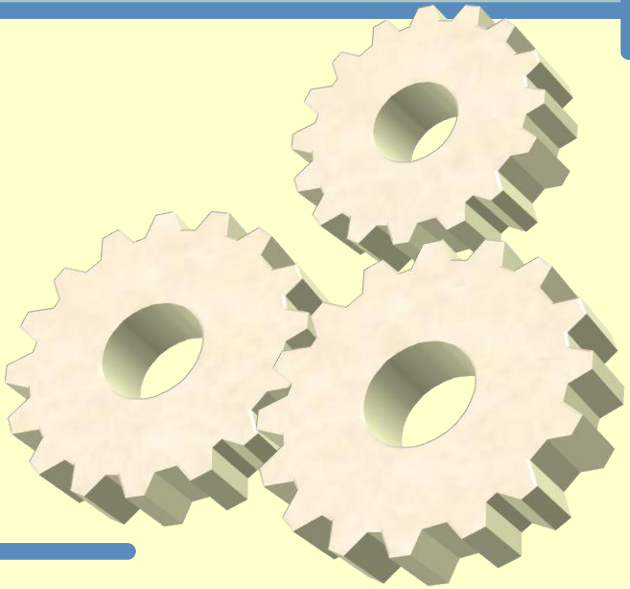
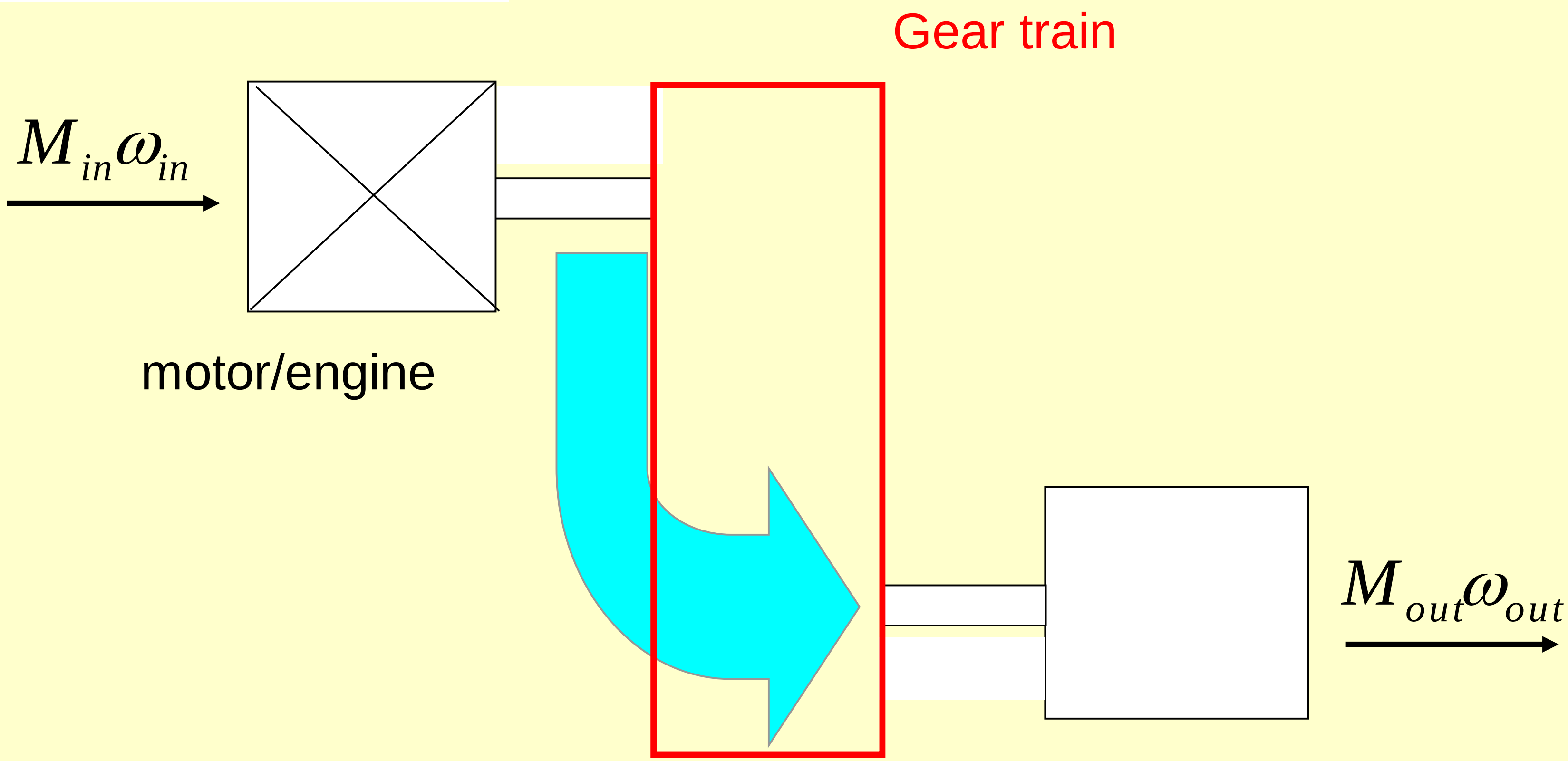
Planetary gear trains



Gear train - tasks

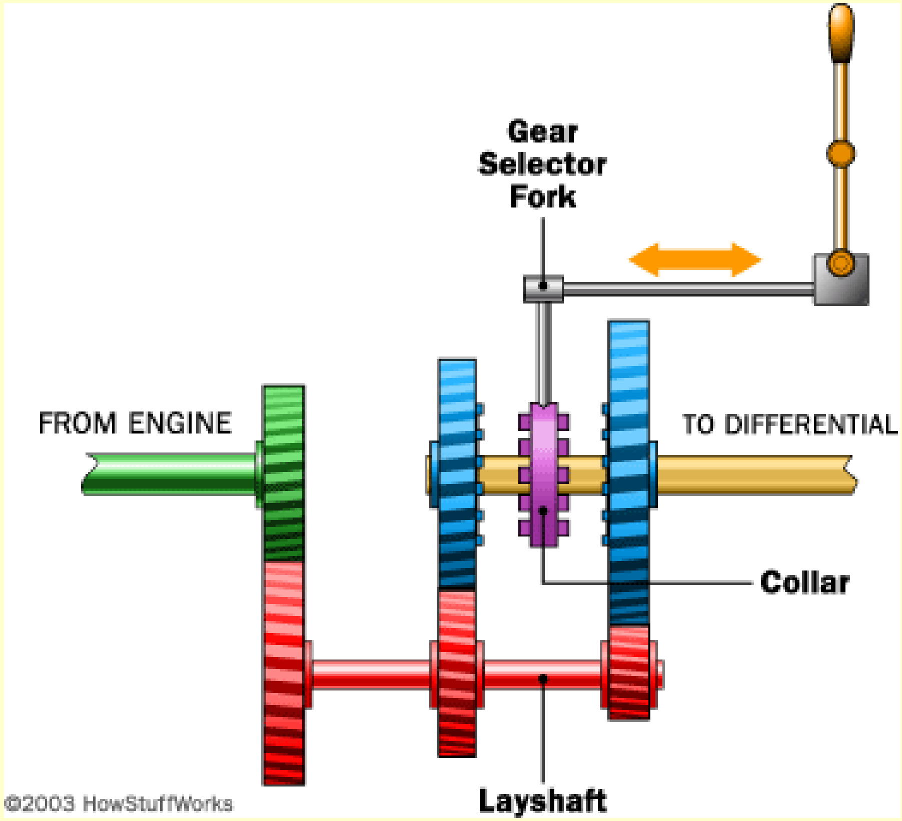


driven link (links) from which required forces and motions are obtained

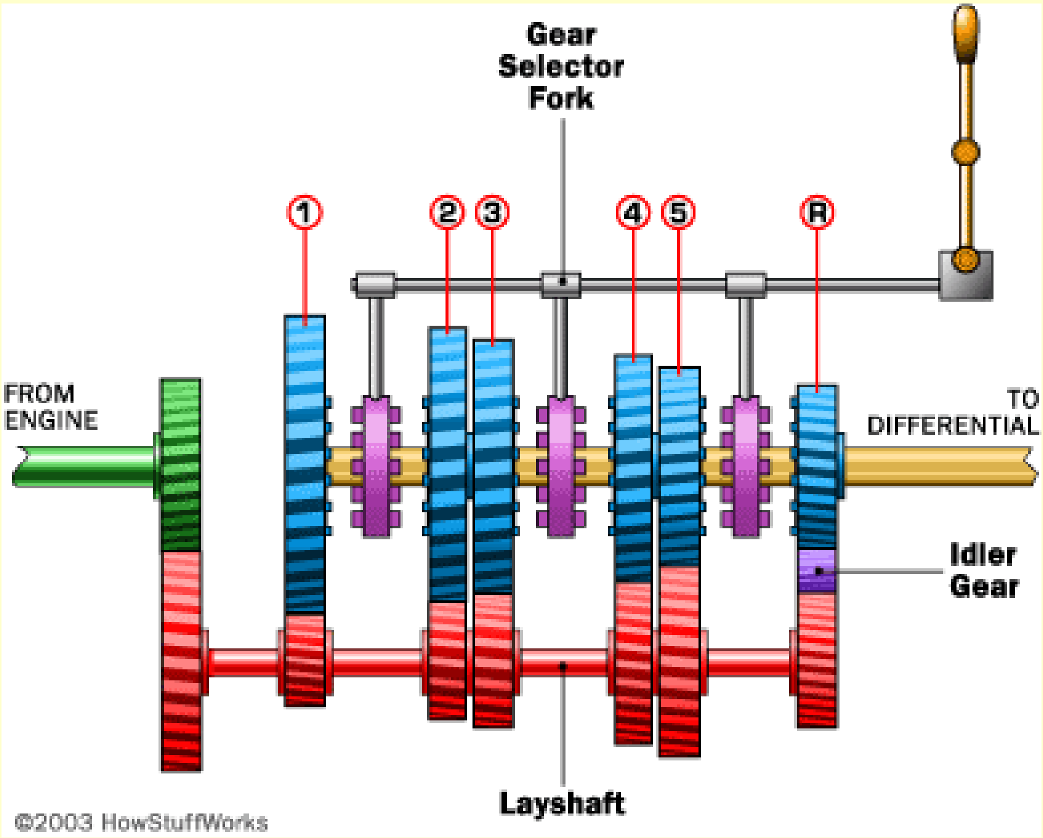
If efficiency $\eta = 1$

$$M_{in}\omega_{in} = M_{out}\omega_{out}$$

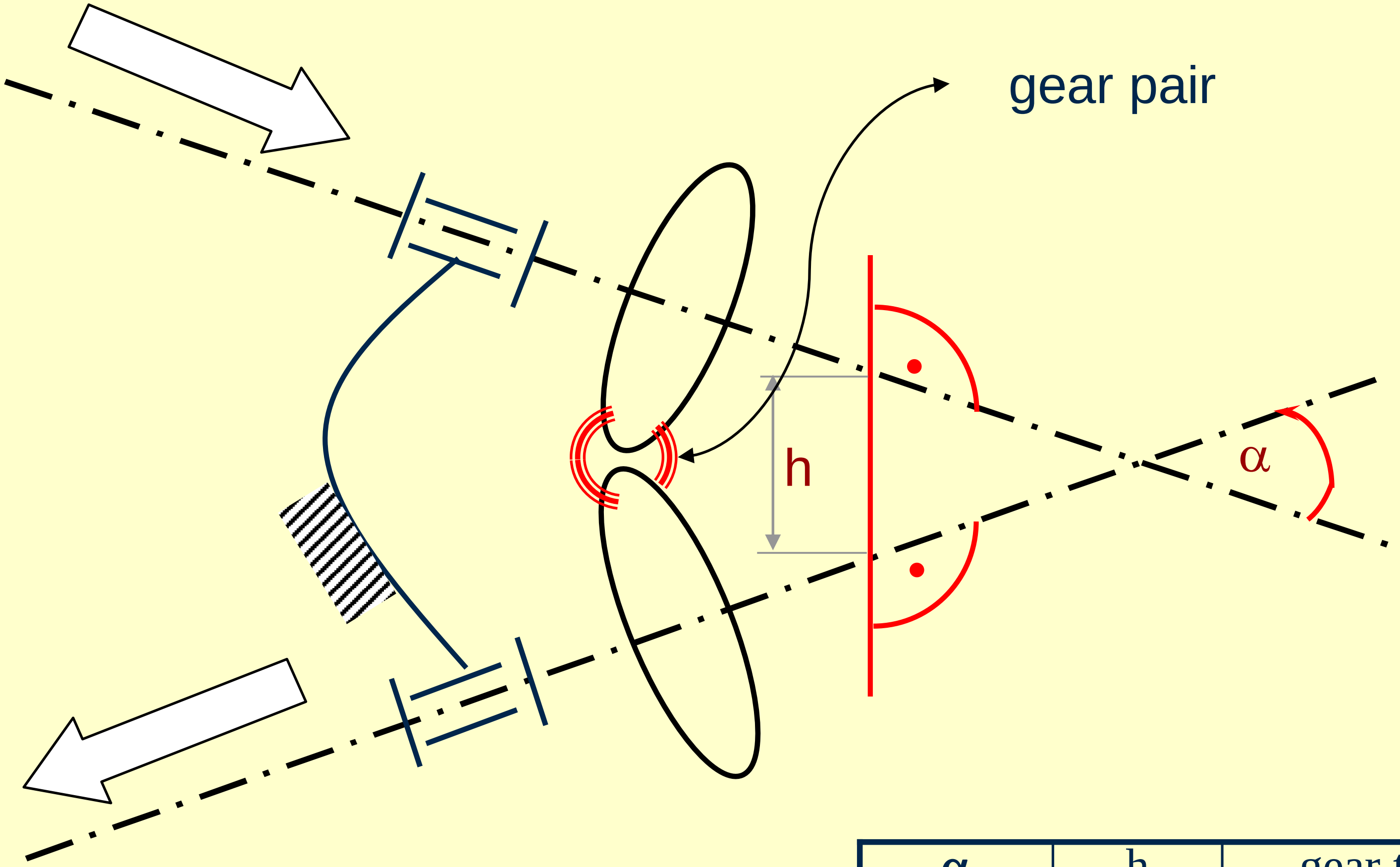
Gear box – example 1



Gear box – example 2



Gear train types



α	h	gear type
$\alpha = 0$	$h \neq 0$	cylindrical
$\alpha \neq 0$	$h = 0$	bevel (conical)
$\alpha = \pi/2$	$h \neq 0$	worm
$\alpha \neq 0$	$h \neq 0$	helical

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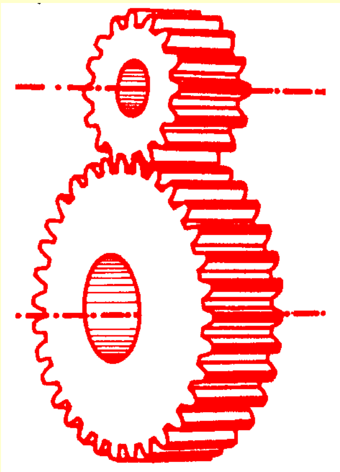
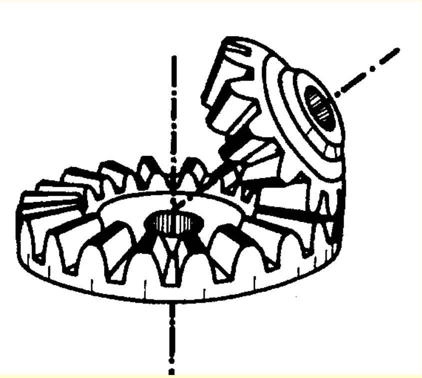
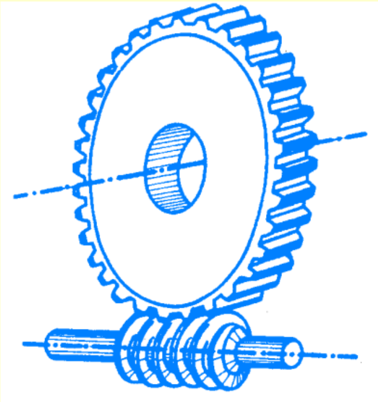
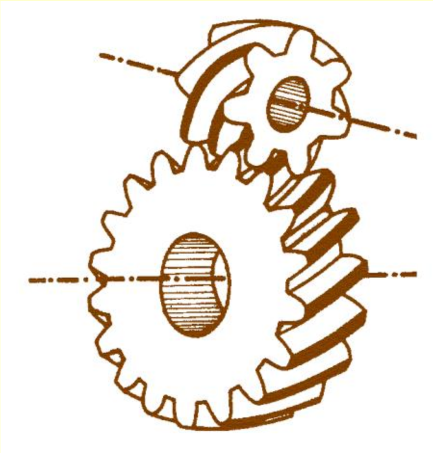
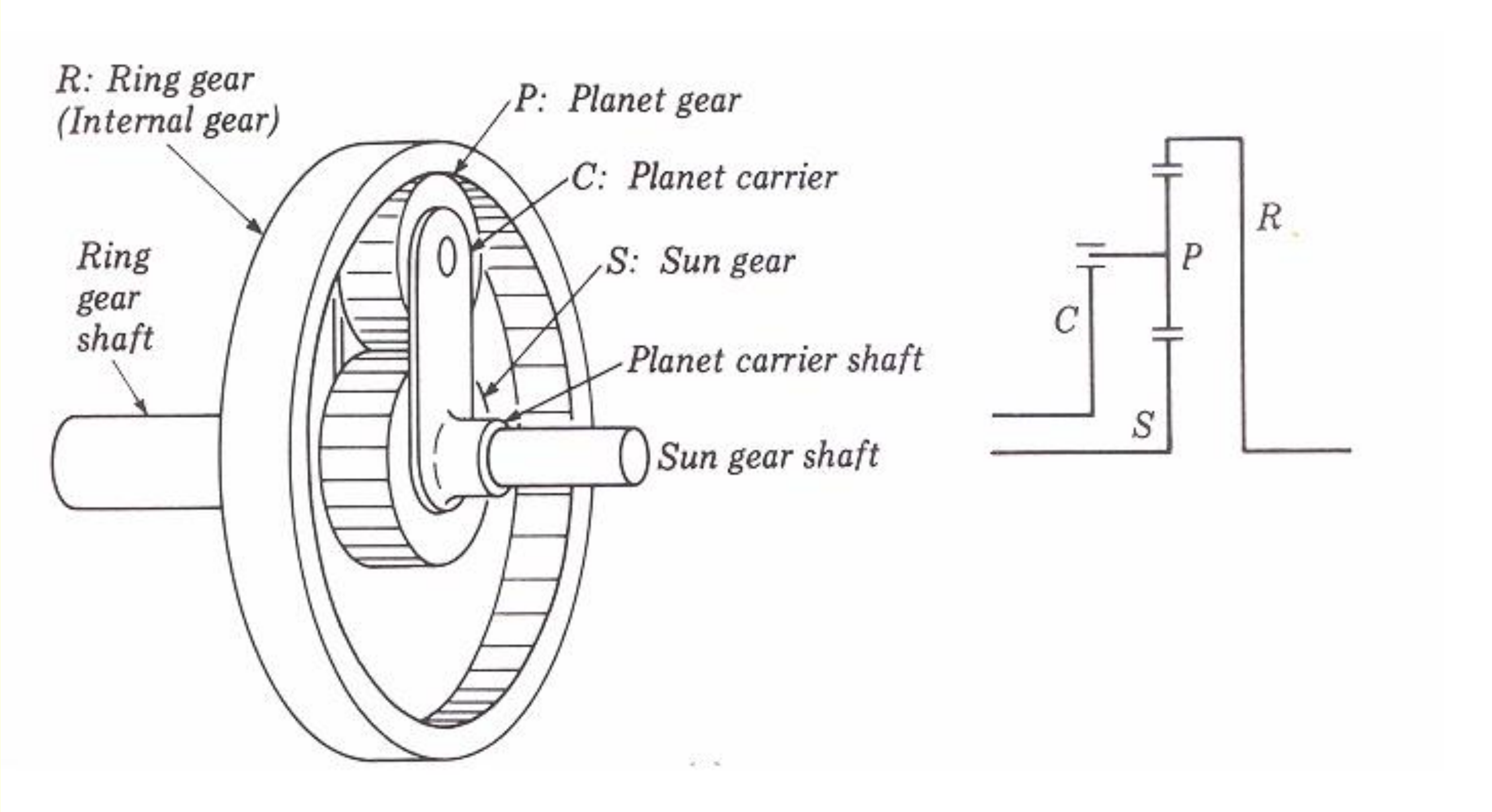





Planetary gear train definition

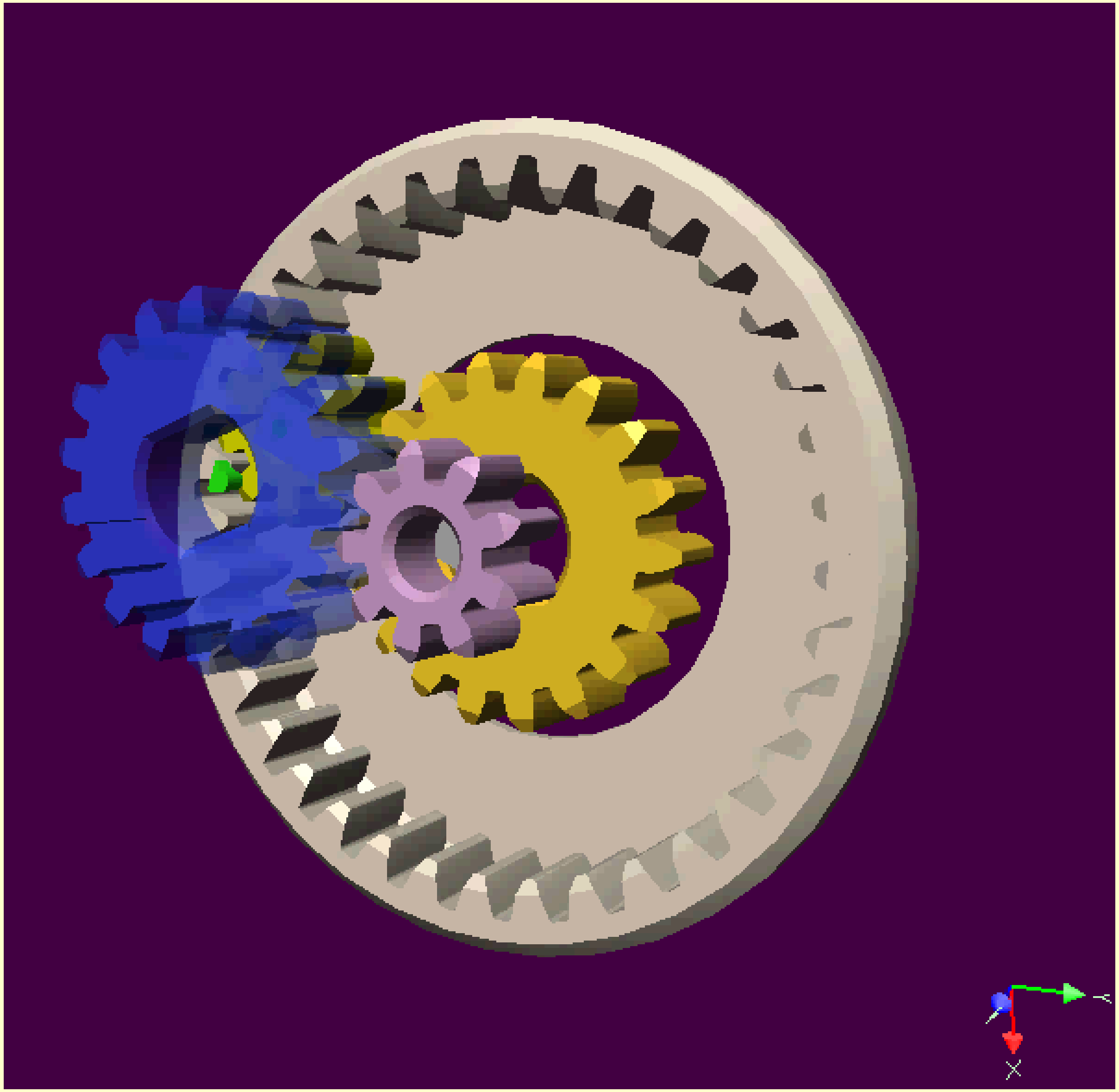
Most of simple and compound gear trains have the restriction that their gear shafts may rotate in bearings fixed to the frame.

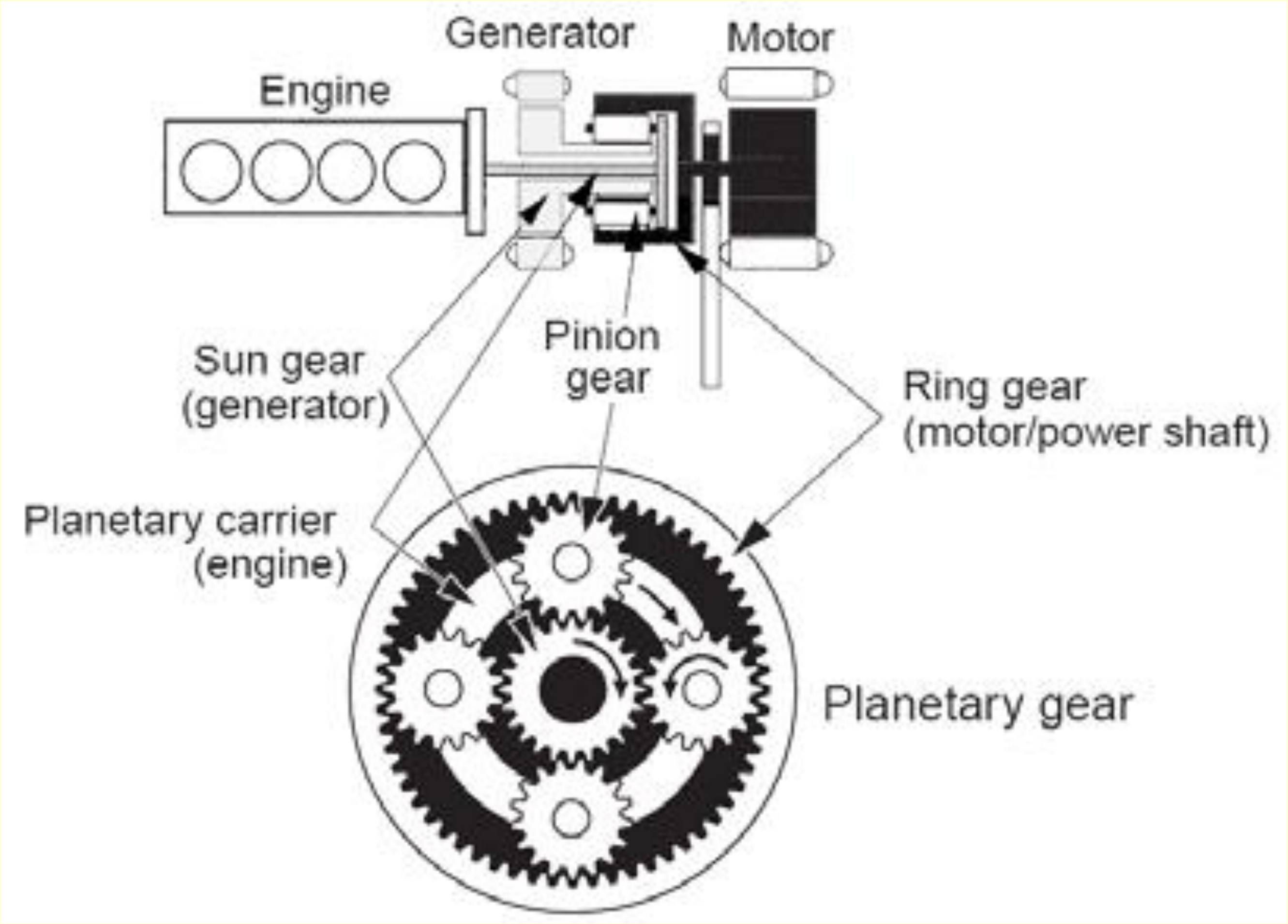
If one or more shafts rotate around another shaft a gear train is called a planetary (or epicyclic) gear train

Planetary gear nomenclature



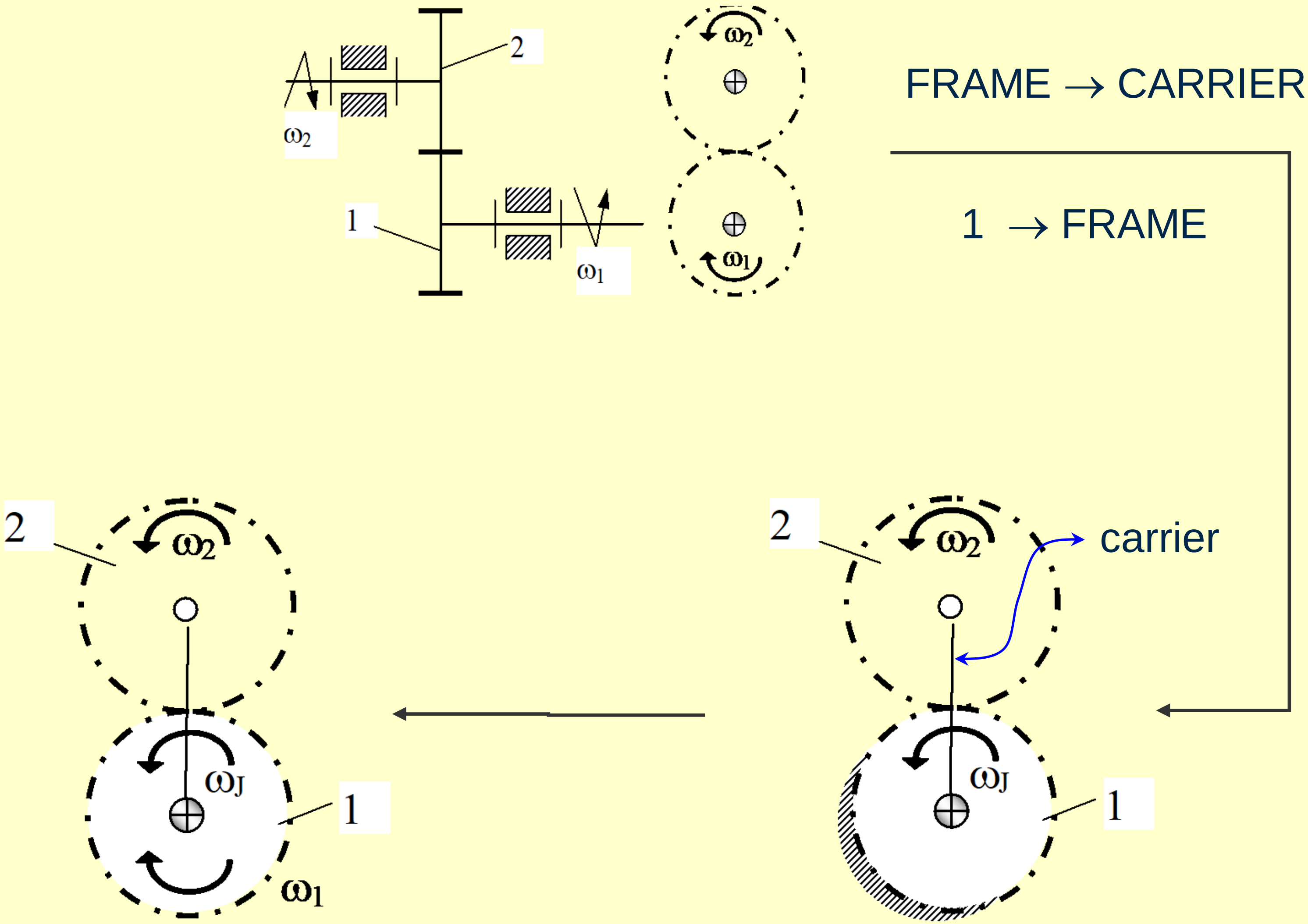
A simple planetary gear



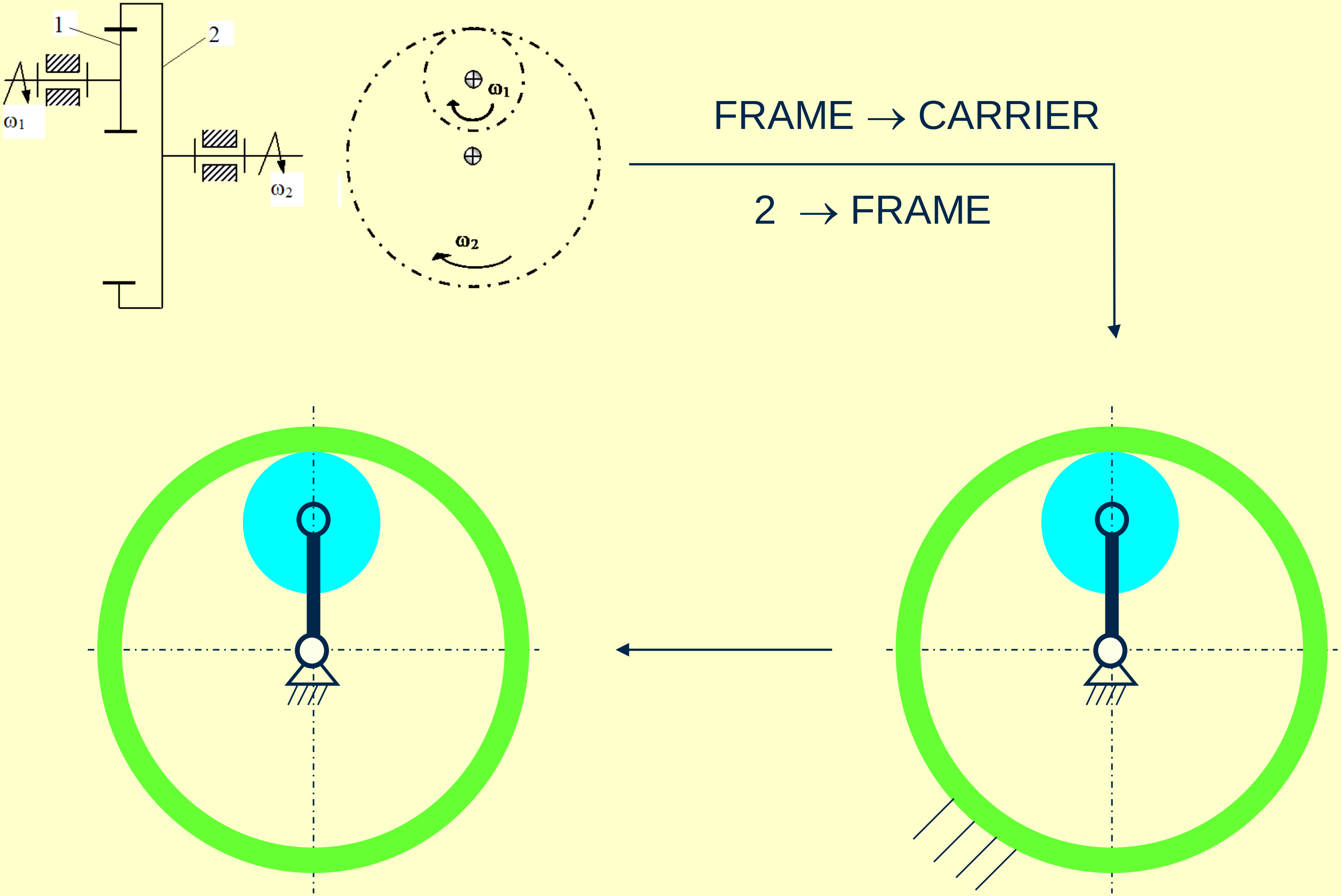


Planetary gear box of the power split device

Simple planetary gear train (obtained from unmovable axes train)



Simple planetary gear train (obtained from unmovable axes train)

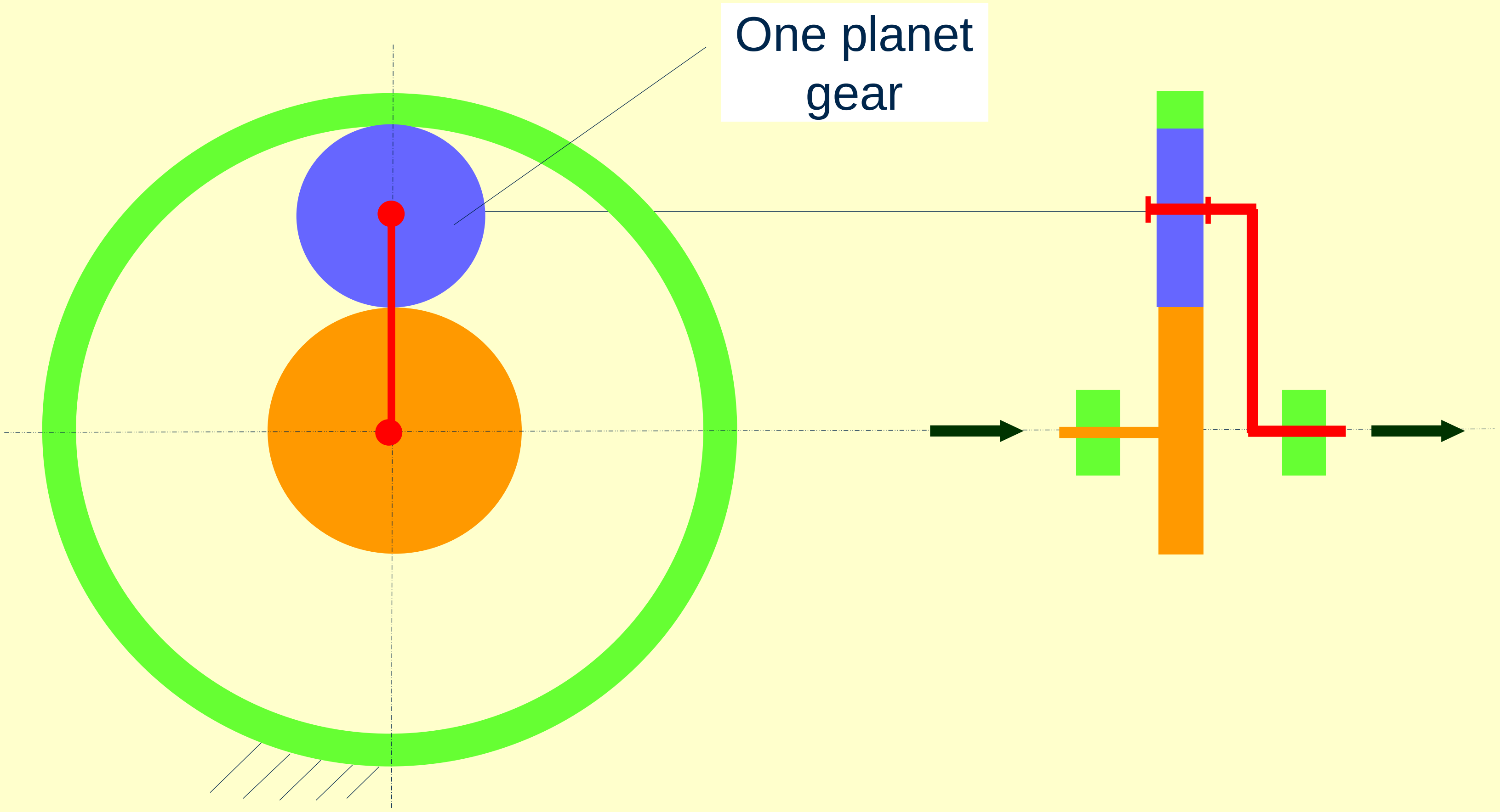


Properties of planetary gear train

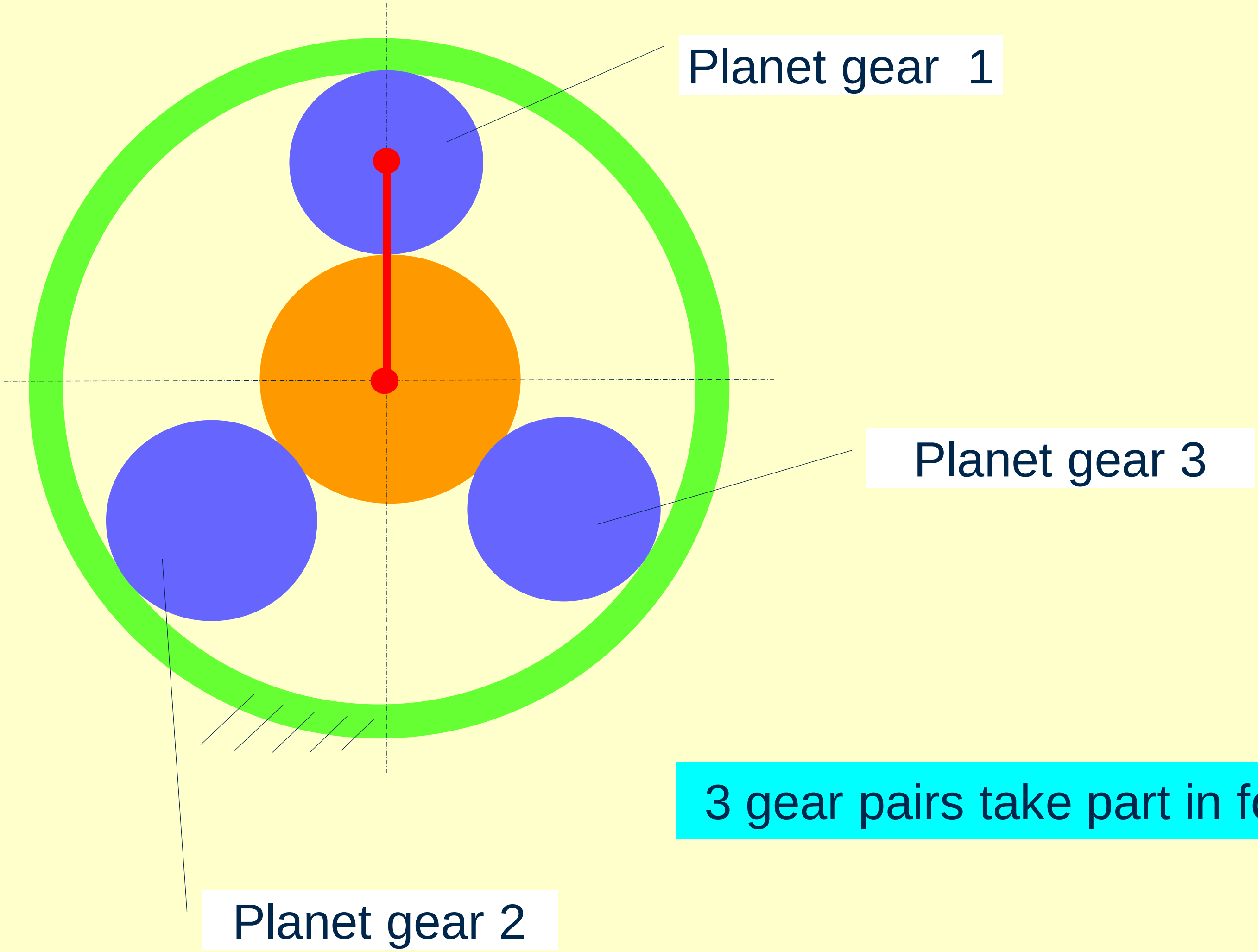
- Large velocity ratio (for compact gear train)
- Ability to transfer large forces (and power)
  - One motor can drive few links (car differentials)
  - A few motors can drive one machine
- Interesting trajectories of planet gear points

- Gears and other parts must be manufactured in very high accuracy $\rightarrow$ COSTS !!!

- Ability to transfer large forces (power)



⌗ Ability to transfer large forces (power)

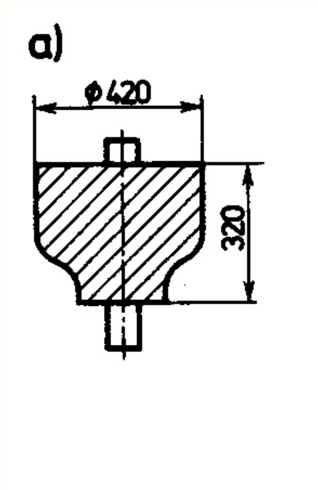


3 gear pairs take part in force transfer

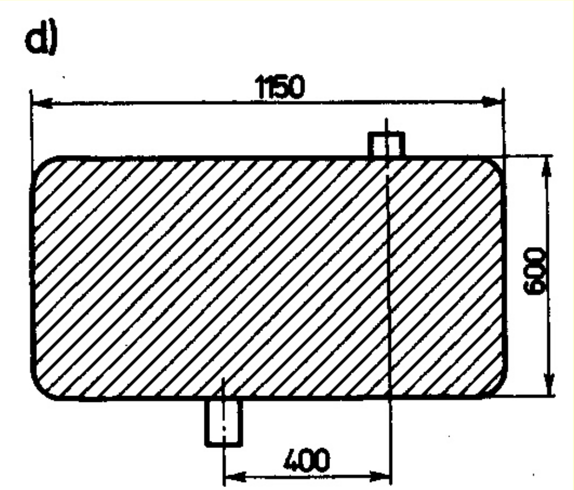
The same power and ratio !

420x320

1150x600



Compare



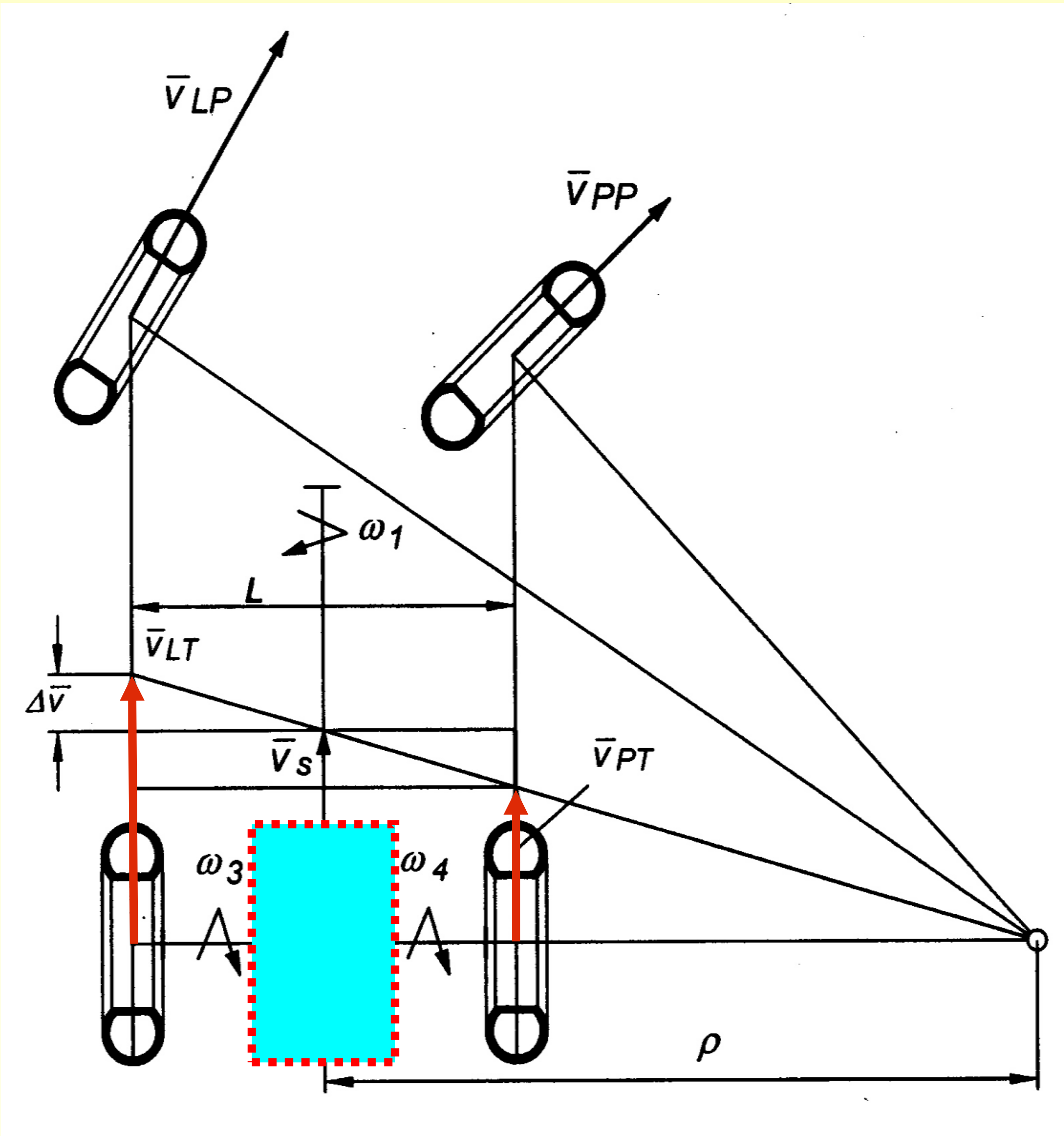
Mass 87 kg

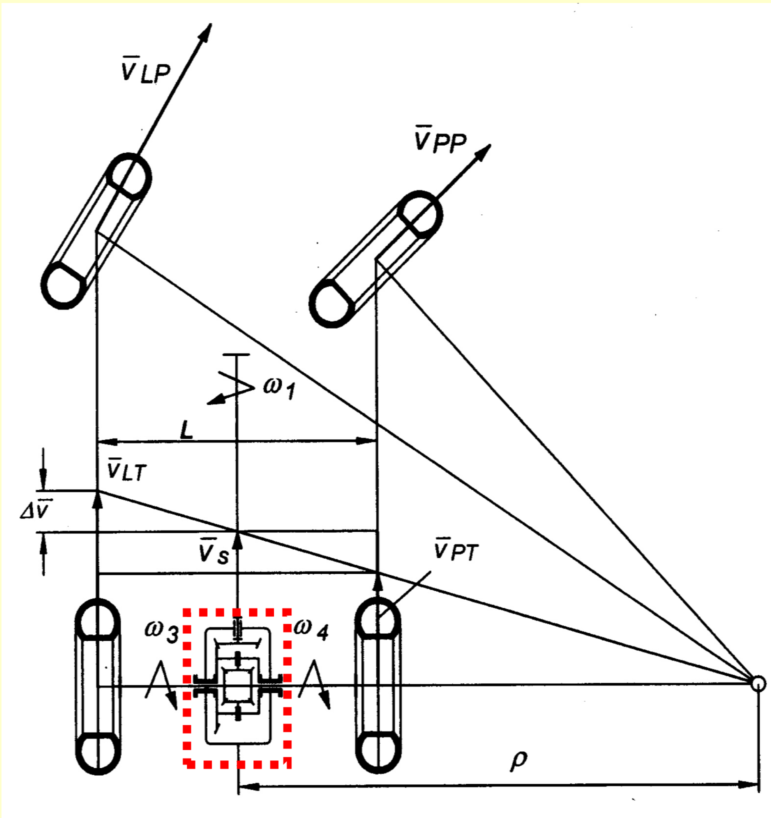
Mass 1400 kg

Planet gear trains

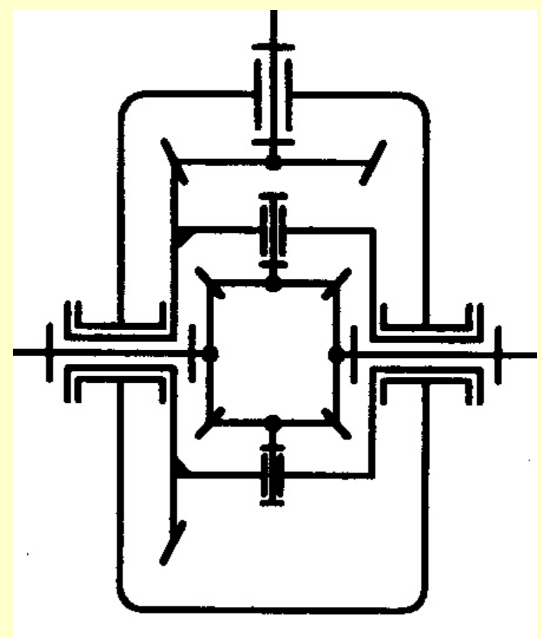
Gears with unmovable
gear axes

⌗ One motor can drive few links (two wheels)

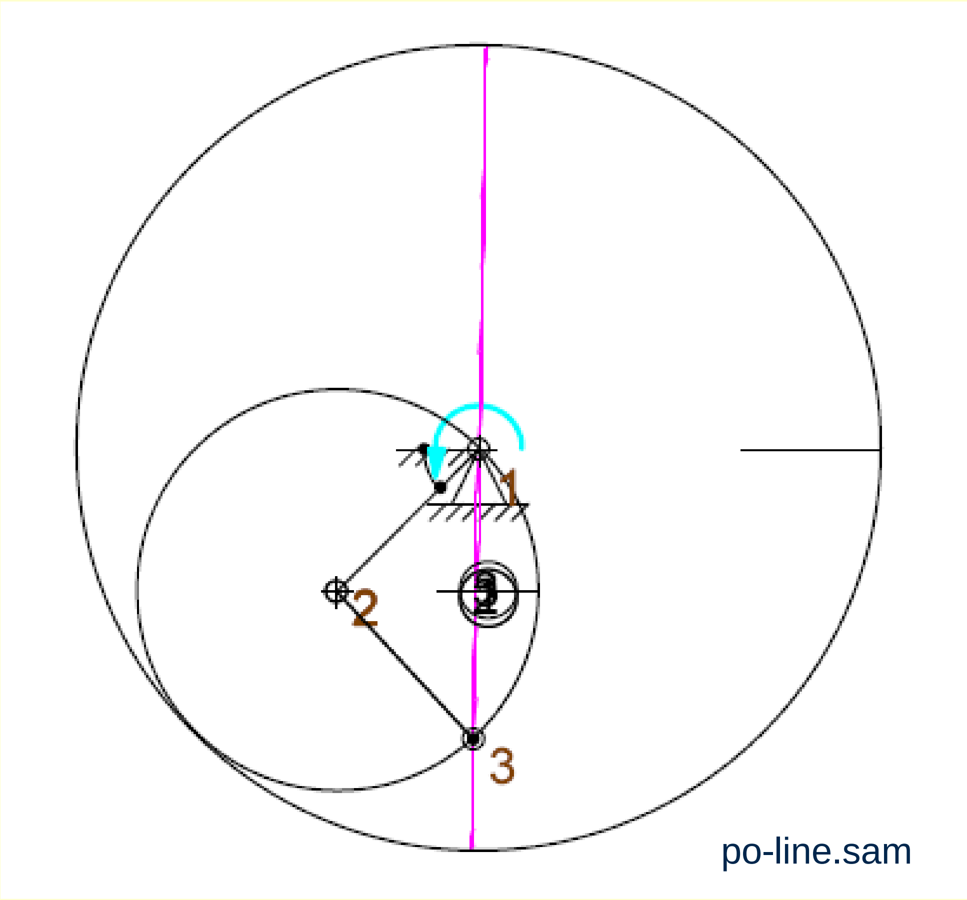




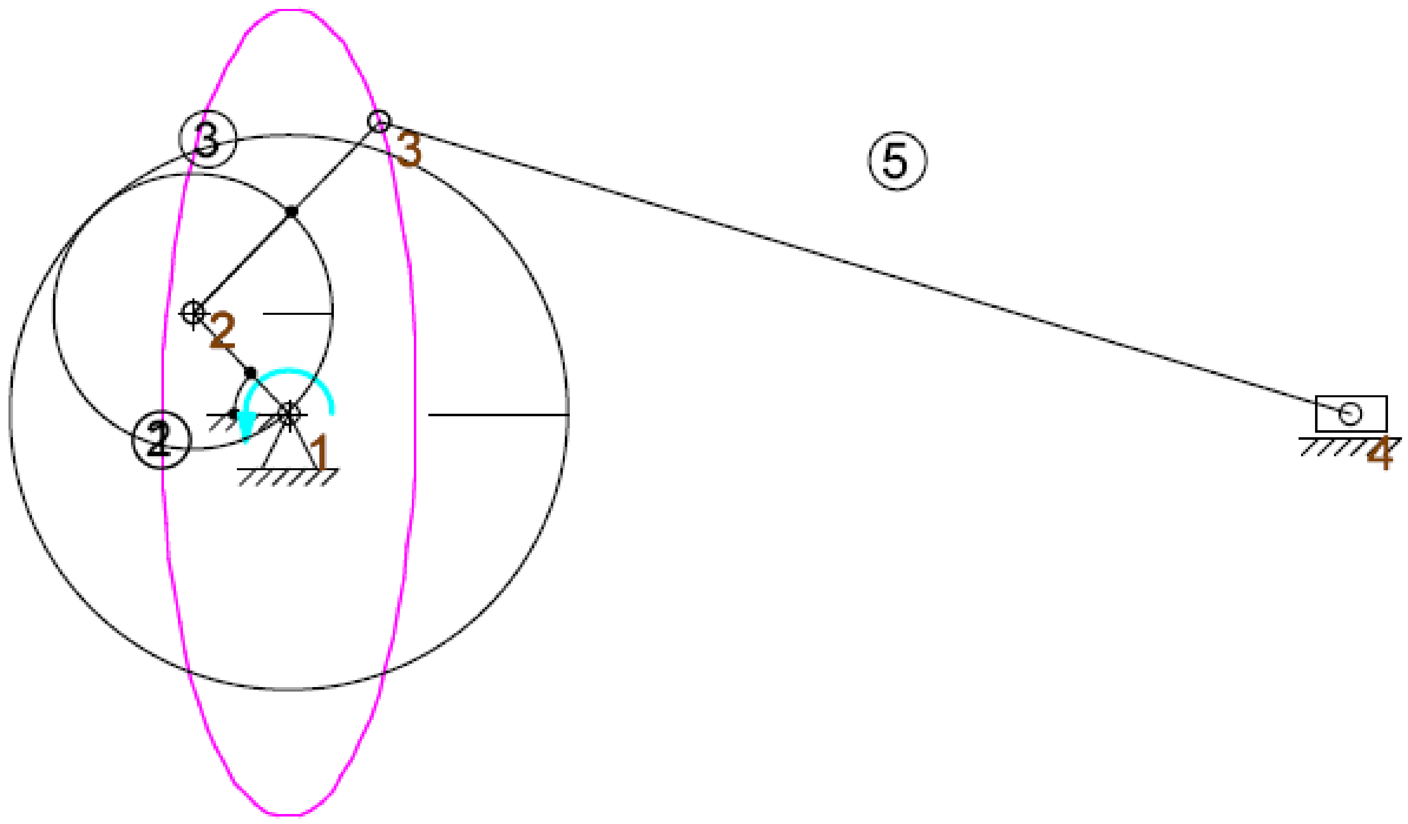
engine



Planetary mechanism – trajectory (1)

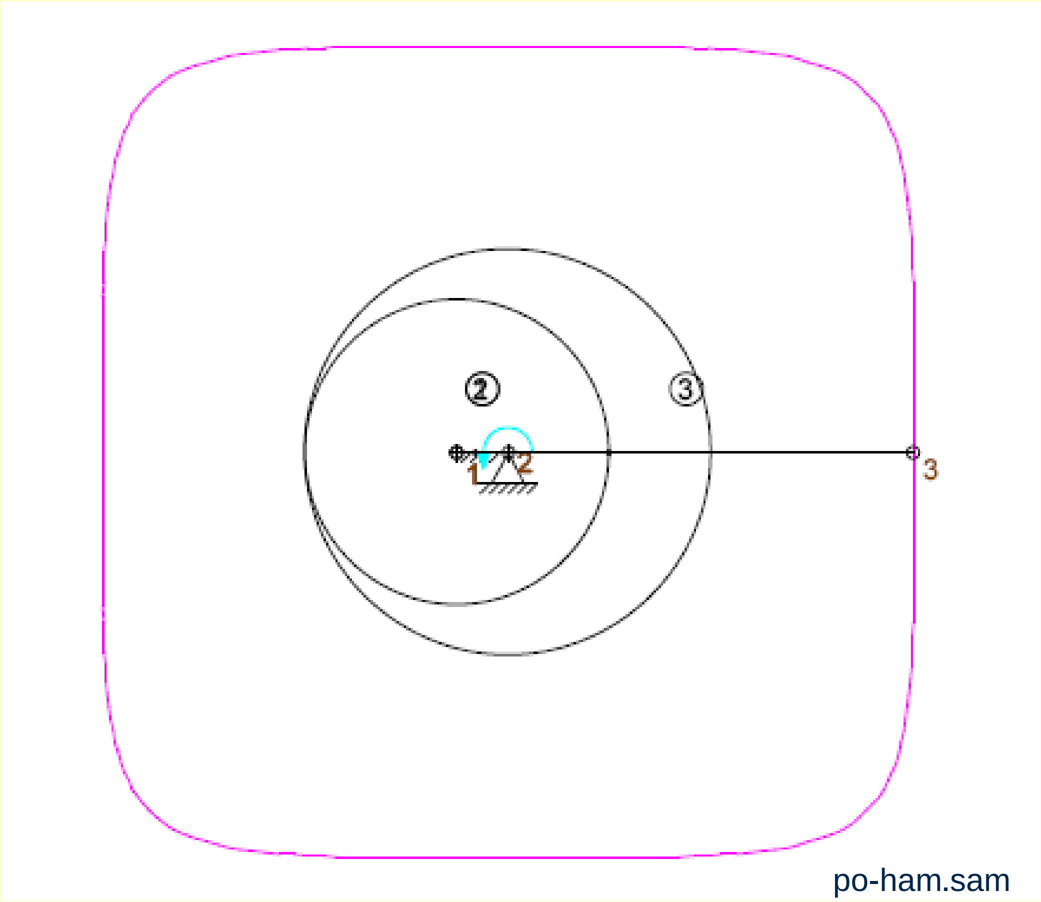


Planetary mechanism – trajectory (2)

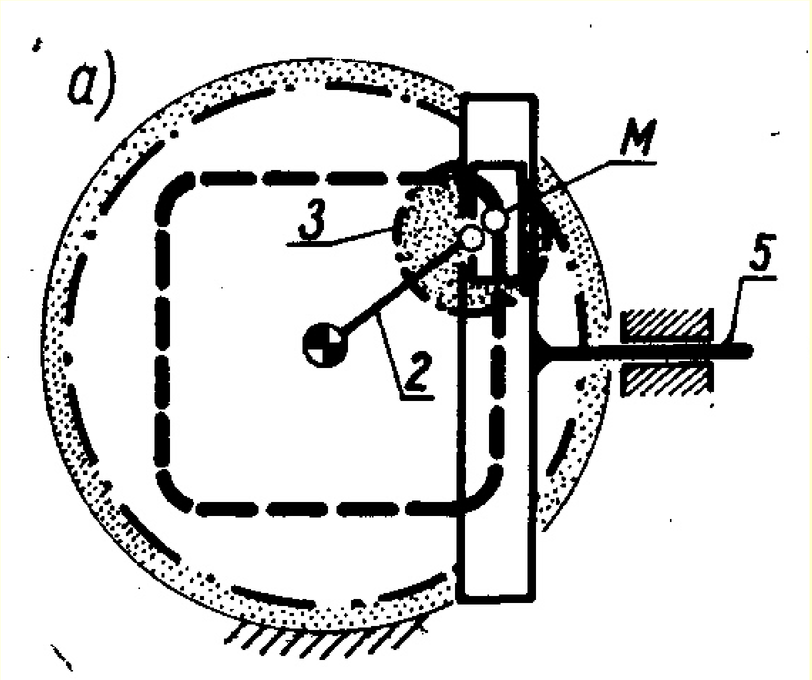


po-stop.sam

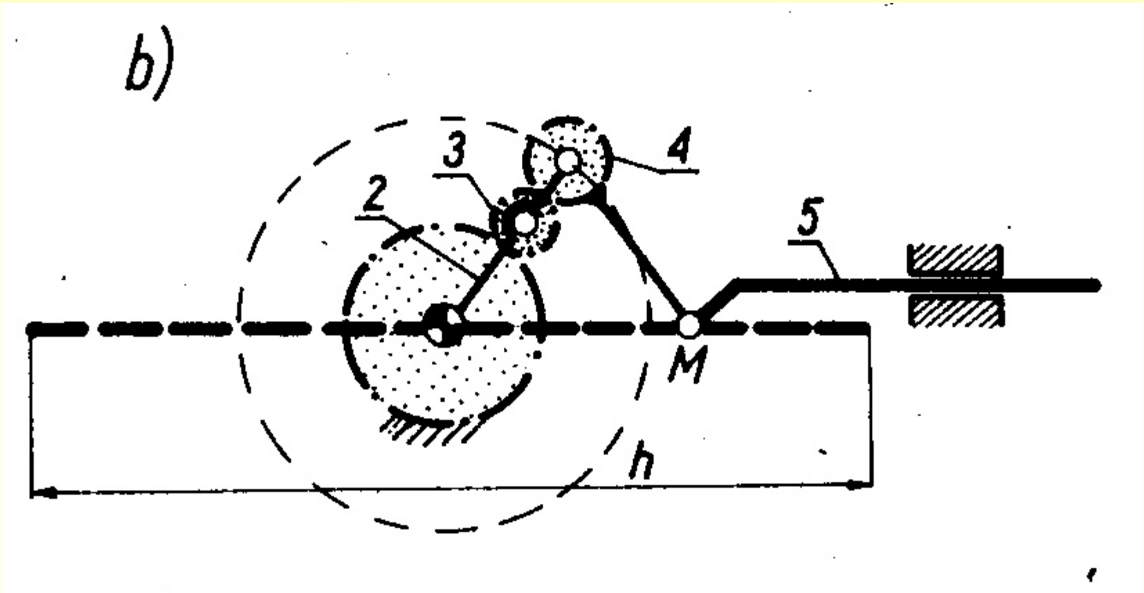
Planetary mechanism – trajectory (3)



Examples of trajectories

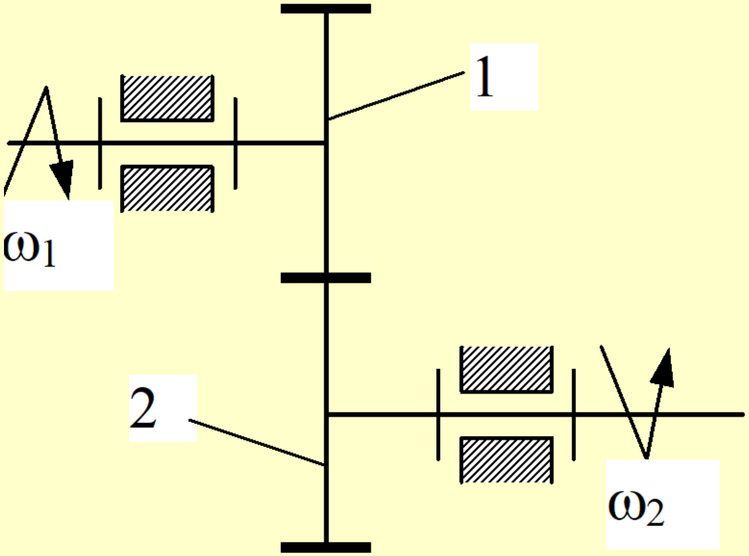


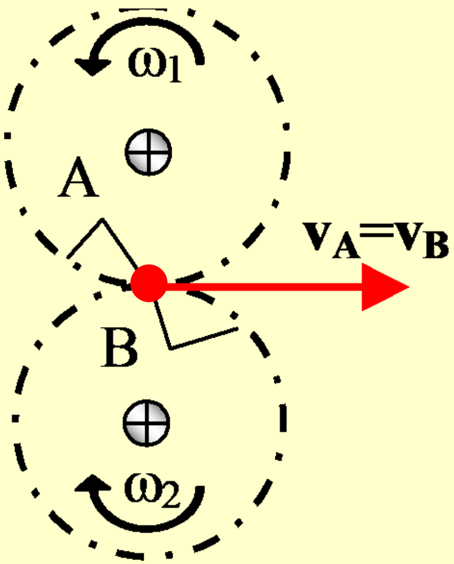
Examples of trajectories



Velocity ratio

External gear



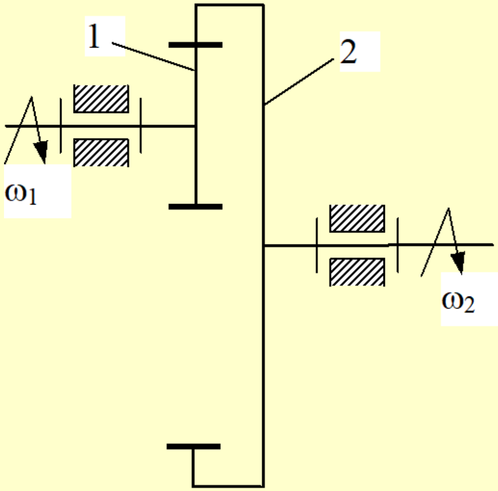


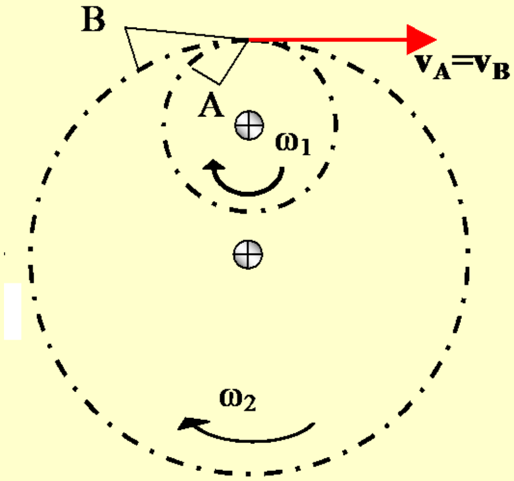
$$\left.\begin{aligned} \omega_1 &= \frac{v}{R_1} \\ \omega_2 &= \frac{v}{R_2} \end{aligned}\right\} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} = \frac{z_2}{z_1}(-1)$$

$$\leftarrow R = \frac{m}{2} z$$

Velocity ratio

Internal gear

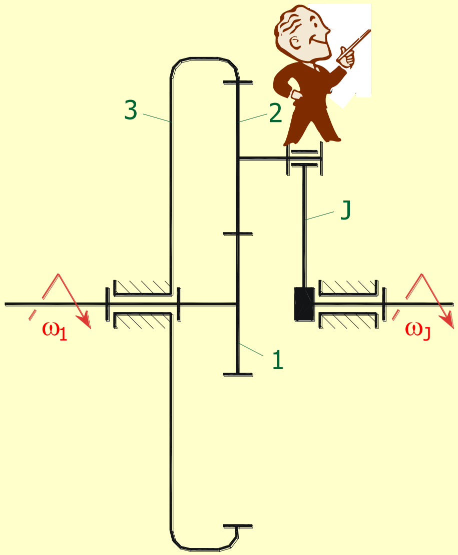




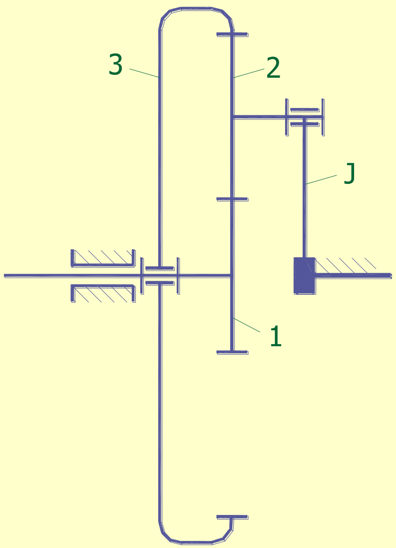
$$\left.\begin{aligned} \omega_1 &= \frac{v}{R_1} \\ \omega_2 &= \frac{v}{R_2} \end{aligned}\right\} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} = \frac{z_2}{z_1} (+1)$$

Analytical method

Idea of analytical method

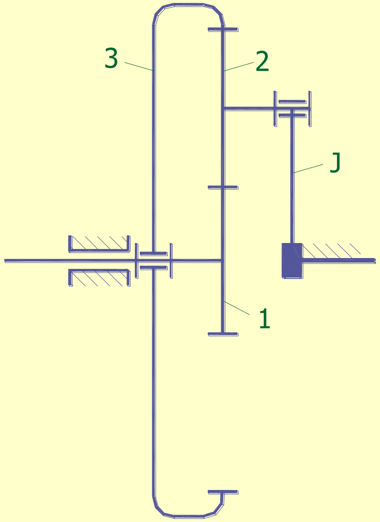


Gear train seen from carrier



	Revolutions in frame (gear 3)	Revolutions seen from carrier J
gear 1	n_1	$n_{1J} = n_1 - n_J$
gear 2	n_2	$n_{2J} = n_2 - n_J$
gear 3	$n_3 = 0$	$n_{3J} = n_3 - n_J$
Carrier J	n_J	0

$$\omega\left[\frac{1}{s}\right] = \frac{\pi\, n\left[\frac{\text{rev}}{\text{min}}\right]}{30}$$



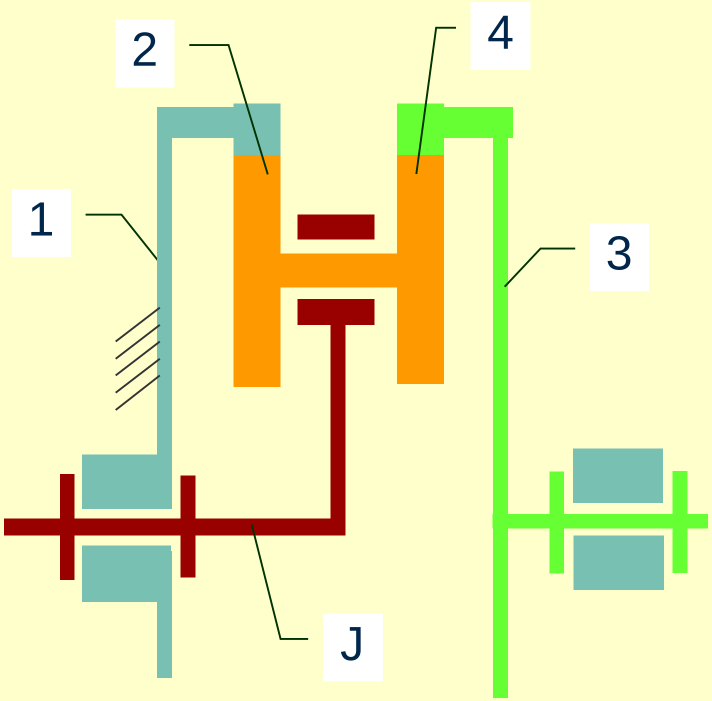
$$\frac{\omega_{uJ}}{\omega_{sJ}} = \frac{\omega_u - \omega_J}{\omega_s - \omega_J} = f(z_i)$$

$$\frac{\omega_{1J}}{\omega_{3J}} = \frac{\omega_1 - \omega_J}{\omega_3 - \omega_J} = \left[\frac{\omega_{1J}}{\omega_{2J}}\right] \bullet \left[\frac{\omega_{2J}}{\omega_{3J}}\right] =$$

$$= \left[\frac{z_2}{z_1}(-1)\right] \cdot \left[\frac{z_3}{z_2}(+1)\right]$$

$$\frac{\omega_1 - \omega_J}{\omega_3 - \omega_J} = \frac{z_3}{z_1}(-1)$$

$$\omega_3 = 0 \longrightarrow \frac{\omega_1 - \omega_J}{-\omega_J} = \frac{z_3}{z_1}(-1) \Longrightarrow \omega_1 = \omega_J\left(\frac{z_3}{z_1} + 1\right)$$



$$\omega_3 / \omega_J = ?$$

„seen” from the carrier J:

$$\frac{\omega_3 - \omega_J}{\omega_1 - \omega_J} = \frac{z_1}{z_2}(+1)\frac{z_4}{z_3}(+1)$$

Since:

$$\omega_1 = 0$$

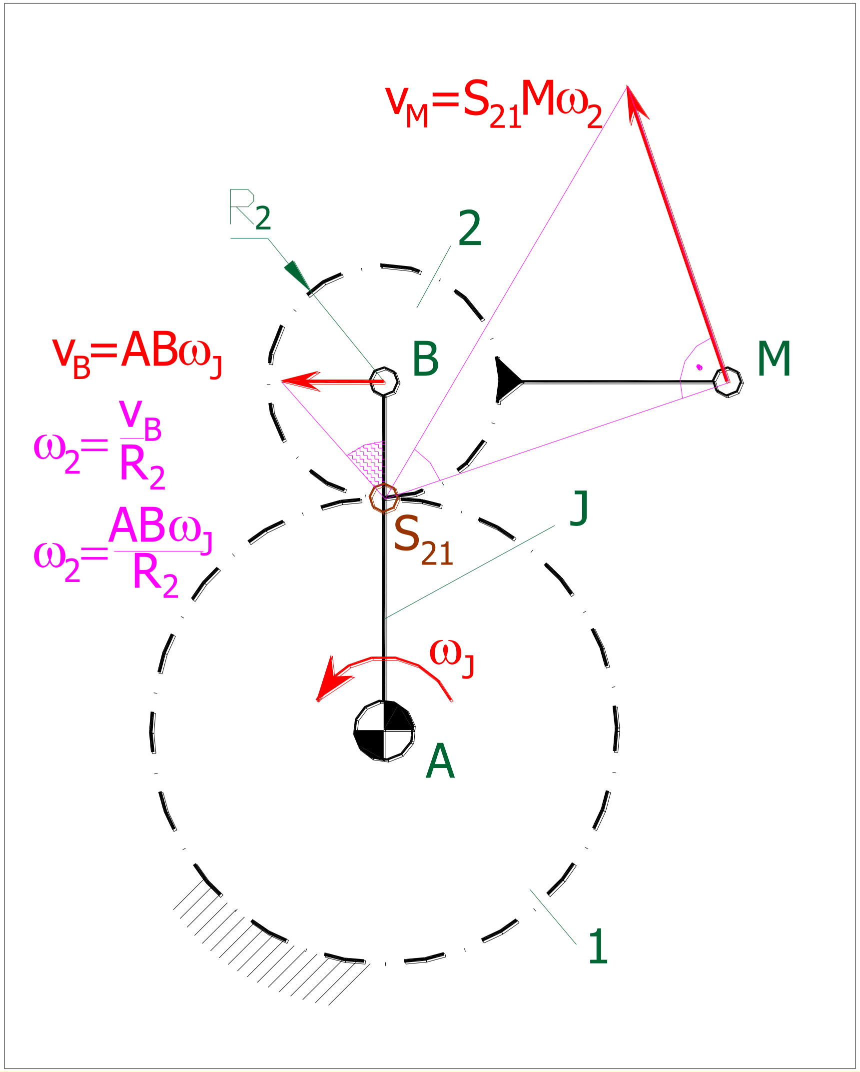
Then:

$$\frac{\omega_3}{\omega_J} = 1 - \frac{z_1 z_4}{z_3 z_2}$$

$$\text{Assume toothnumbers: } z_1 = 101;\ z_2 = 51;\ z_3 = 99;\ z_4 = 50$$

$$\frac{\omega_3}{\omega_J} = 1 - \frac{101 \cdot 50}{99 \cdot 51} = \frac{-1}{5049}$$

Graphical method (Velocity analysis)



$$\omega_2 = \frac{\omega_J AB}{R_2}$$

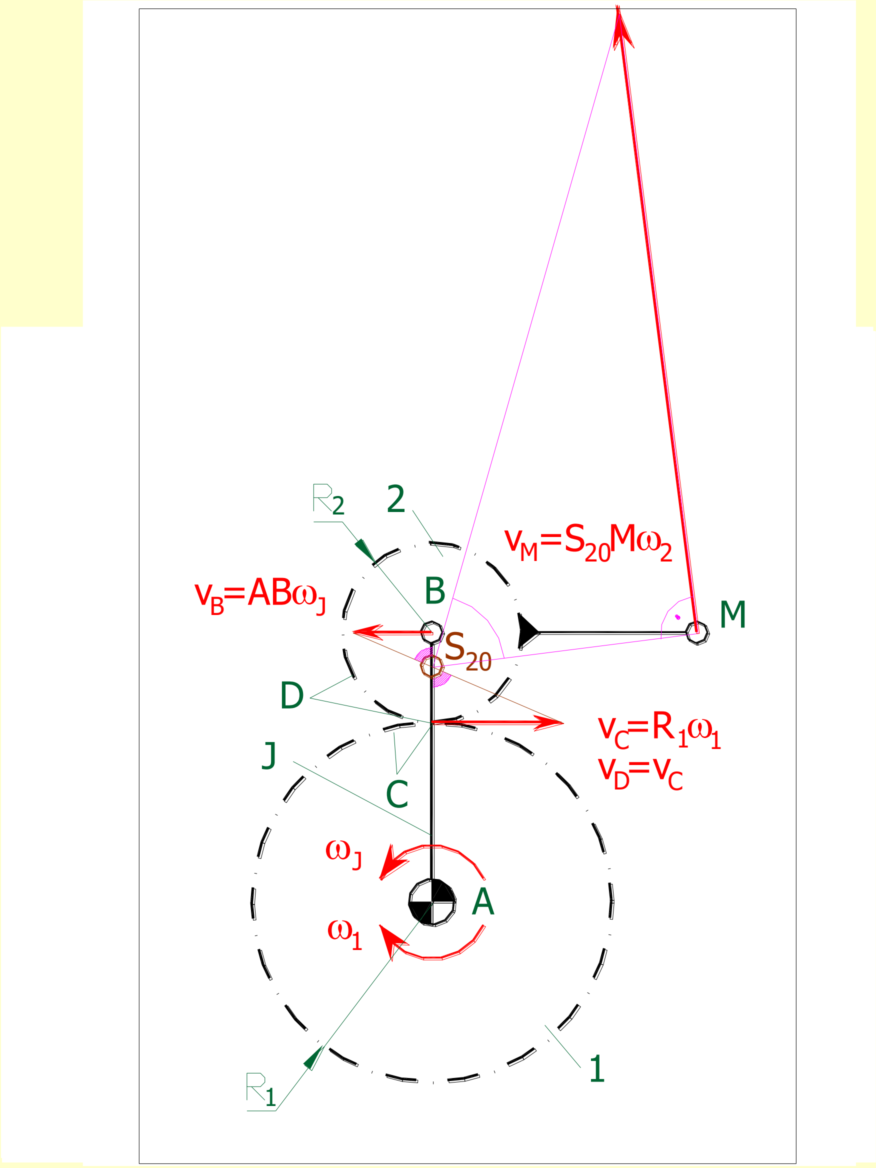
$$AB = R_1 + R_2$$

$$\omega_2 = \frac{\omega_J \left(R_1 + R_2\right)}{R_2}$$

$$\omega_2 = \frac{\omega_J \left(\frac{1}{2} m z_1 + \frac{1}{2} m z_2\right)}{\frac{1}{2} m z_2}$$

$$\omega_2 = \frac{\omega_J \left(z_1 + z_2\right)}{z_2}$$

Two driving gears (gear 1 and carrier)

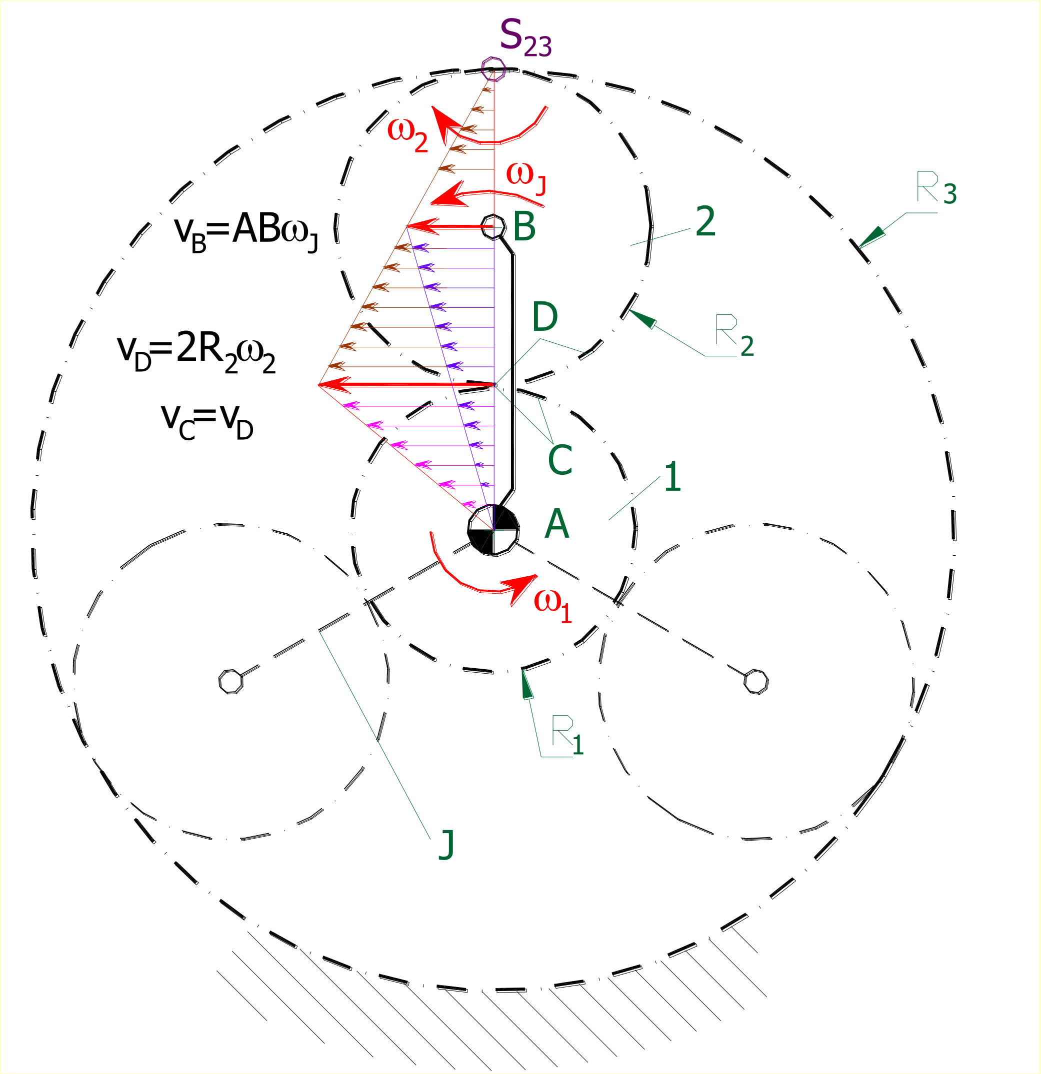


$$1.\ \omega_J \rightarrow v_B$$

$$2.\ \omega_1 \rightarrow v_C = v_D$$

$$3.\ S_{20}\ \ (0-\text{frame})$$

Planetary gear train – graphical method



$$\mathbf{v}_B = AB\omega_J$$

$$AB = R_1 + R_2$$

$$\mathbf{v}_B = (R_1 + R_2)\omega_J$$

$$\omega_2 = \frac{\mathbf{v}_B}{R_2}$$

$$\omega_2 = \frac{R_1 + R_2}{R_2}\omega_J$$

$$\mathbf{v}_D = 2R_2\omega_2$$

$$\mathbf{v}_D = 2(R_1 + R_2)\omega_J$$

$$\mathbf{v}_C = \mathbf{v}_D$$

$$\mathbf{v}_C = 2(R_1 + R_2)\omega_J$$

$$\omega_1 = \frac{\mathbf{v}_C}{R_1}$$

$$\omega_1 = \frac{2(R_1 + R_2)\omega_J}{R_1}$$

$$\omega_1 = \left(1 + \frac{R_3}{R_1}\right)\omega_J$$