KINEMATICS OF MECHANISMS

position – velocity – acceleration; <u>driving forces are neglected</u>

For a point K we have linear displacement $s_K(t)$, velocity v_K and acceleration a_K

$$\mathbf{v}_K = \dot{s}_K = \frac{ds_K}{dt} \qquad a_K = \ddot{s}_K = \frac{d\mathbf{v}_K}{dt} = \frac{d^2 s_K}{dt^2}$$

For a link *k* we have angular displacement $\theta_k(t)$, angular velocity ω_k and angular acceleration ε_k

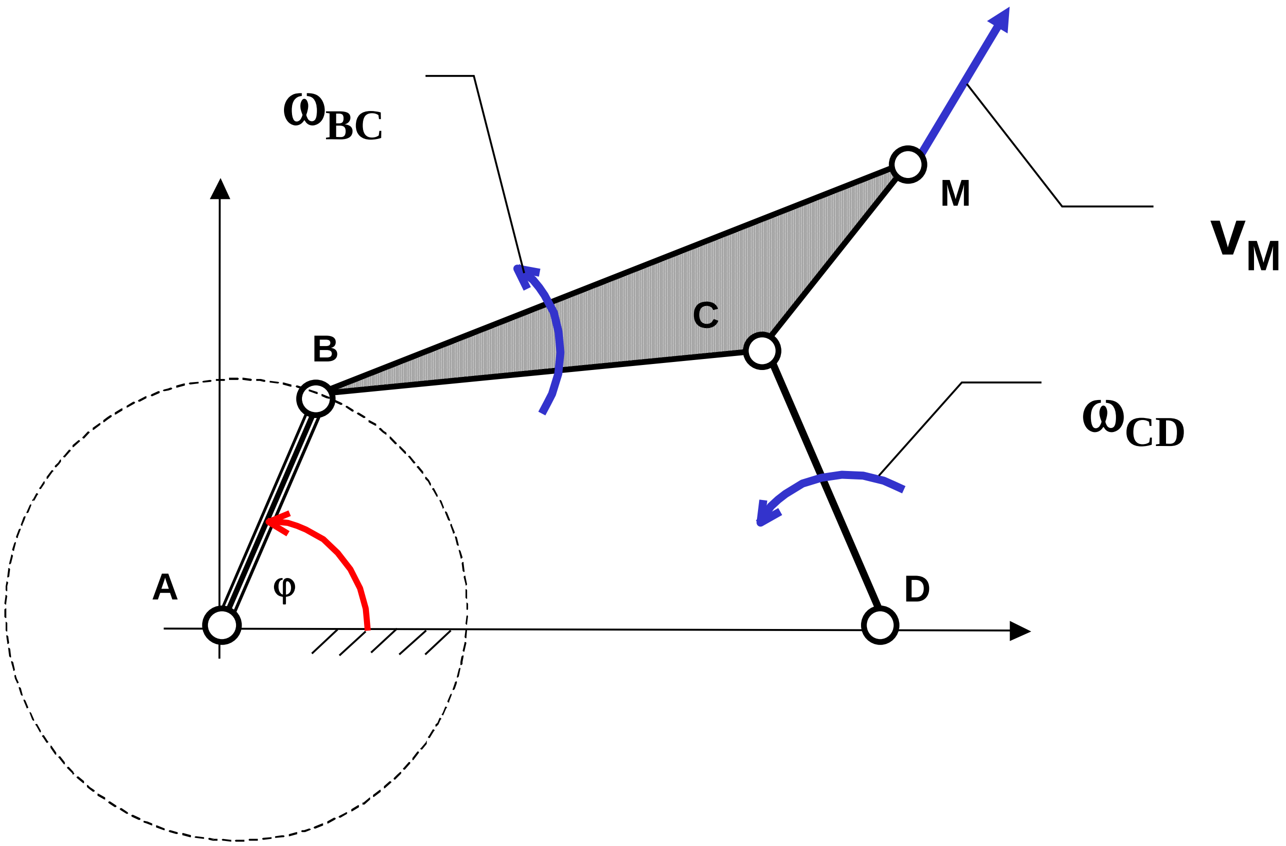
$$\omega_k = \dot{\Theta}_k = \frac{d\Theta_k}{dt} \qquad \varepsilon_k = \ddot{\Theta}_k = \frac{d\omega_k}{dt} = \frac{d^2\Theta_k}{dt^2}$$

METHODS OF KINEMATIC ANALYSIS

graphical (use only: pencil, ruler, compass, calculator)

analytical

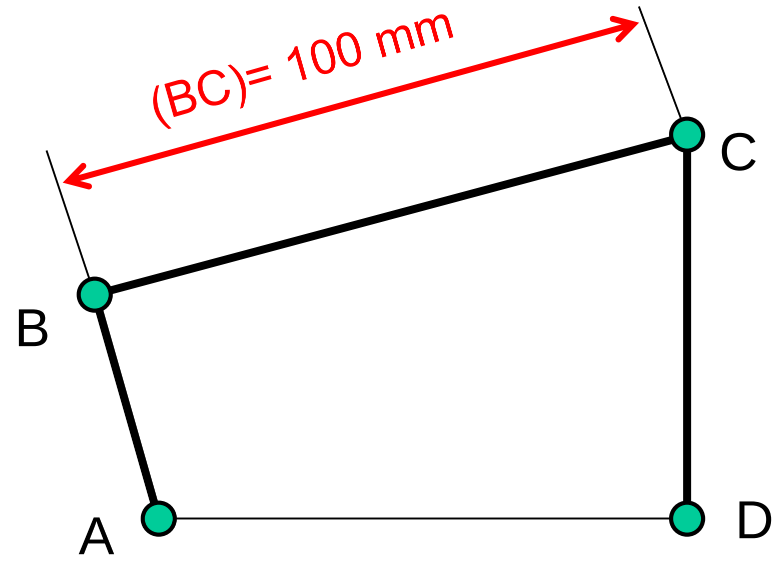
numerical



DOF = 1 → 1 driver, this case: crank AB – known φ(t)

SCALE DEFINITION (for graphical methods)

LENGTH SCALE



Real length

BC = 1 m

Graphical (drawn) length

(BC) = 100 mm

Length scale:

$$\kappa_l = \frac{BC}{(BC)} = \frac{1}{100}\left[\frac{m}{mm}\right]$$

Scale (general)

$$\kappa_i = \frac{i}{(i)}$$

$$\frac{\text{real value}}{\text{graphical value}}$$

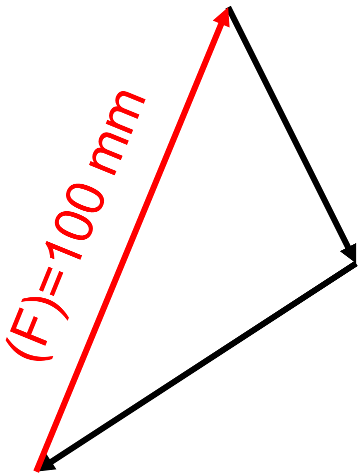
Velocity scale

$$\kappa_v = \frac{v}{(v)} \left[\frac{ms^{-1}}{mm} \right]$$

Acceleration scale

$$\kappa_a = \frac{a}{(a)} \left[\frac{ms^{-2}}{mm} \right]$$

FORCE SCALE



Real value

F= 100 N

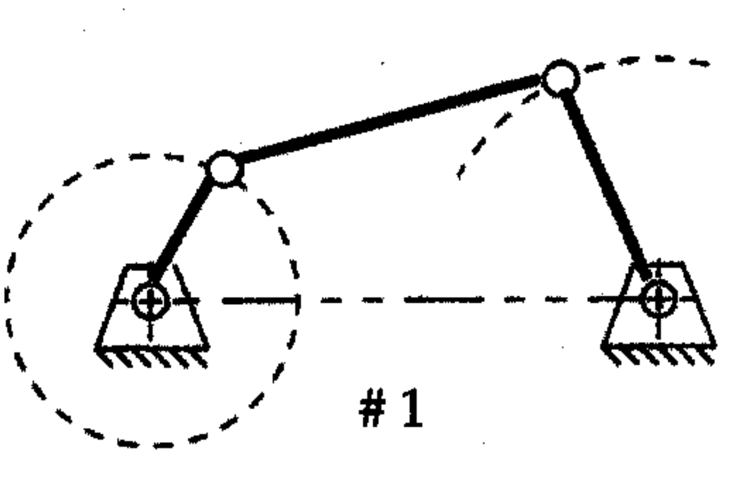
Graphical value

(F)= 100 mm

Force scale:

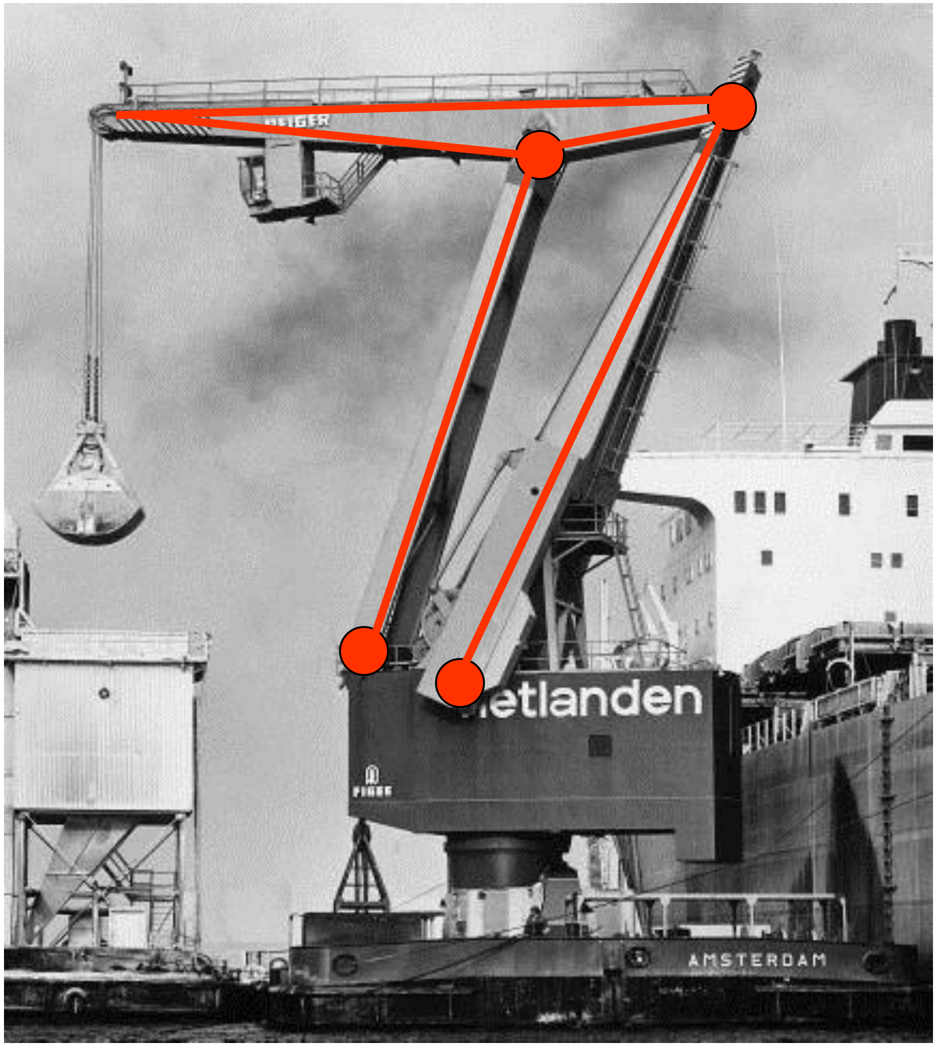
$$\kappa_F = \frac{F}{(F)} = \frac{100}{100} = 1\left[\frac{N}{mm}\right]$$

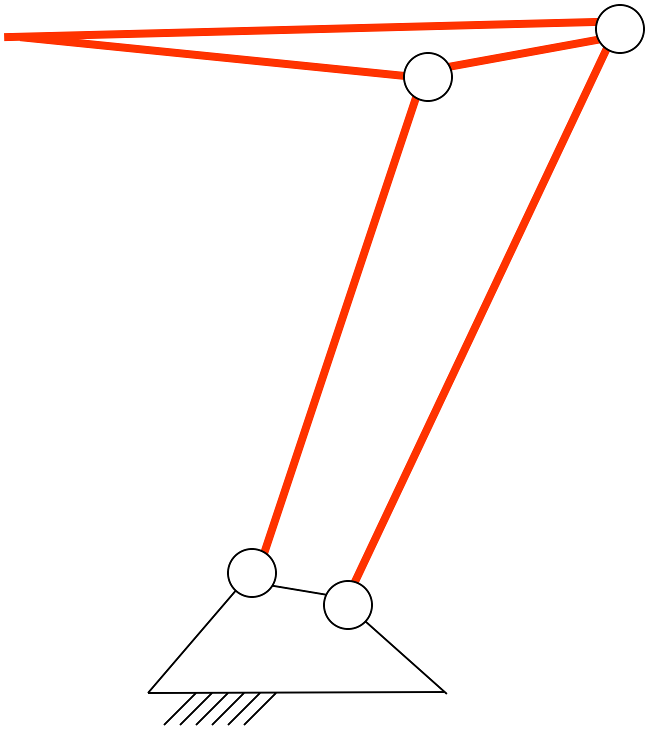
4 BAR LINKAGE



4 bar linkage has a few inversions

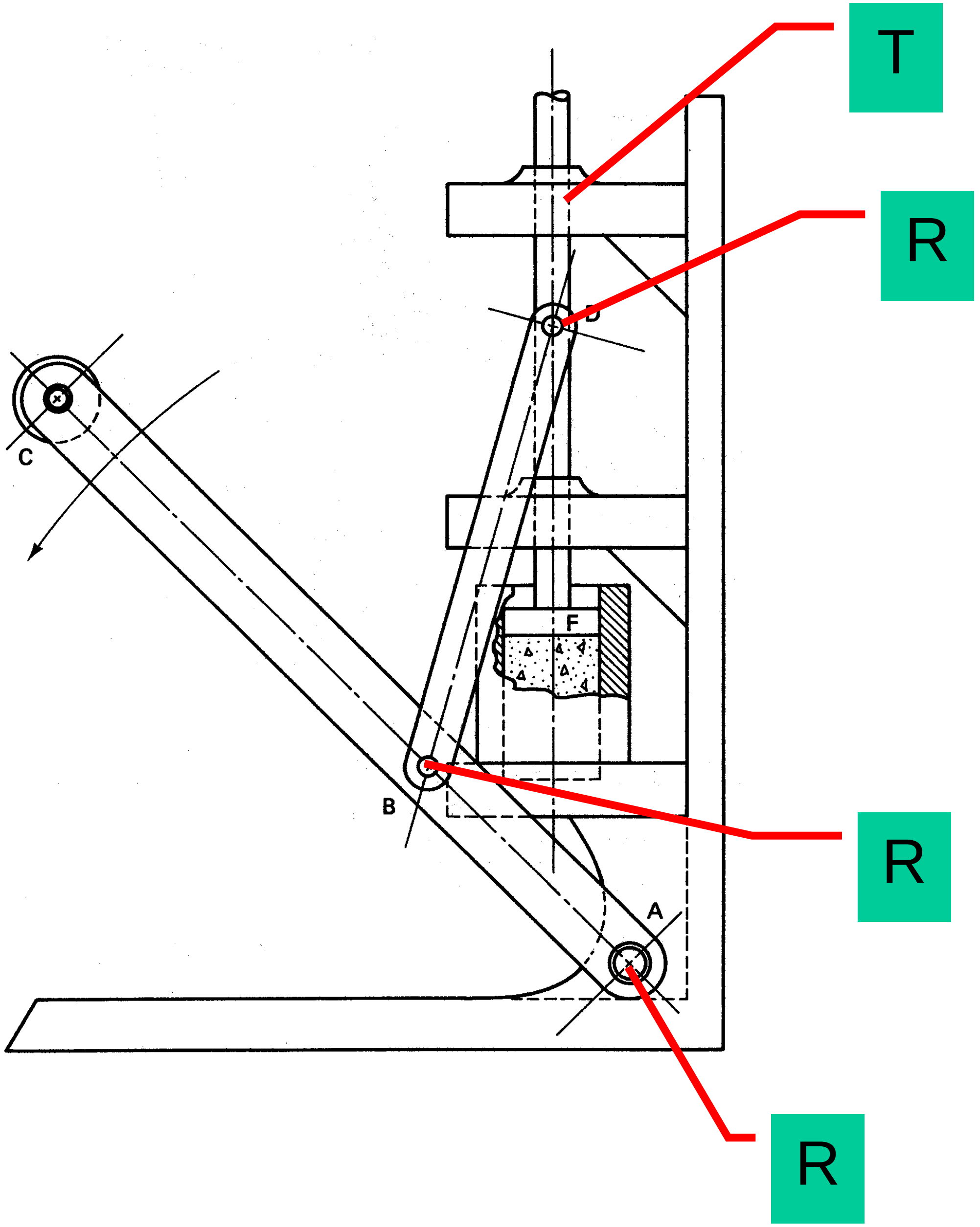
a lot of applications

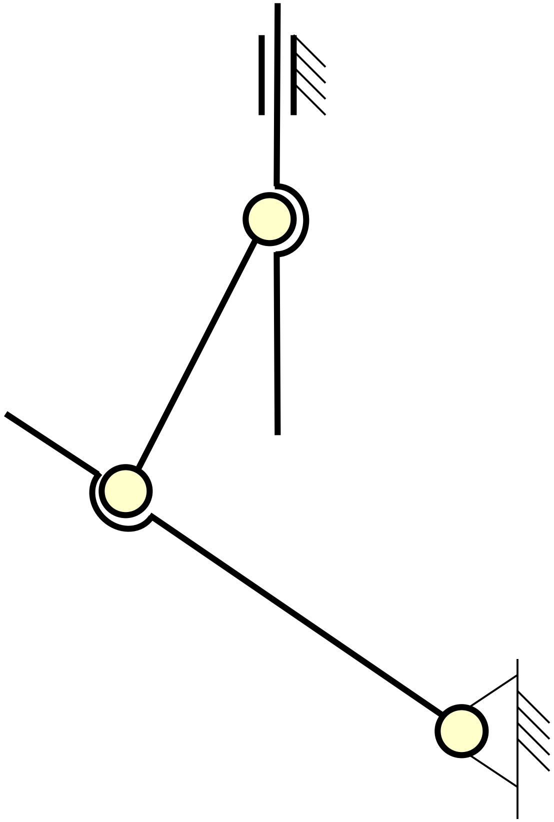


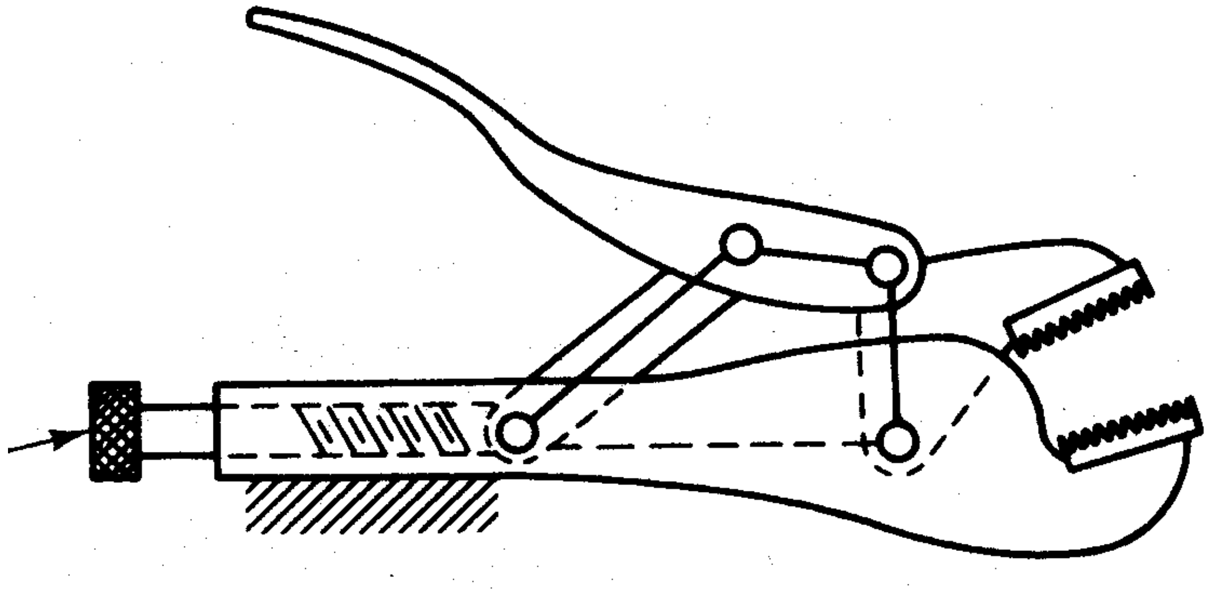


Harbor crane

hand press

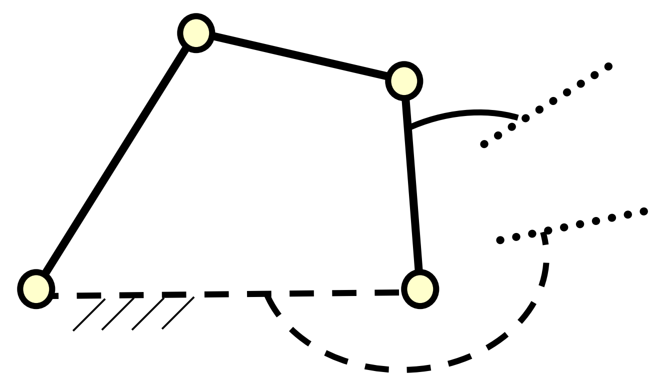


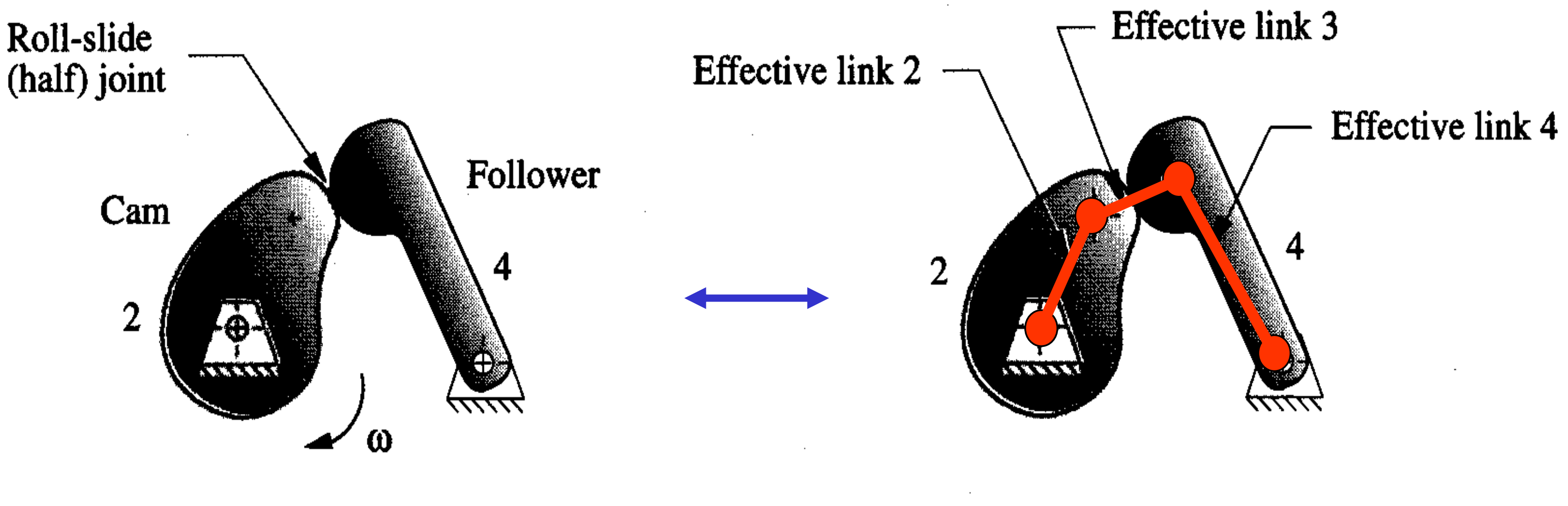




pipe spanner,

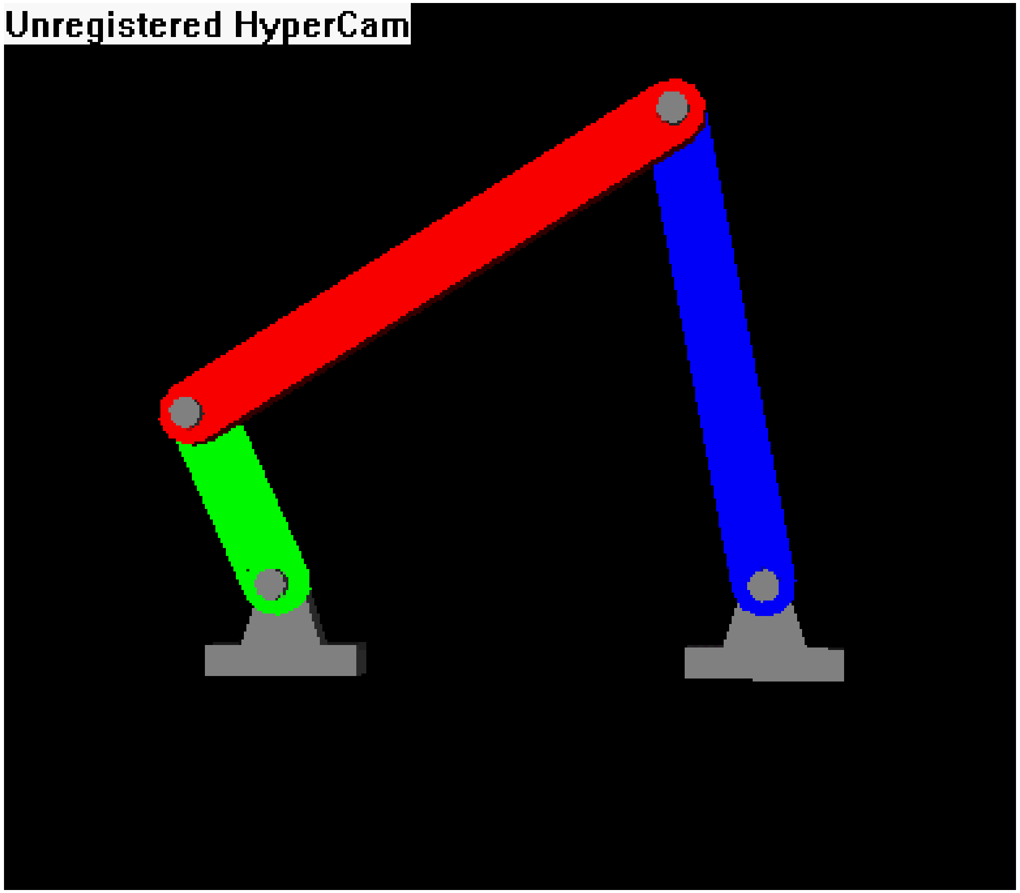
pipe wrench



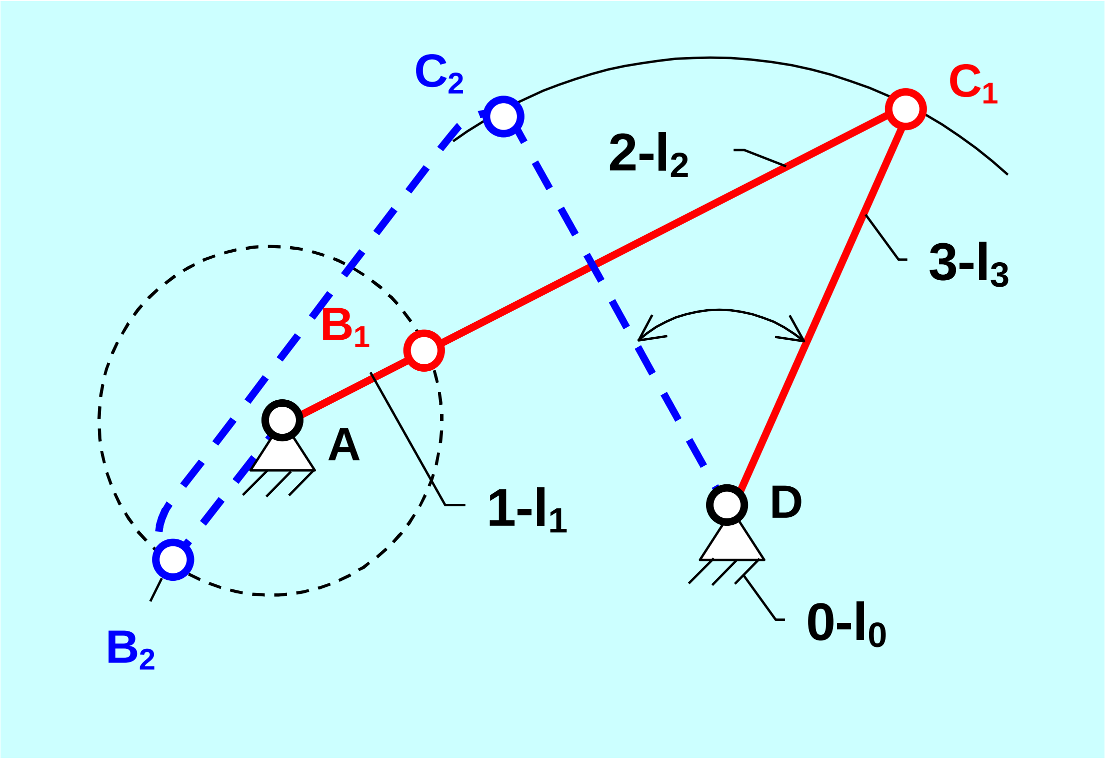


(c) The cam-follower mechanism has an effective fourbar equivalent

Four-bar linkage: crank $\rightarrow$ rocker



4 BAR LINKAGE (4R)



Grashof inequalities

$$\left.\begin{aligned} l_1 + l_0 &\le l_2 + l_3 \\ l_1 + l_2 &\le l_3 + l_0 \\ l_1 + l_3 &\le l_2 + l_0 \end{aligned}\right\}$$

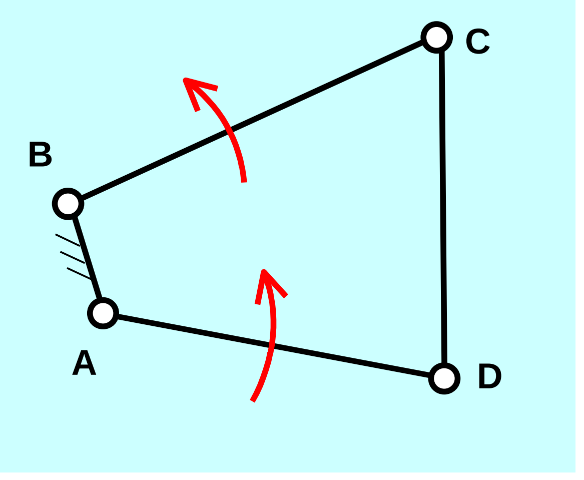
⇓

$$l_1 \langle l_0 \quad l_1 \langle l_2 \quad l_1 \langle l_3$$

For the crank-rocker we have:

1. Grashof inequalities fulfilled,
2. shortest link (l_1) is a crank (rotates around frame)

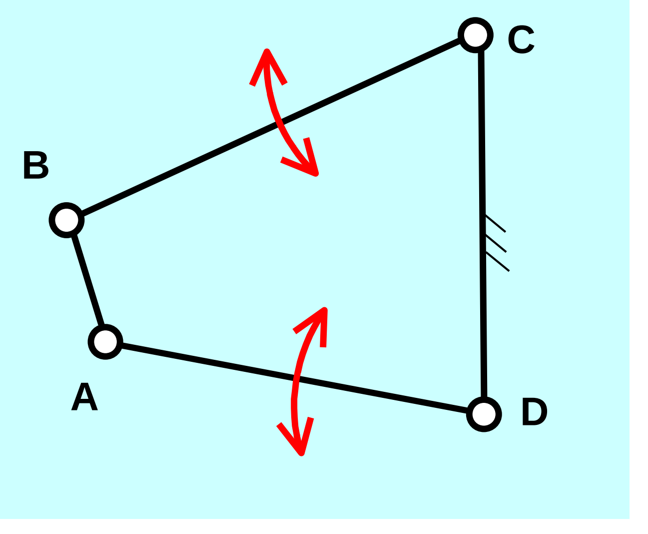
DOUBLE CRANK



Grashof inequalities fulfilled,

shortest link (l_1) is a frame

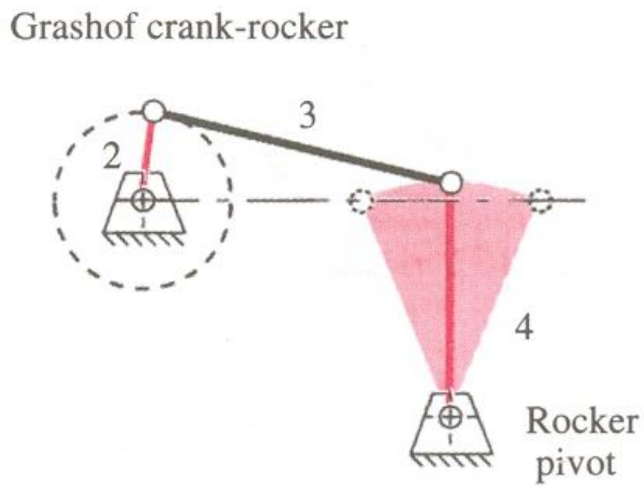
DOUBLE ROCKER

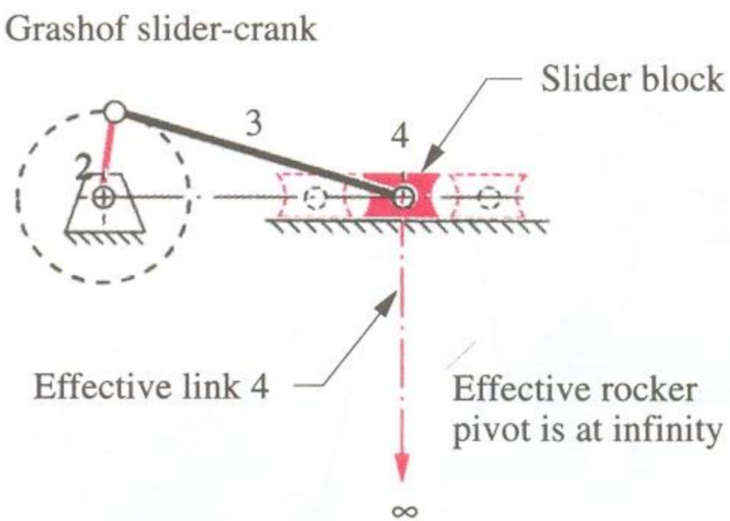


Grashof inequalities fulfilled,

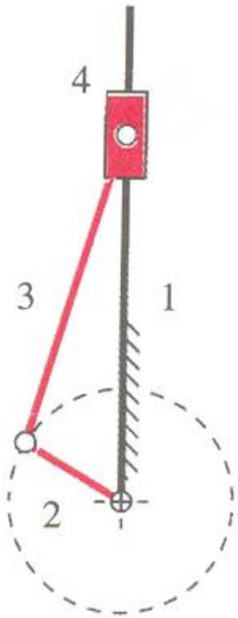
shortest link (l_1) is a coupler

Transforming a fourbar crank-rocker to crank-slider



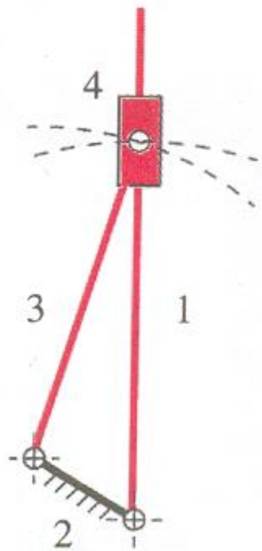


4 BAR - crank-slider - INVERSIONS



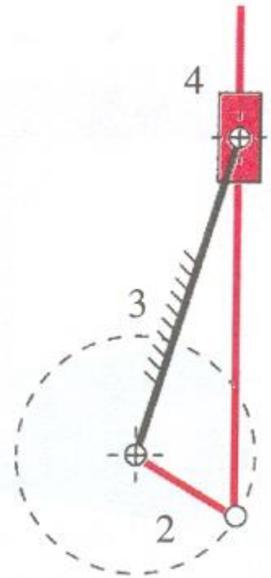
R → T

Slider block translates



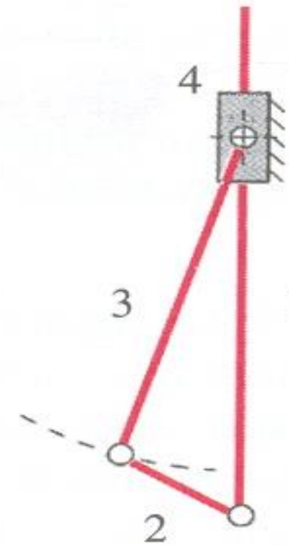
R → R

Slider block has complex motion



R → R

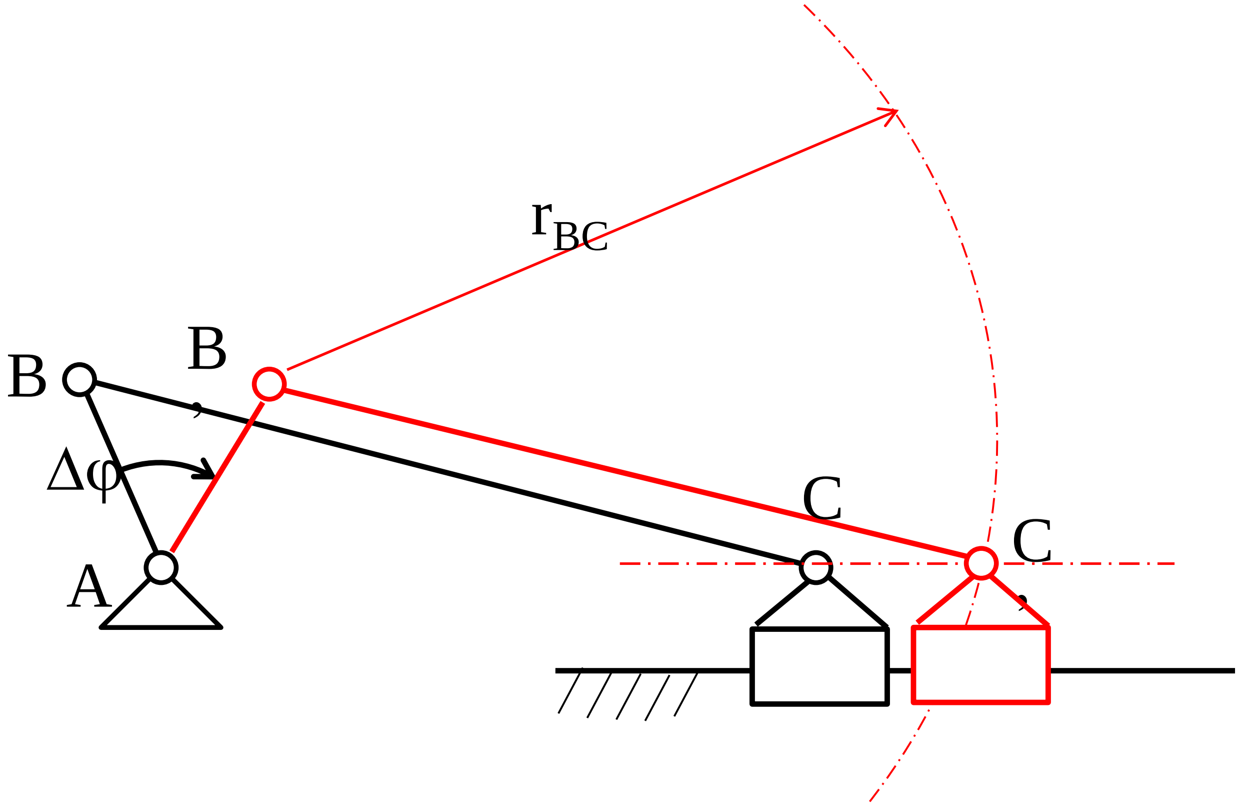
Slider block rotates



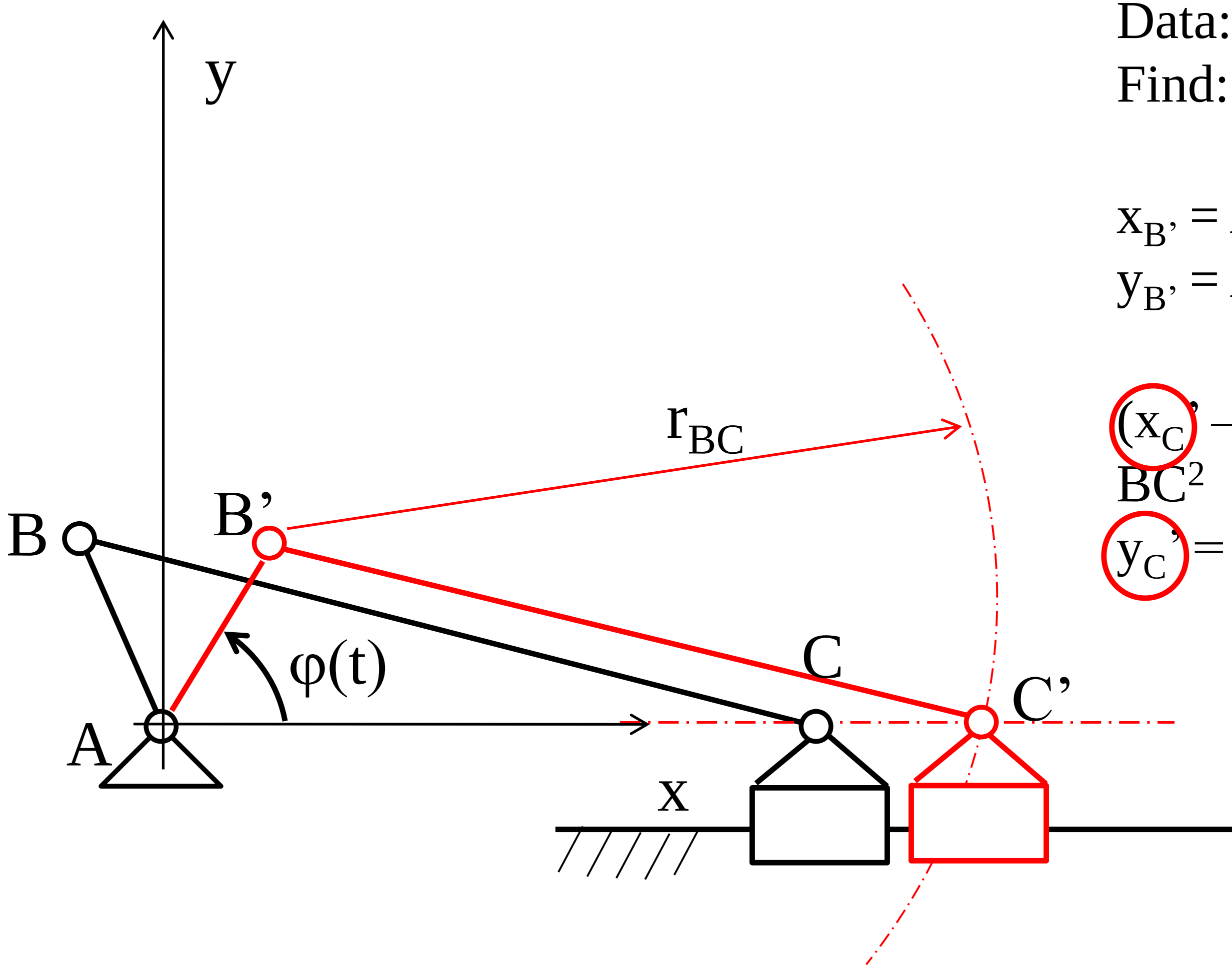
R → T

Slider block is stationary

GRAPHICAL POSITION ANALYSIS



GRAPHICAL POSITION ANALYSIS



Data: $\varphi(t)$

Find: x_c', y_c'

$x_{B'} = AB \cos\varphi$

$y_{B'} = AB \sin\varphi$

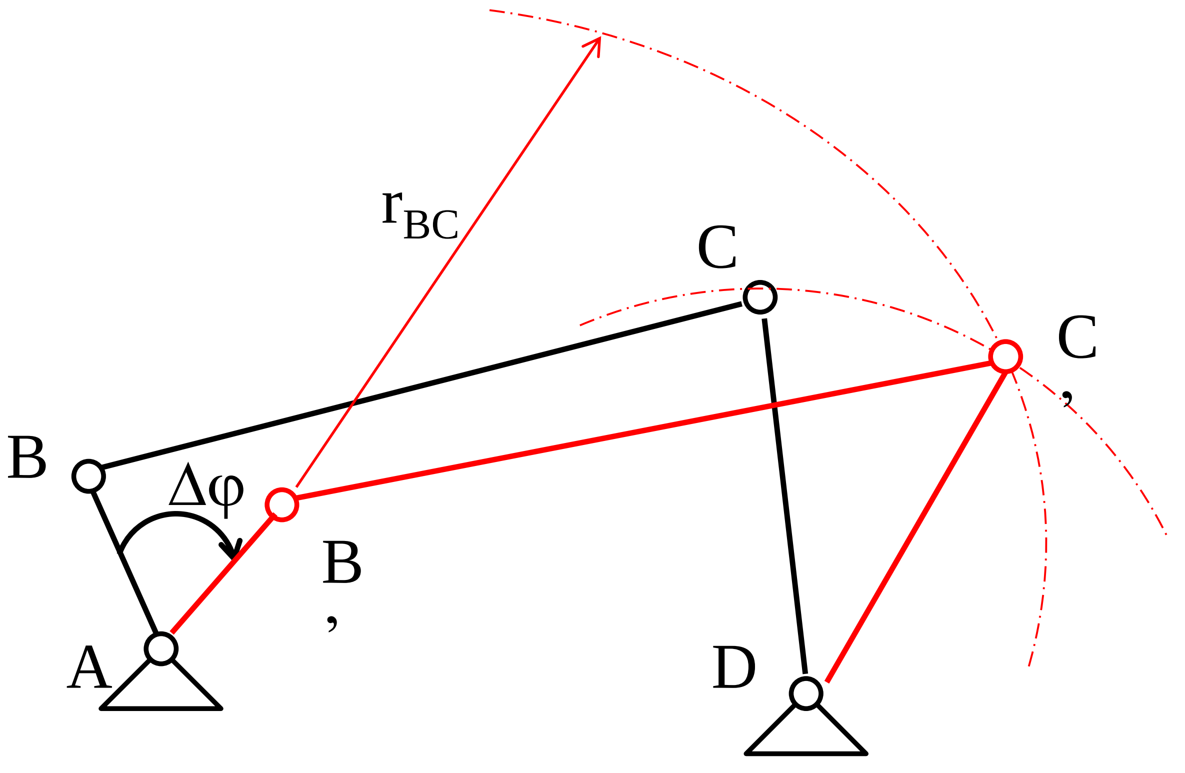
$(x_C' - x_B')^2 + (y_C' - y_B')^2 = BC^2$

$y_C' = 0$

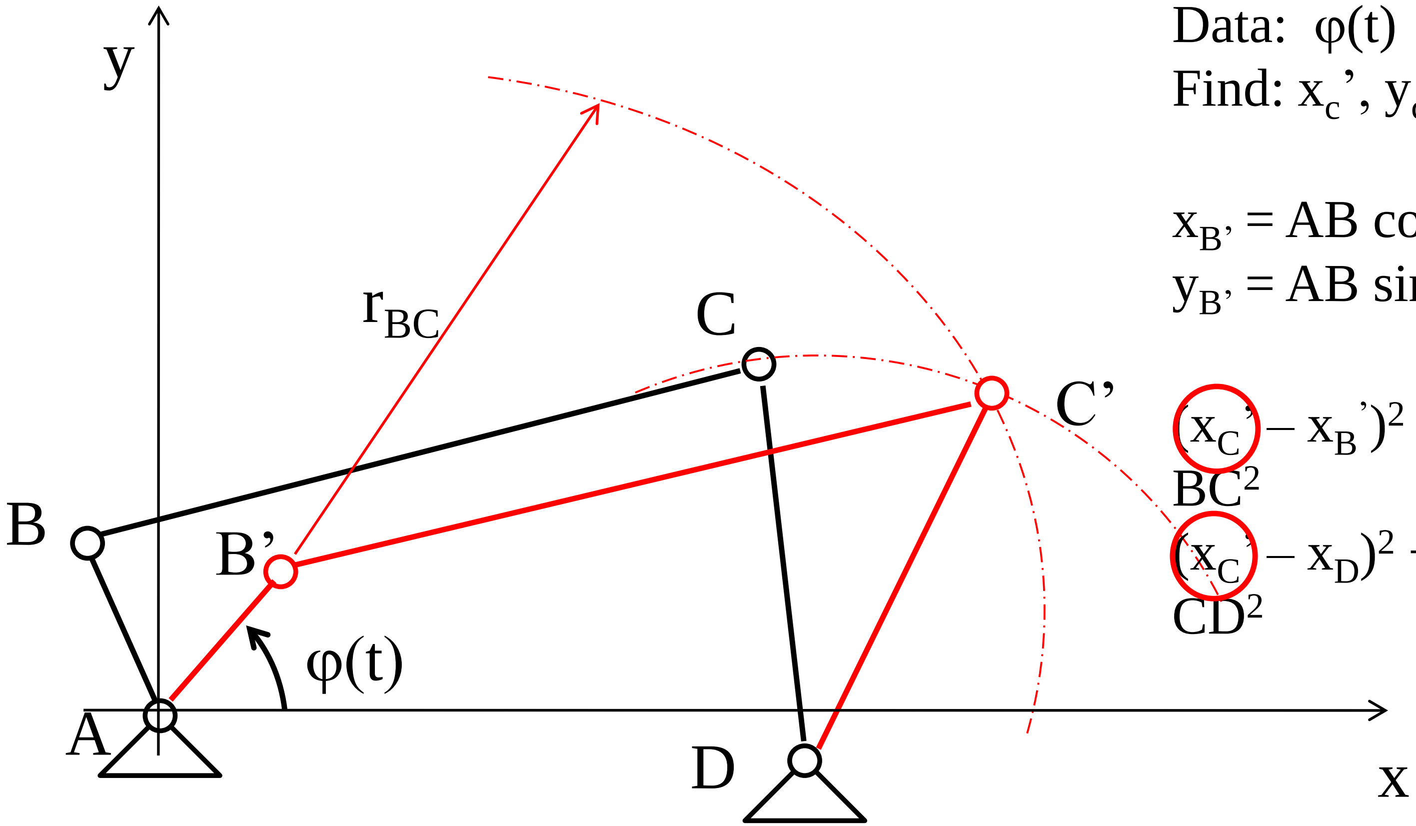
From the system of equations:

x_C' , y_C'

GRAPHICAL POSITION ANALYSIS



GRAPHICAL POSITION ANALYSIS



Data: $\varphi(t)$

Find: x_c', y_c'

$x_{B'} = AB \cos \varphi$

$y_{B'} = AB \sin \varphi$

$(x_C' - x_B')^2 + (y_C' - y_B')^2 = BC^2$

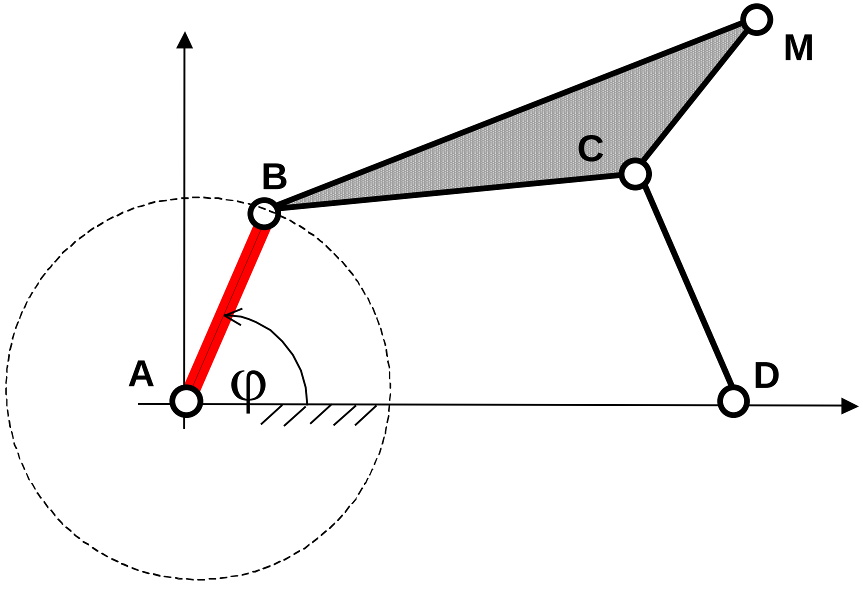
$(x_C' - x_D)^2 + (y_C' - y_D)^2 = CD^2$

From the system of equations : $\mathbf{x_C'}$, $\mathbf{y_C'}$

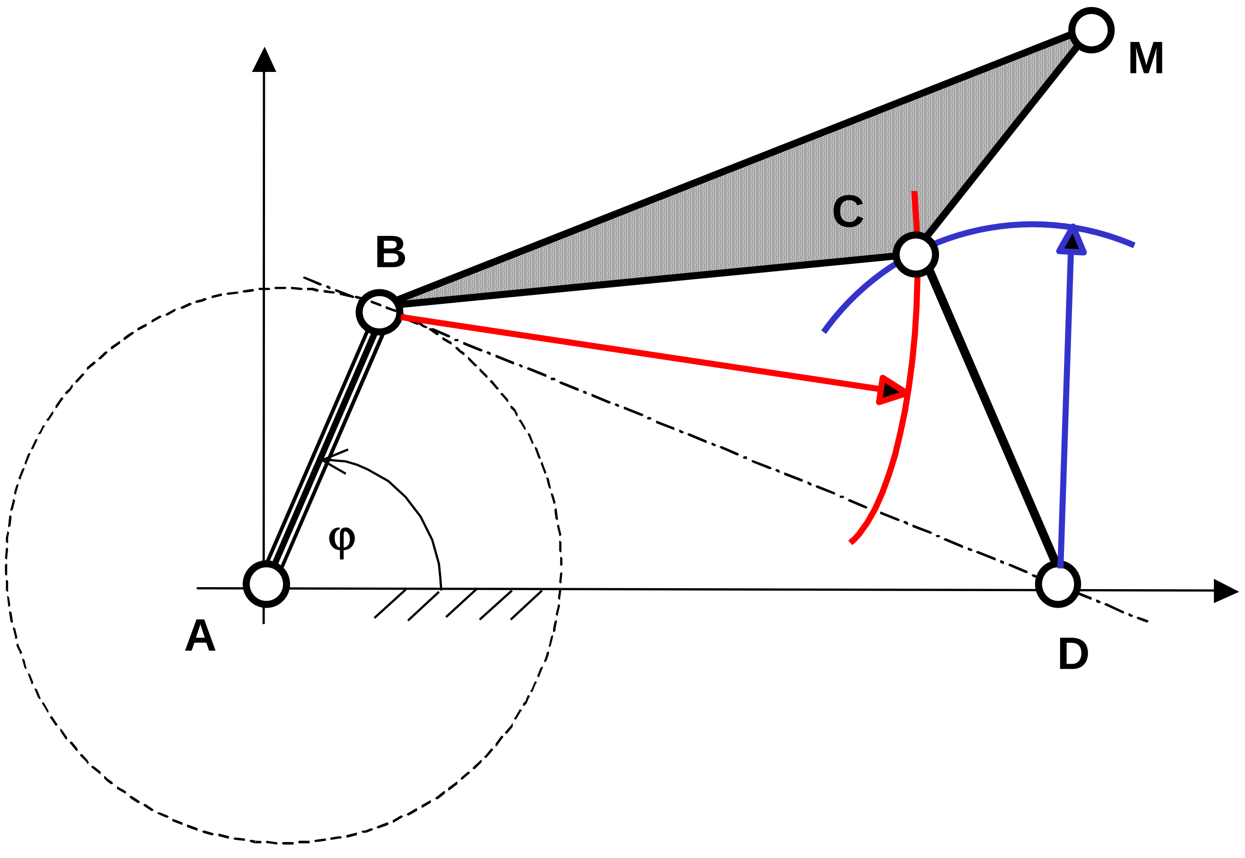
GRAPHICAL POSITION ANALYSIS

Draw a mechanism for a given φ

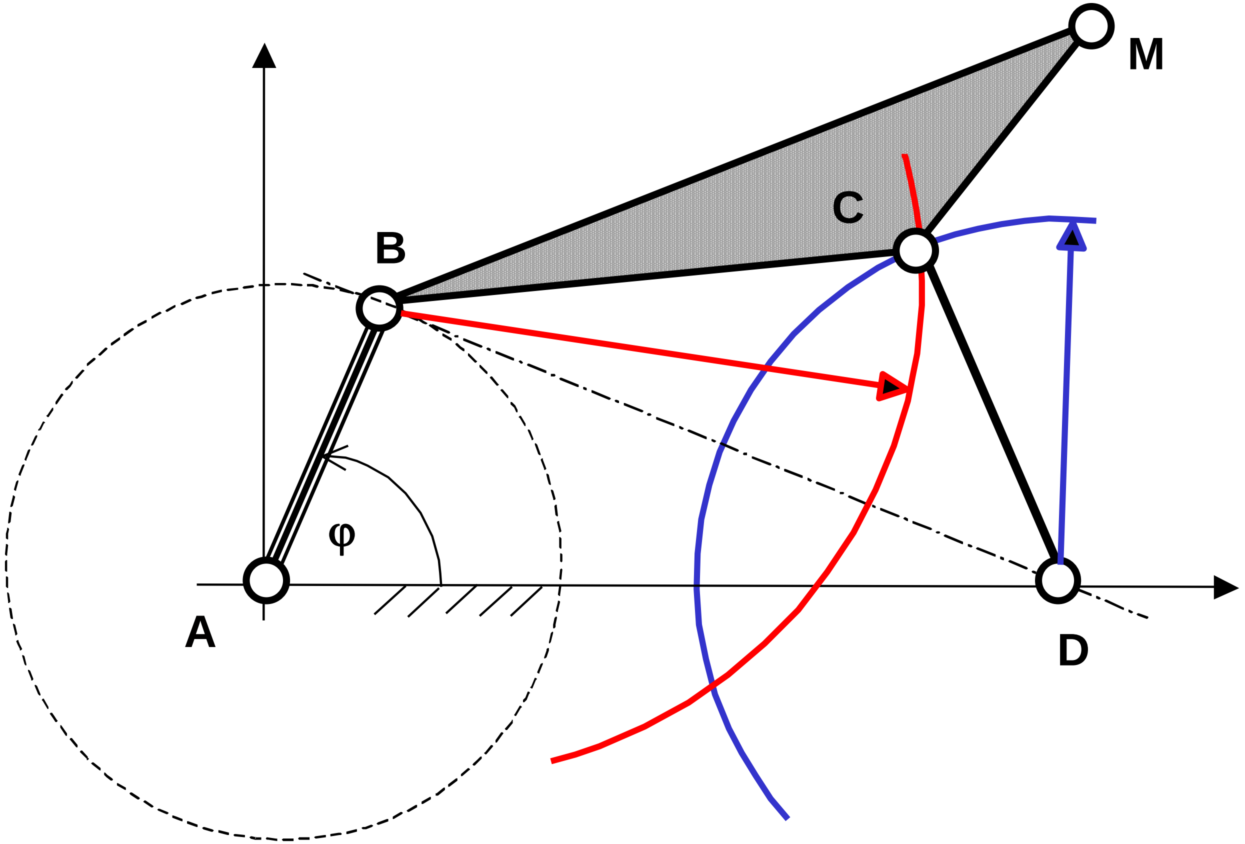
Link dimensions are known: AB, BC, CD, AD and BM, CM



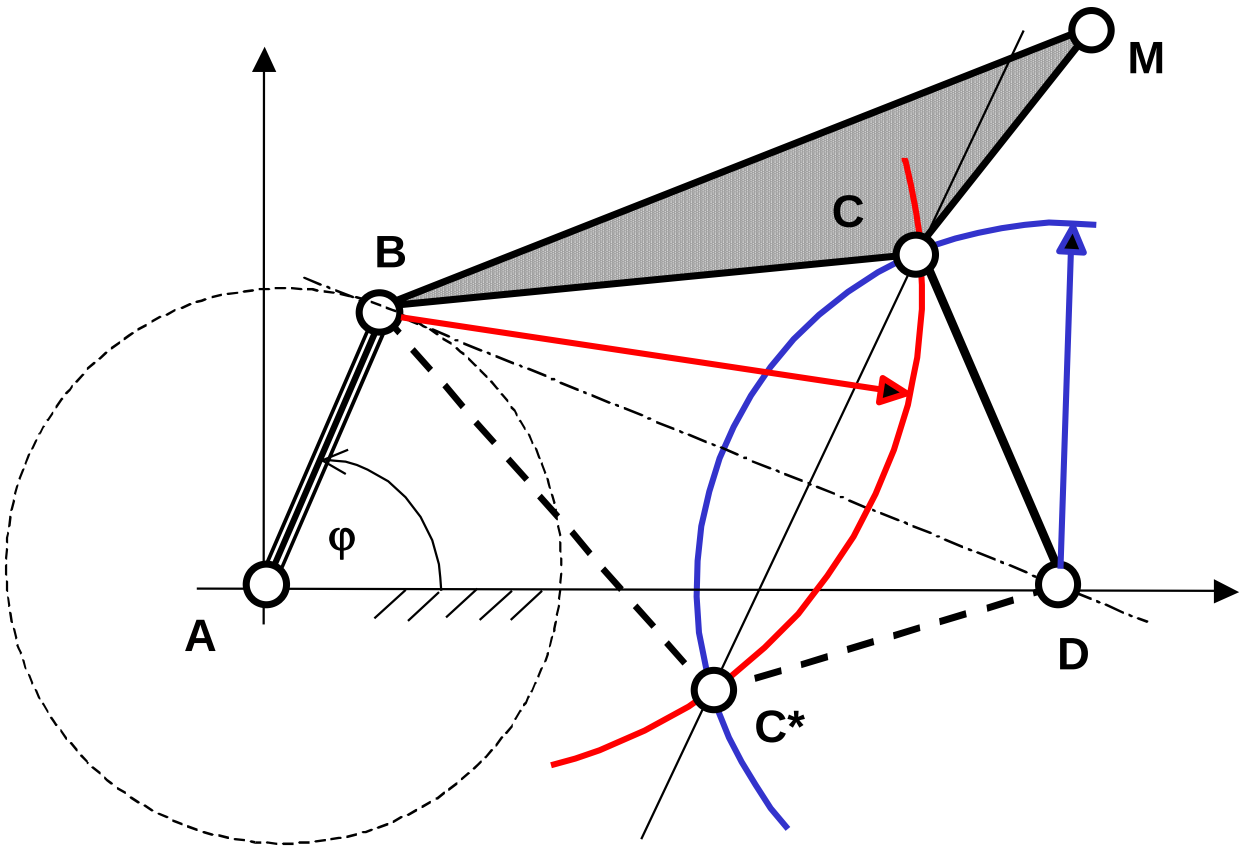
GRAPHICAL POSITION ANALYSIS



GRAPHICAL POSITION ANALYSIS

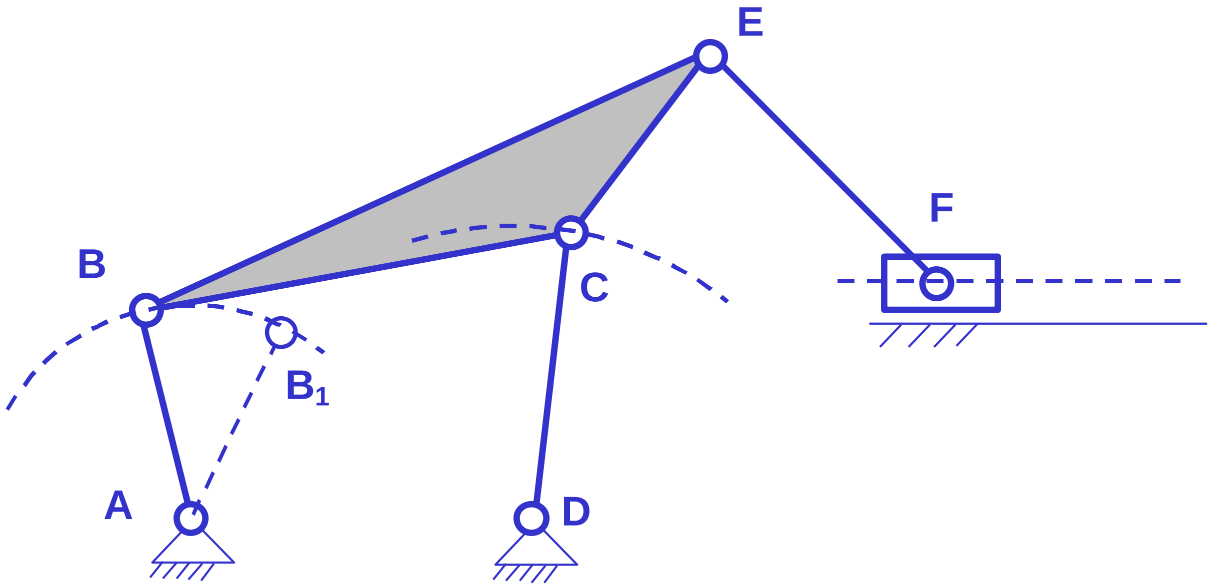


GRAPHICAL POSITION ANALYSIS



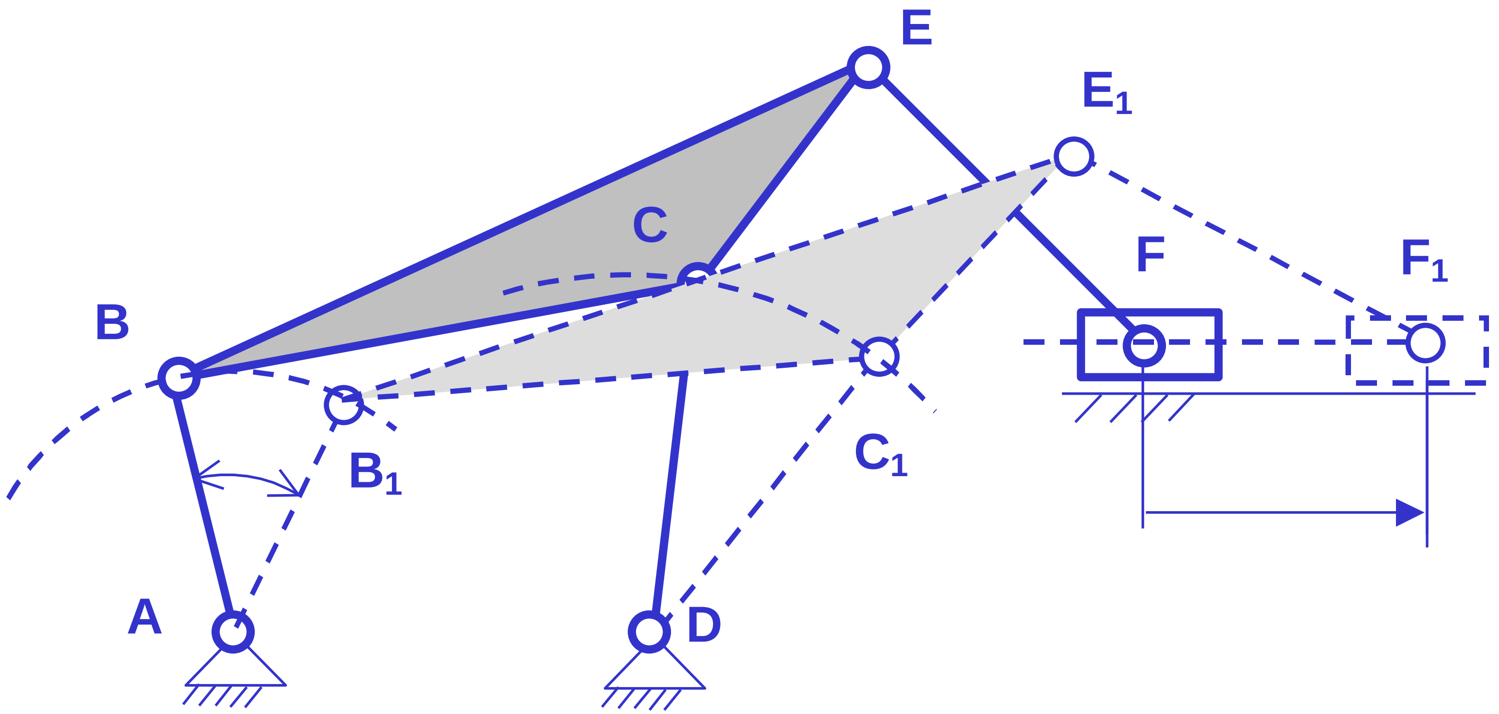
Two positions for given φ

GRAPHICAL POSITION ANALYSIS

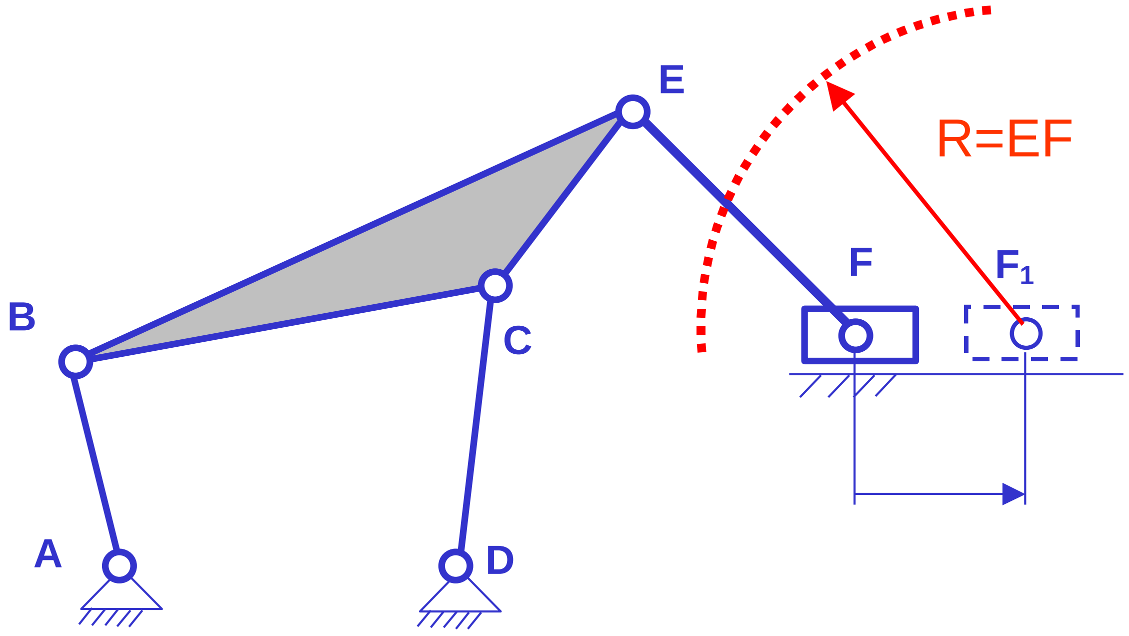


1 driver: AB $\rightarrow$ AB$_1$

GRAPHICAL POSITION ANALYSIS

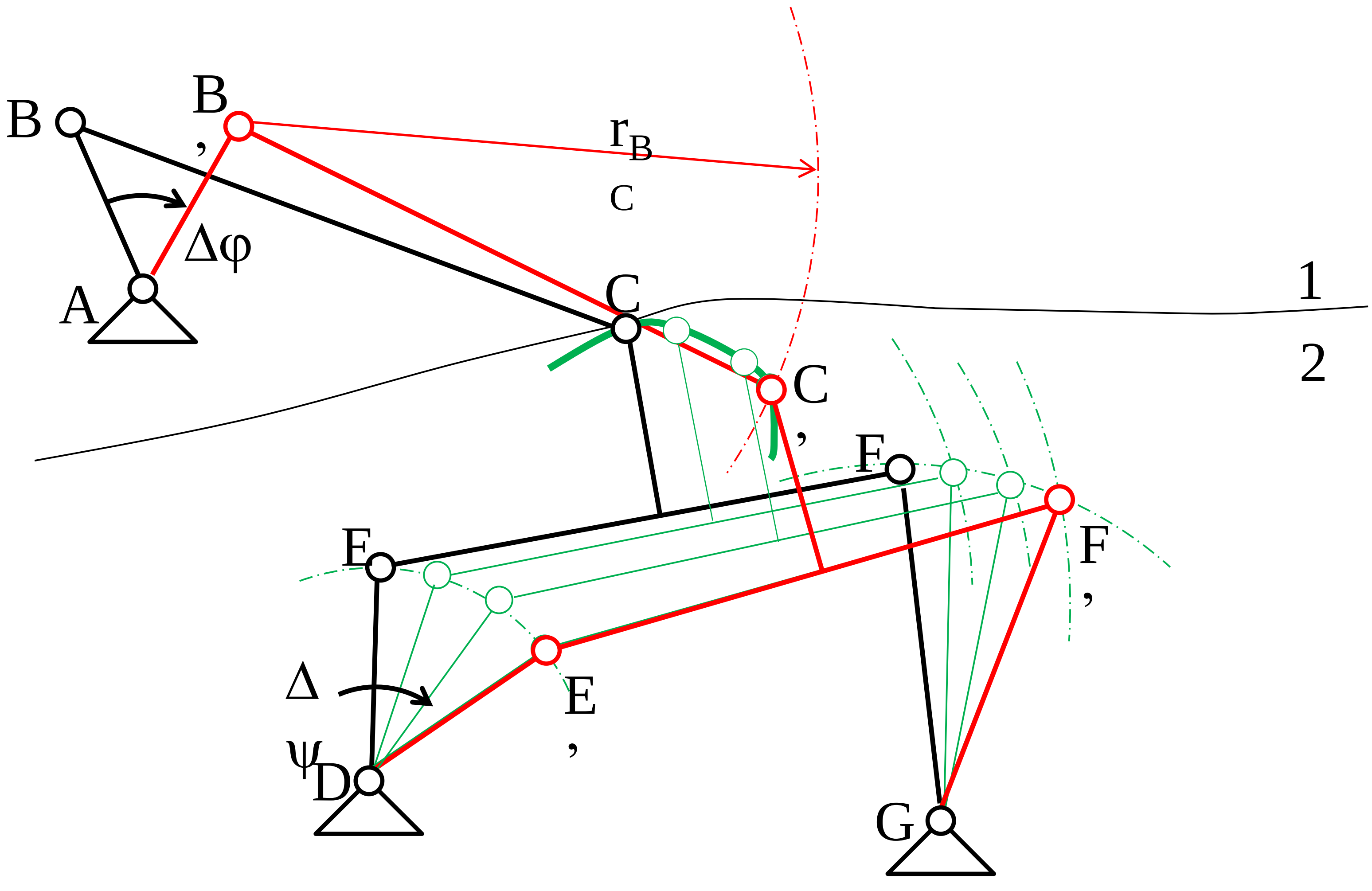


GRAPHICAL POSITION ANALYSIS



1 driver: slider F $\rightarrow$ F_1

GRAPHICAL POSITION ANALYSIS - III class mechanism



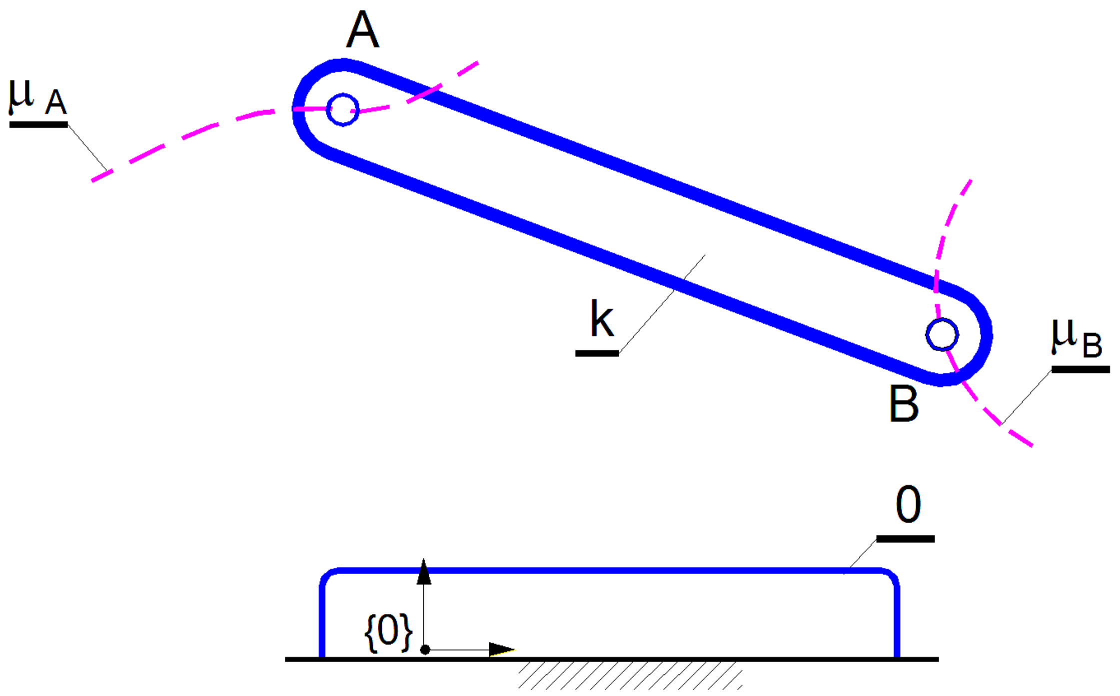
Kinematics – graphical methods

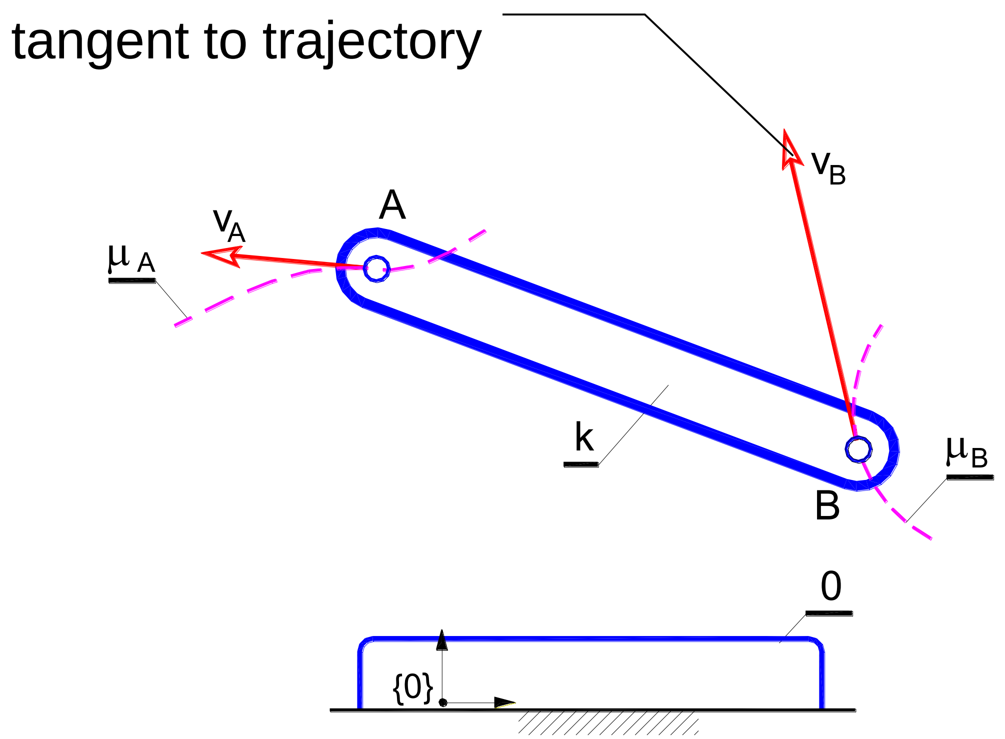
INSTANT CENTERS OF VELOCITY

VELOCITY AND ACCELARATION

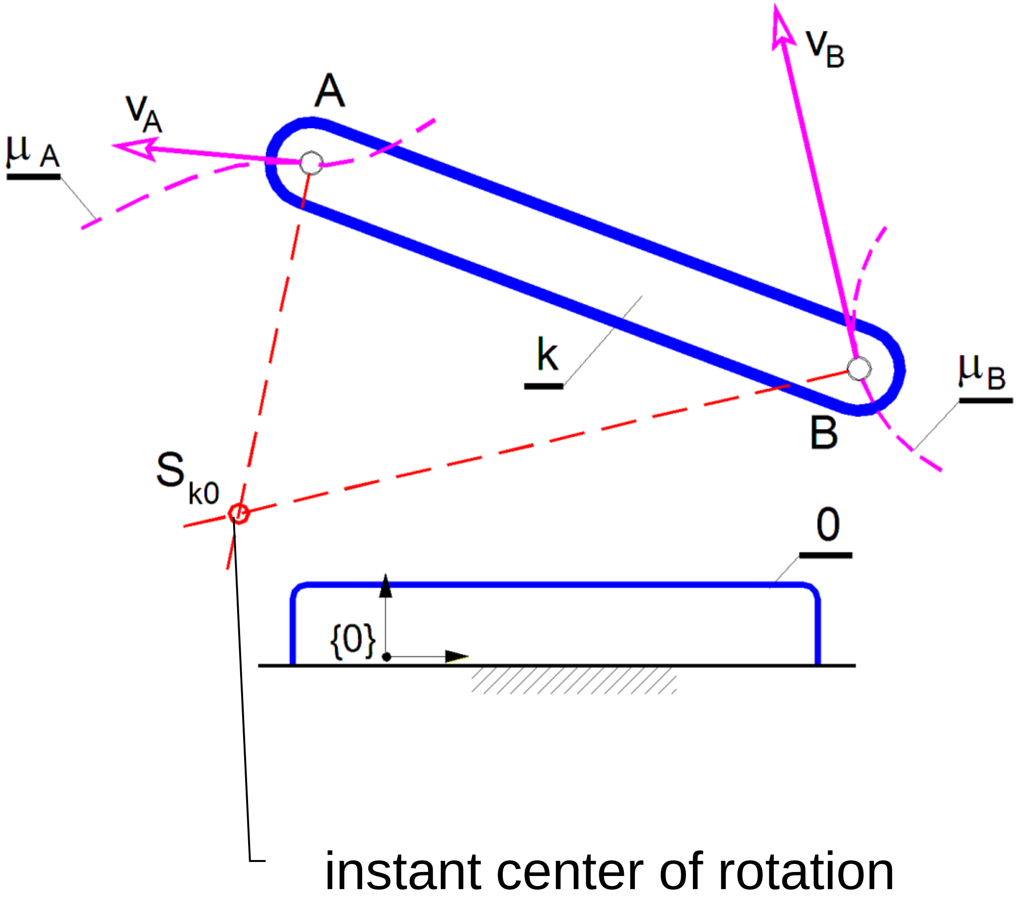
From planar complex motion to rotation

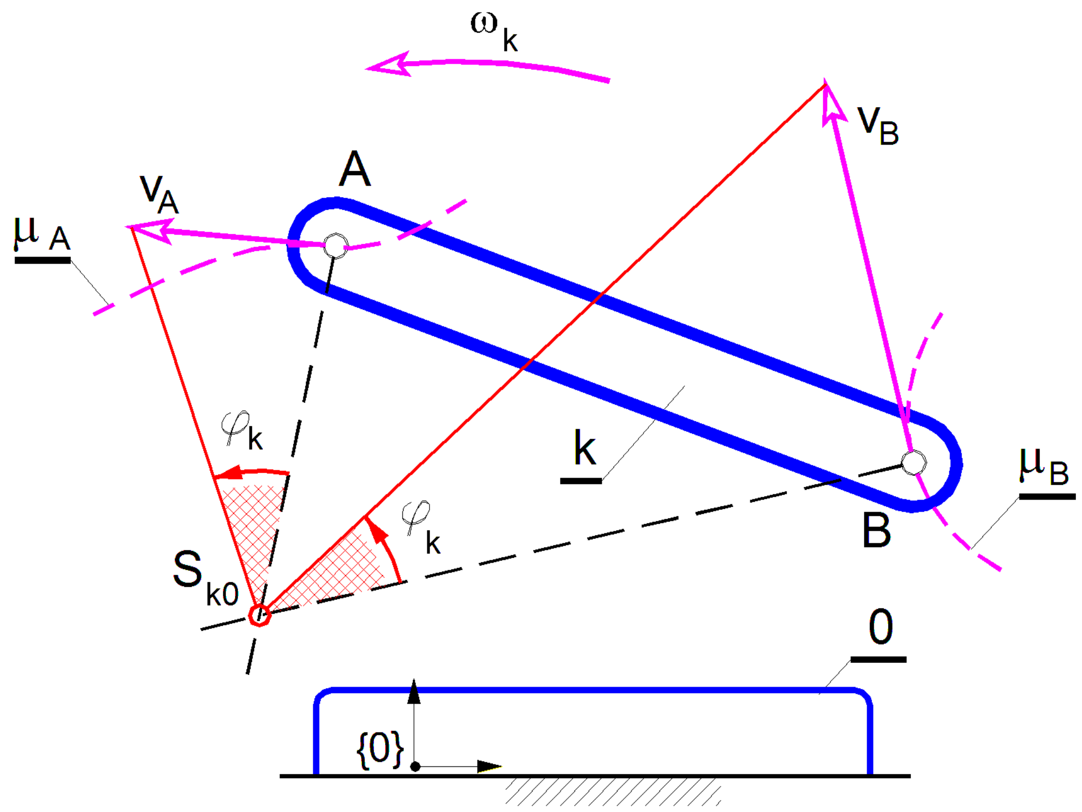
(2 links: frame *0* and moving body *k*)





tangent to trajectory
v_B
v_A
A
μ_A
k
μ_B
B
0
{0}


A
B
v_A
v_B
μ_A
μ_B
k
S_{k0}
0
{0}
instant center of rotation

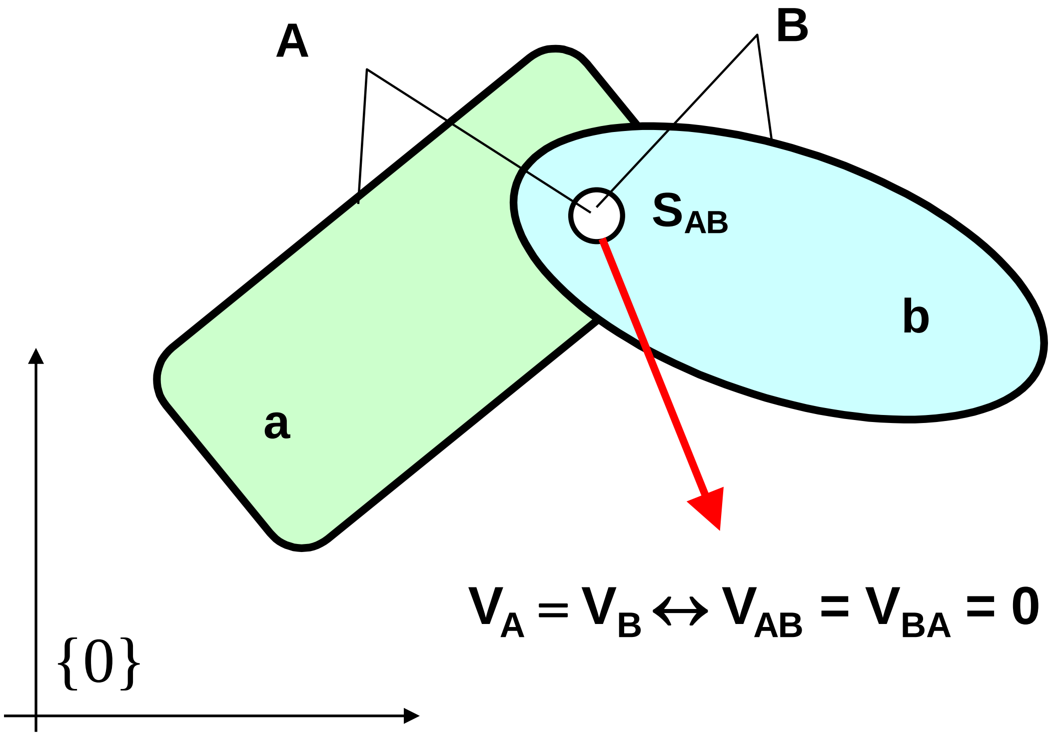


angular velocity:

$$\omega_k = \frac{v_A}{AS_{k0}} = \frac{v_B}{BS_{k0}}$$

$$\frac{(v_A)}{(AS_{k0})} = \frac{(v_B)}{(BS_{k0})} = tg\,\varphi_k$$

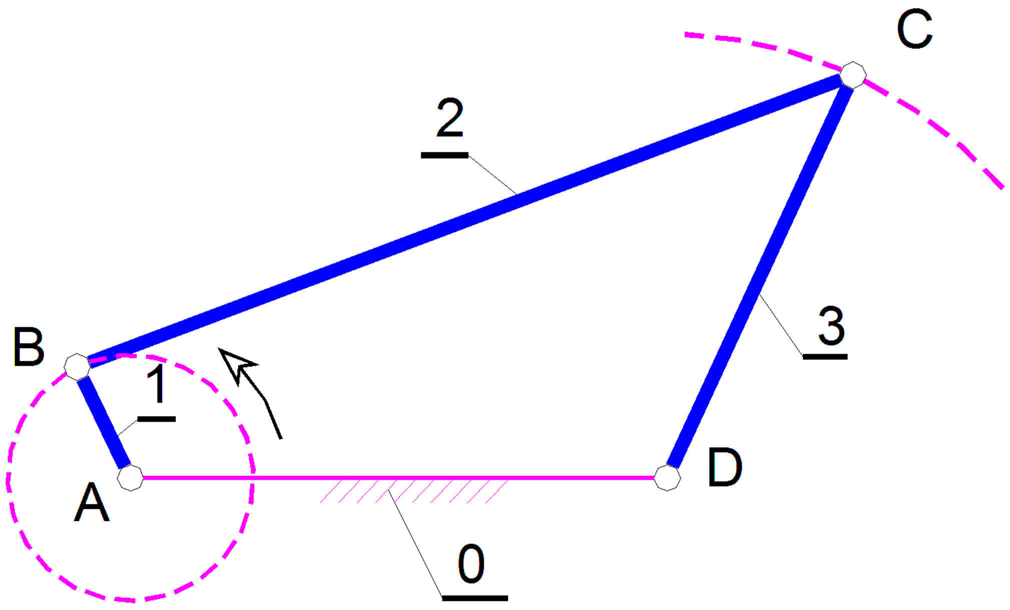
2 bodies in motion



An instant center of velocity is a point, common to two bodies in plane motion, which has the same instantaneous velocity in each body

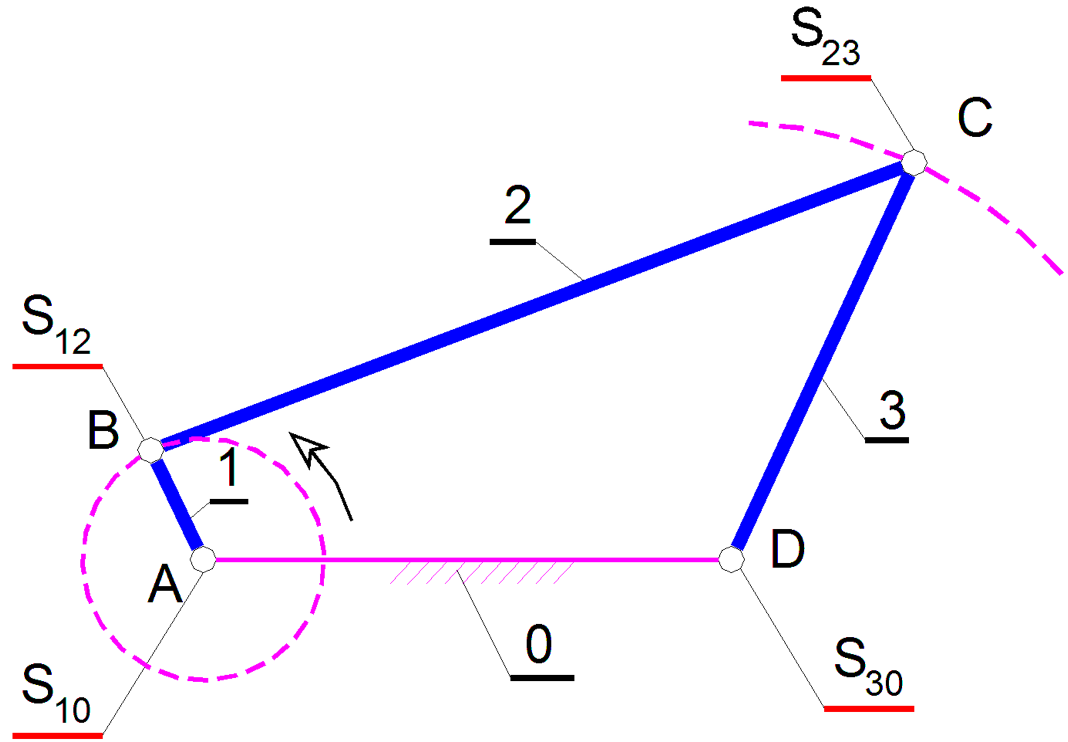
number of IC:

$$i = \binom{n}{2} = \frac{n(n-1)}{2}$$



$$i = \frac{4(4-1)}{2} = 6$$

S_{01}	S_{02}	S_{03}
	S_{12}	S_{13}
		S_{23}



S_{01} S_{03}

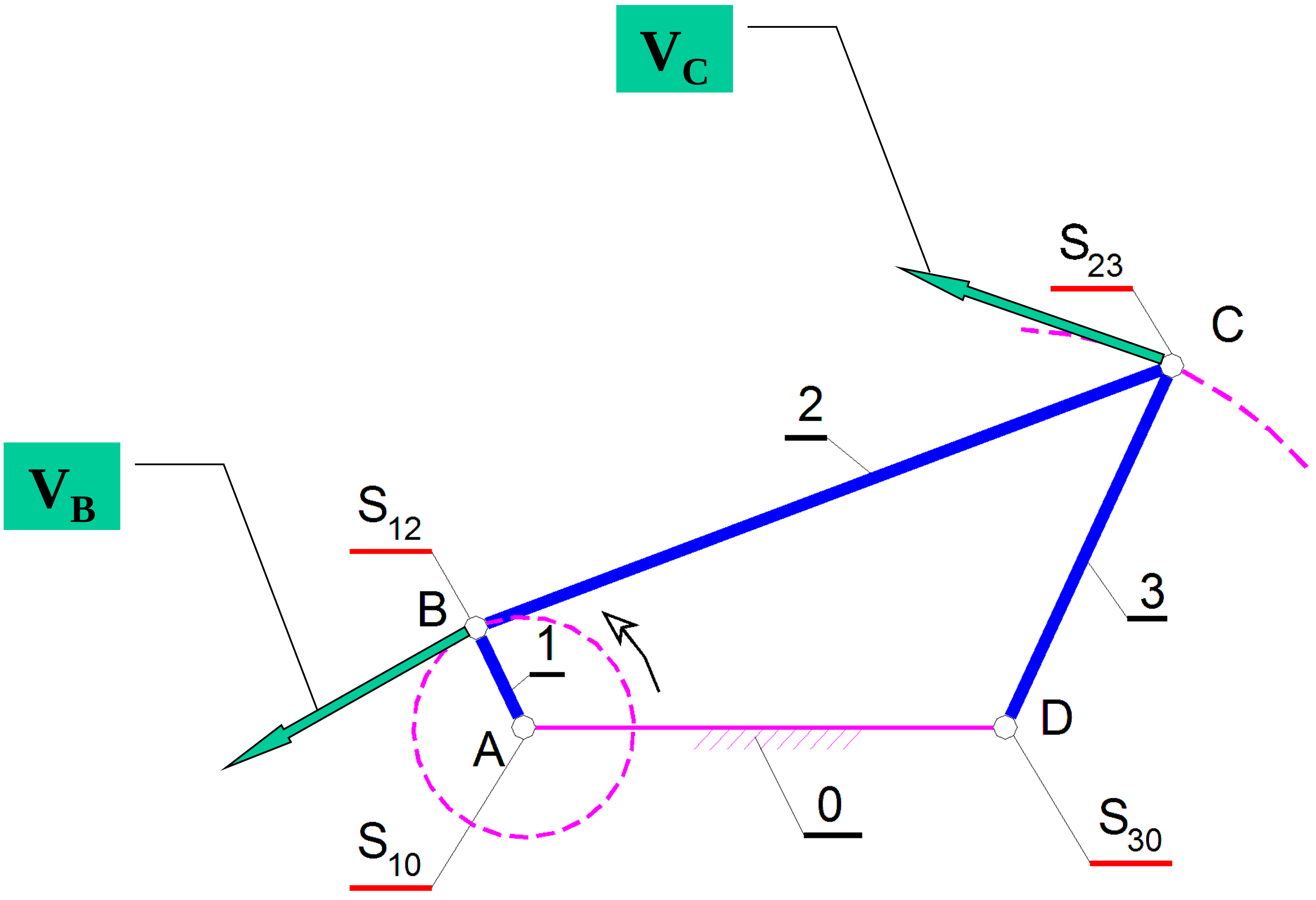
S_{12}

S_{23}

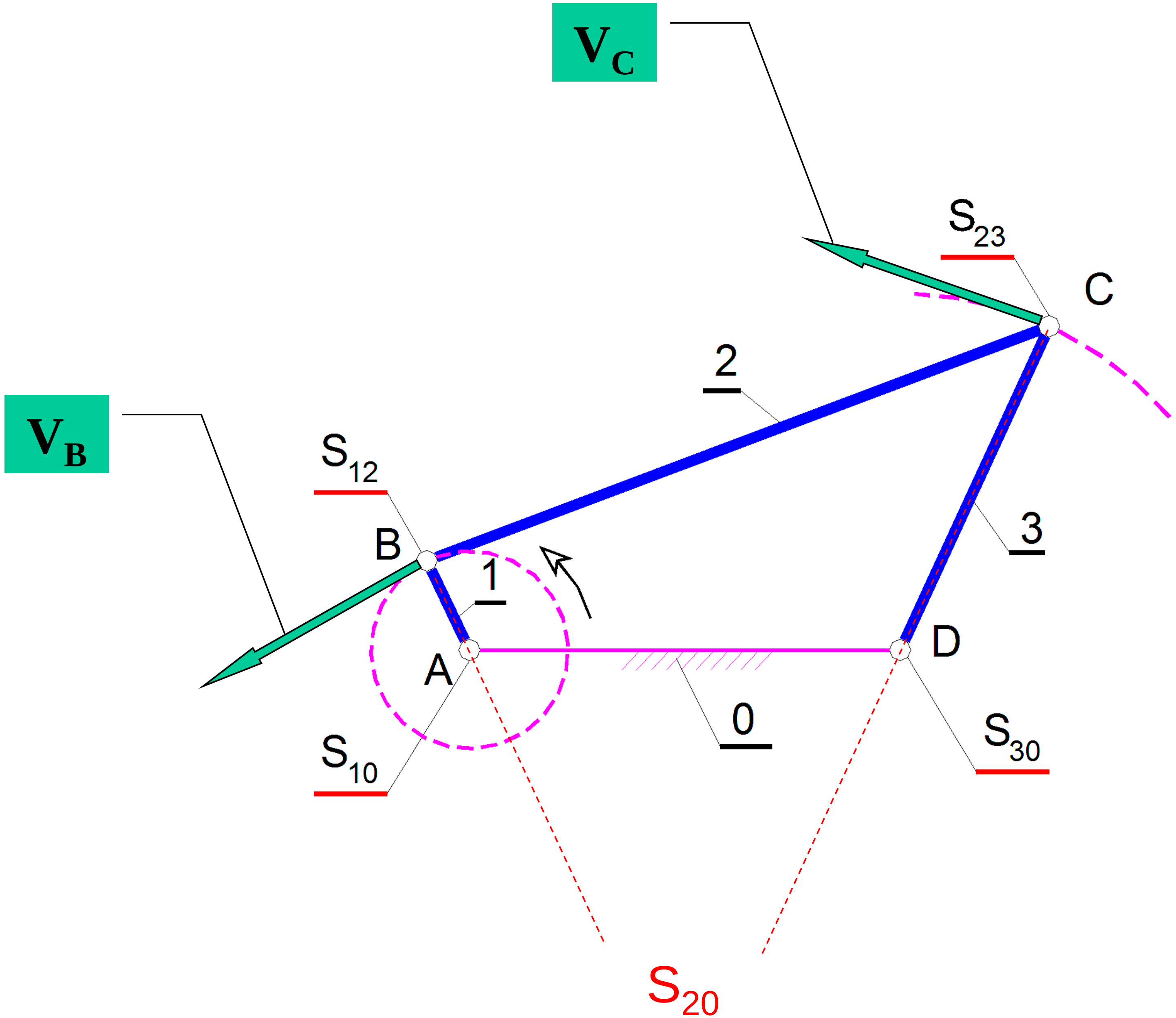
S_{02}

S_{13}

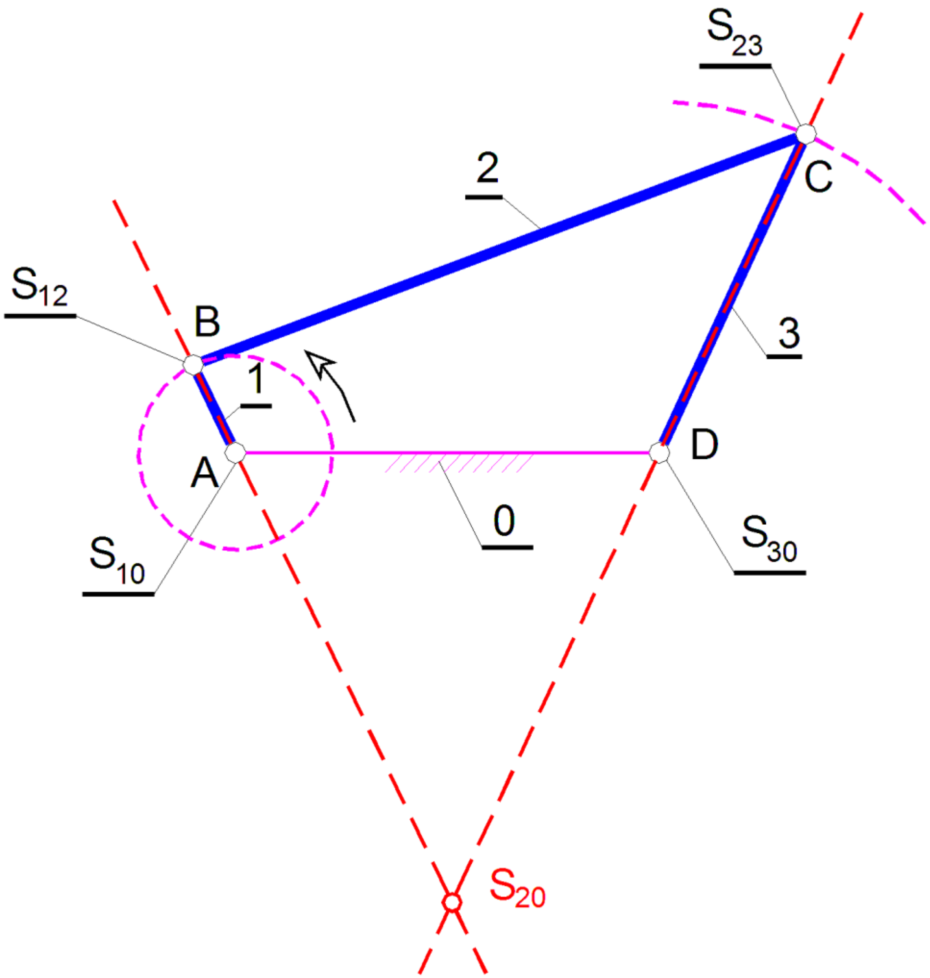
?



V_C
S_{23}
C
2
V_B
S_{12}
B
1
3
D
A
0
S_{10}
S_{30}



V_C
S_{23}
C
2
V_B
S_{12}
B
1
3
D
A
0
S_{10}
S_{30}
S_{20}

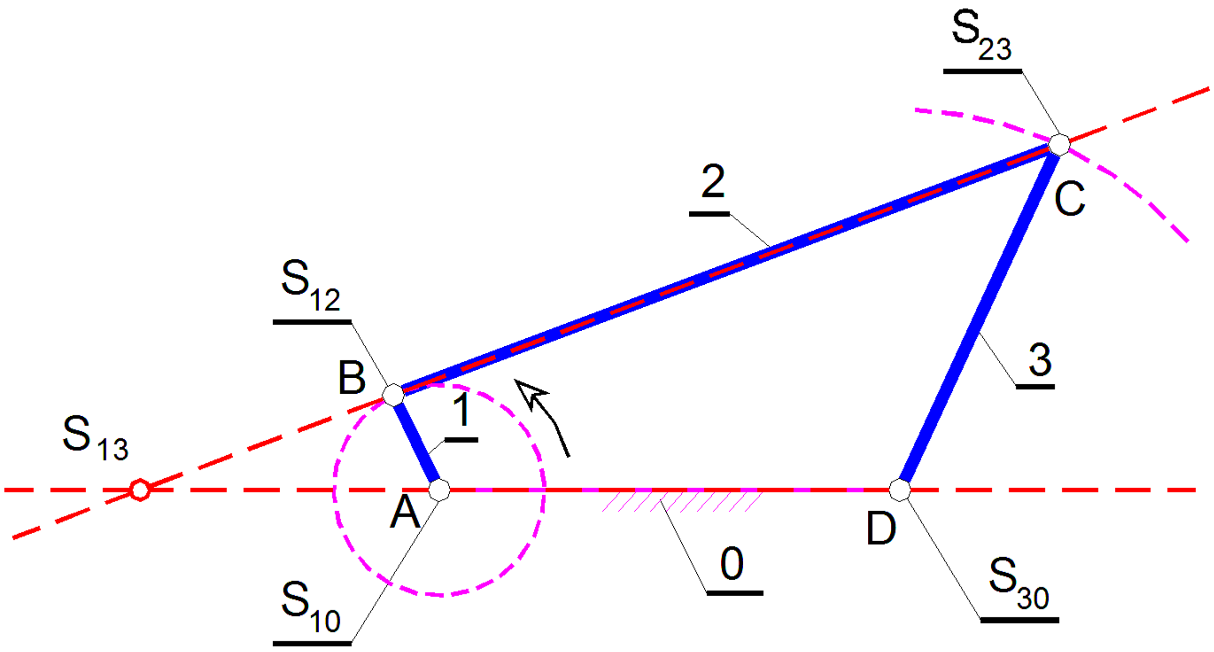


3, 2, 0 $\rightarrow$ S_{32} & $S_{30} \rightarrow S_{20}$

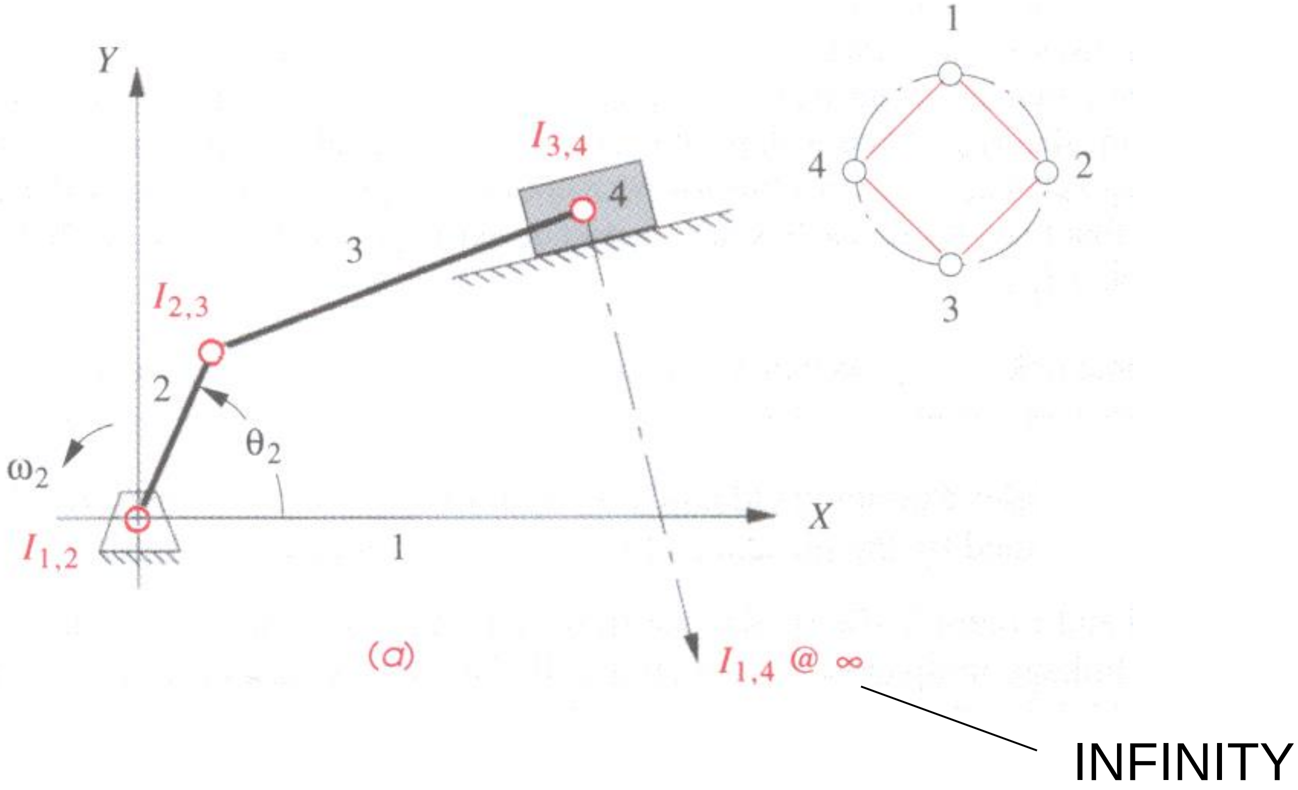
1, 2, 0 $\rightarrow$ S_{12} & $S_{10} \rightarrow S_{20}$

Kennedy's rule:

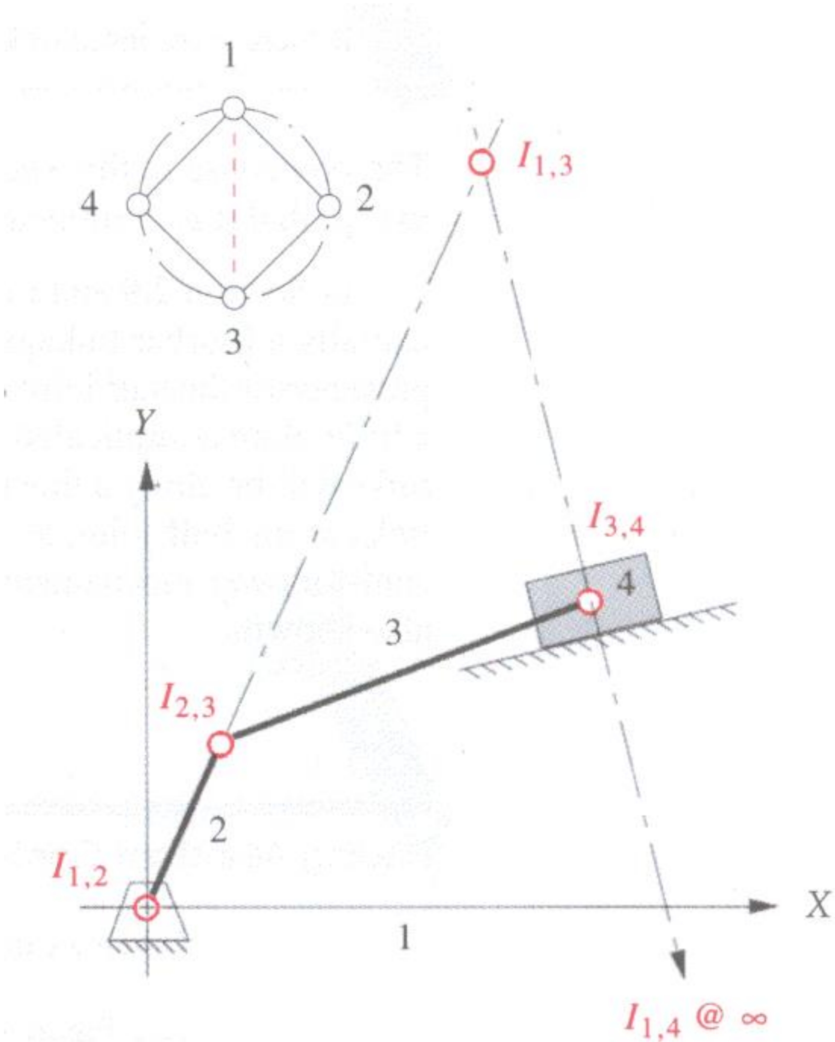
Any three bodies in plane motion will have exactly three instant centers, and ***they will lie on the same straight line****.*

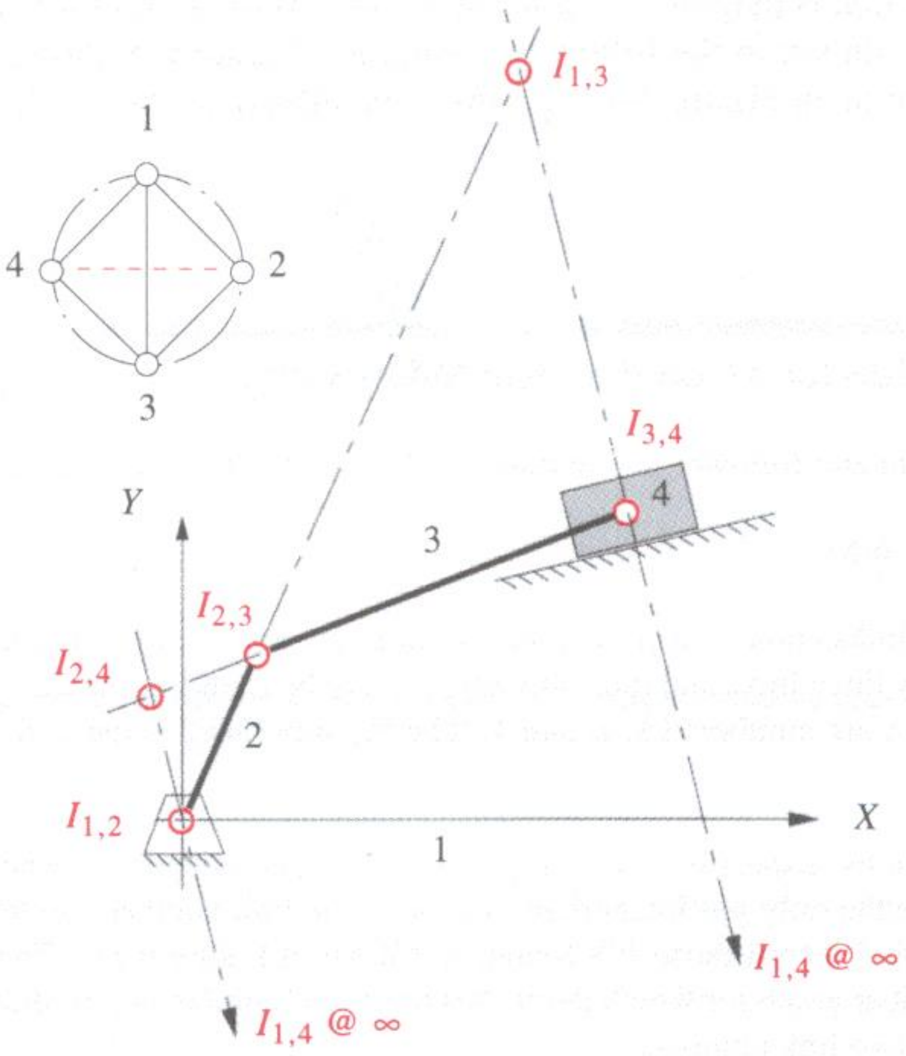


$1, 3, 0 \rightarrow S_{10}$ & $S_{30} \rightarrow S_{31}$



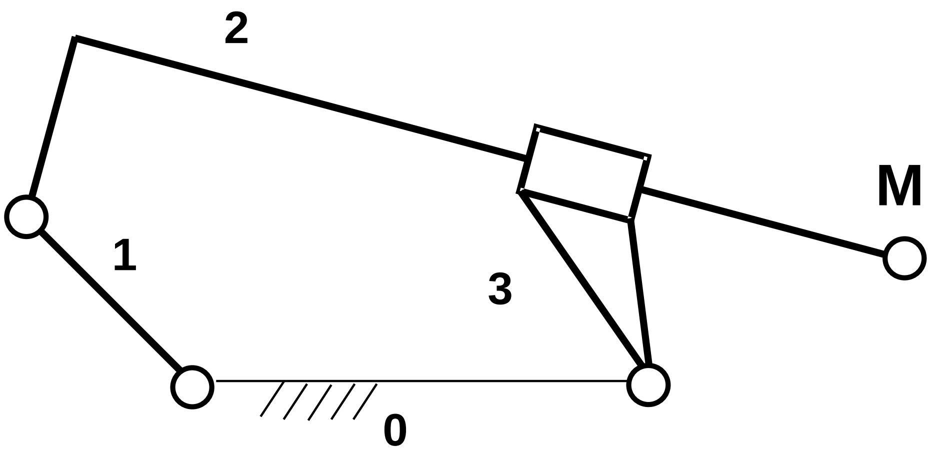
Y
$I_{3,4}$
4
3
$I_{2,3}$
2
ω_2
θ_2
X
$I_{1,2}$
1
(a)
$I_{1,4}$ @ ∞
1
4
2
3
INFINITY


1
2
3
4
$I_{1,3}$
Y
$I_{3,4}$
4
3
$I_{2,3}$
2
$I_{1,2}$
X
1
$I_{1,4}$ @ ∞



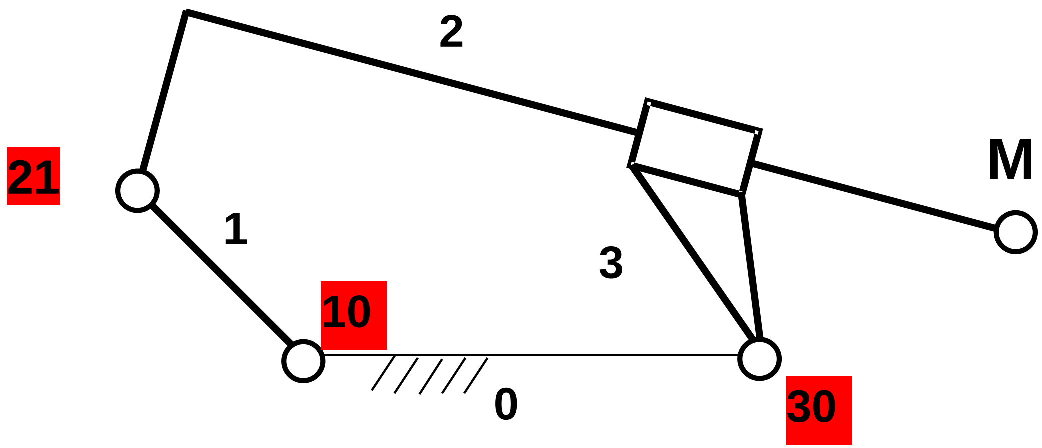
$I_{1,3}$
1
4
2
3
$I_{3,4}$
4
Y
3
$I_{2,3}$
$I_{2,4}$
2
$I_{1,2}$
X
1
$I_{1,4}$ @ ∞
$I_{1,4}$ @ ∞

Find v_M direction

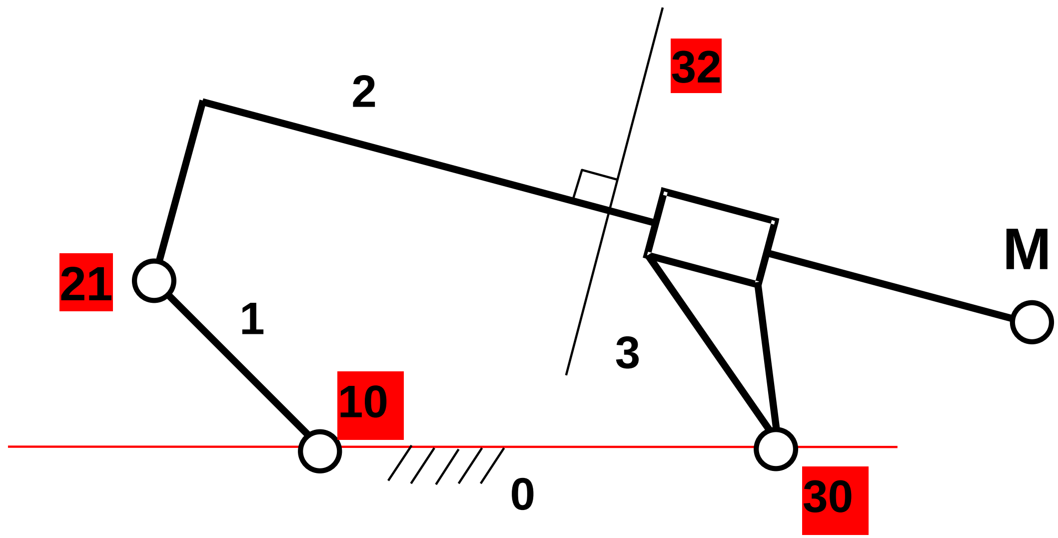


1st method: trajectory of point M and tangent ???

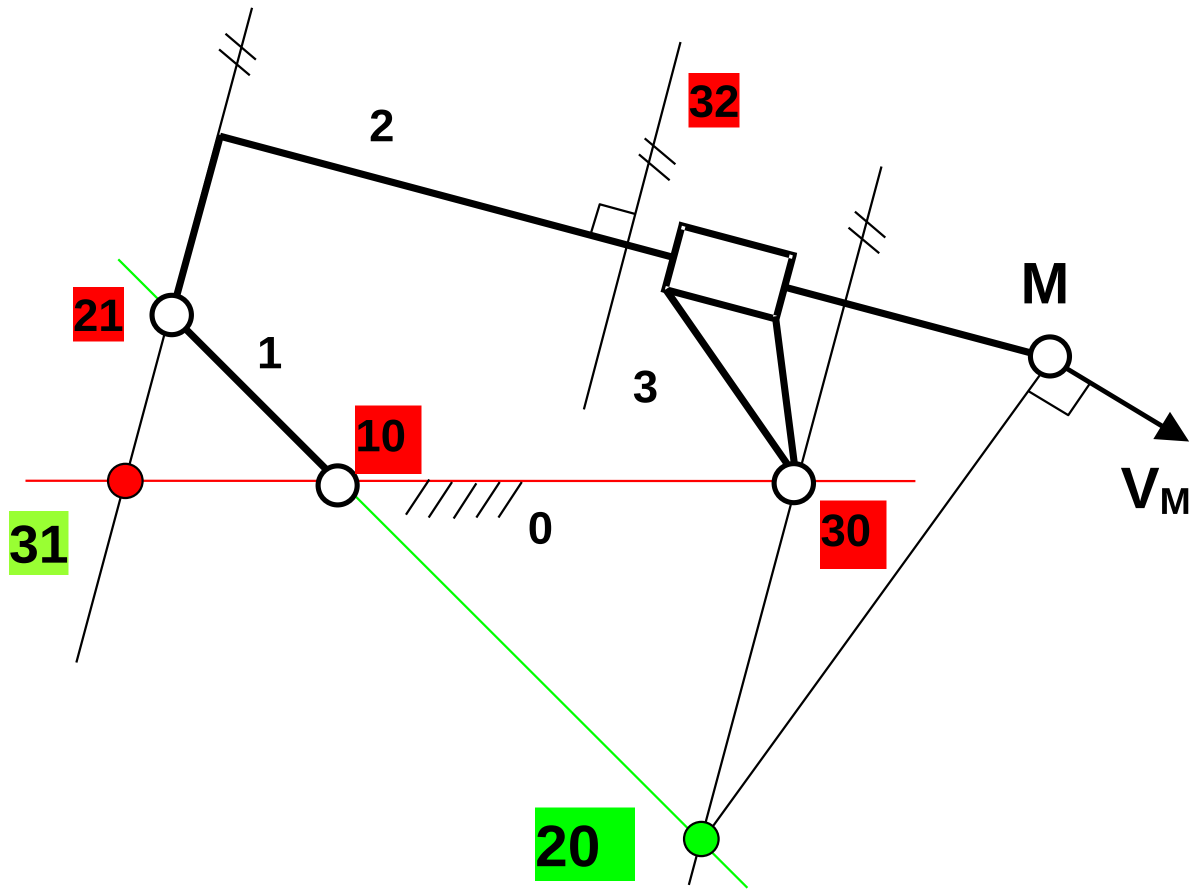
2 method: instant center of velocity S_{20}



S_{01}	S_{02}	S_{03}
	S_{12}	S_{13}
		S_{23}

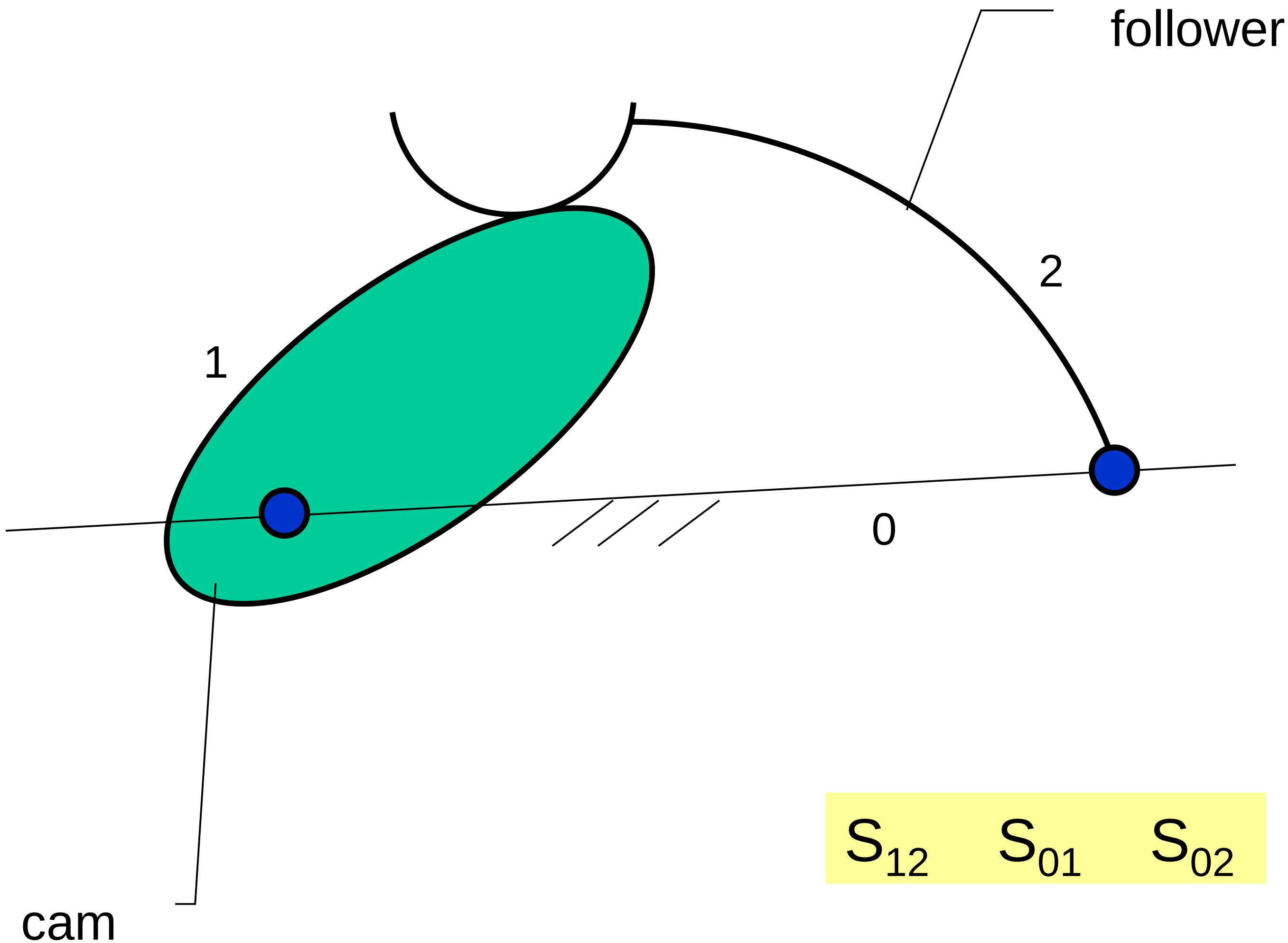


S_{01}	S_{02}	S_{03}
	S_{12}	S_{13}
		S_{23}

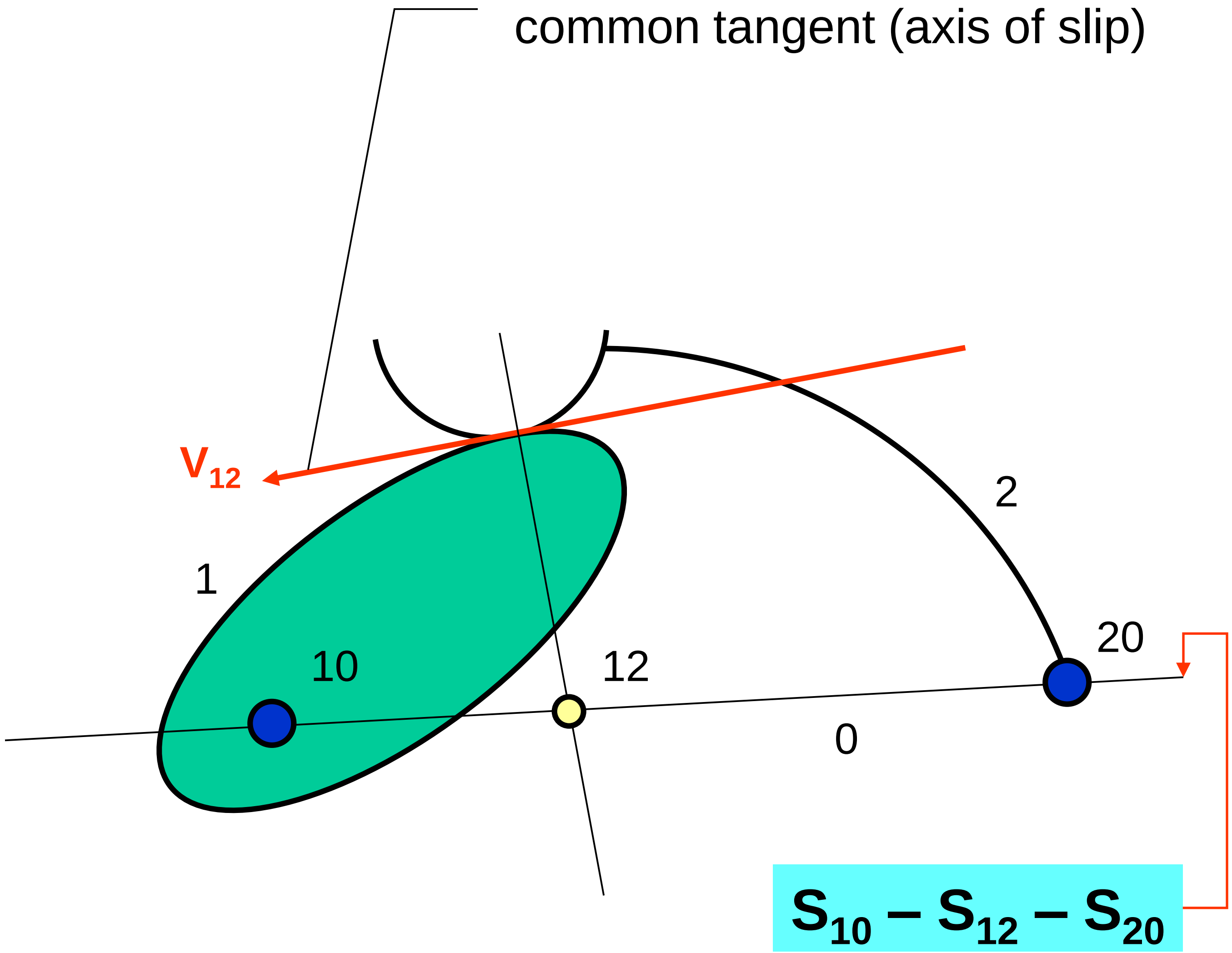


2
32
M
21
1
3
10
0
VM
31
30
20

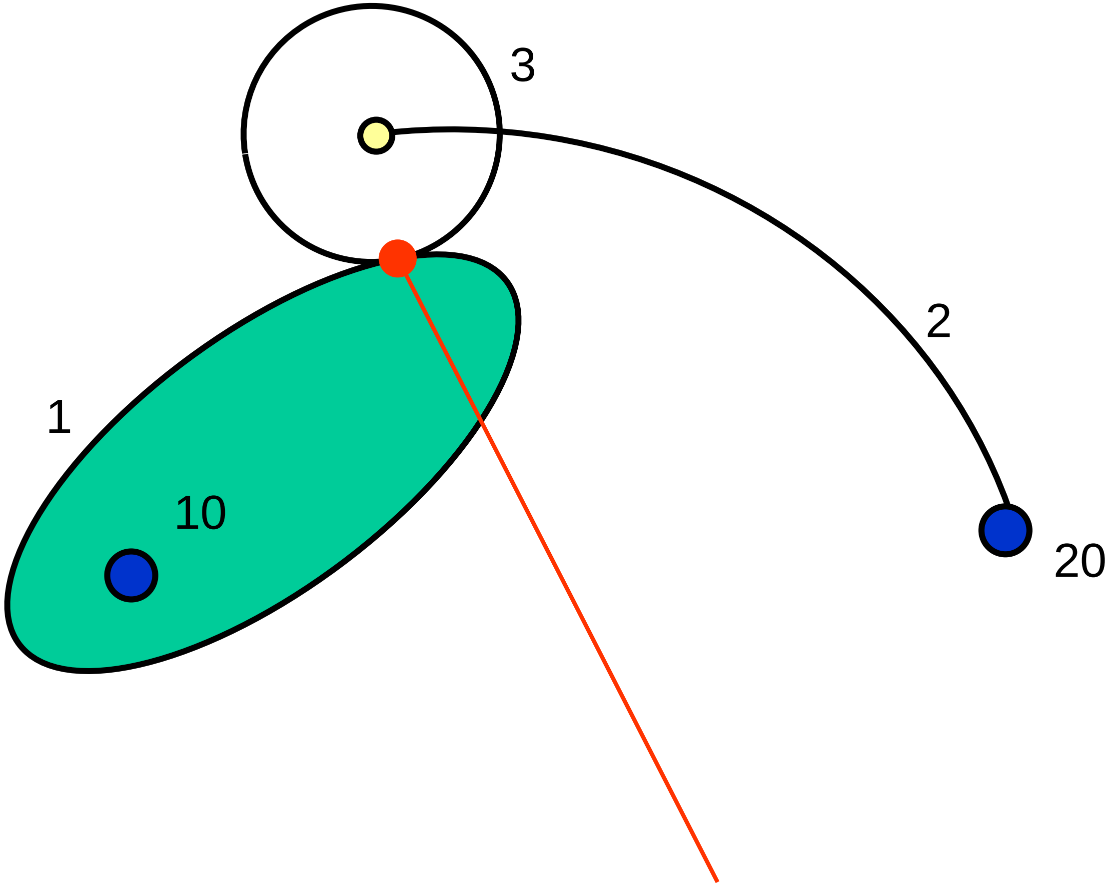
Cam mechanism



S_{12} S_{01} S_{02}


common tangent (axis of slip)
V12
1
2
10
12
20
0
S10 – S12 – S20

Cam with roller



$V_{13} = 0 \rightarrow S_{13}$ at contact point