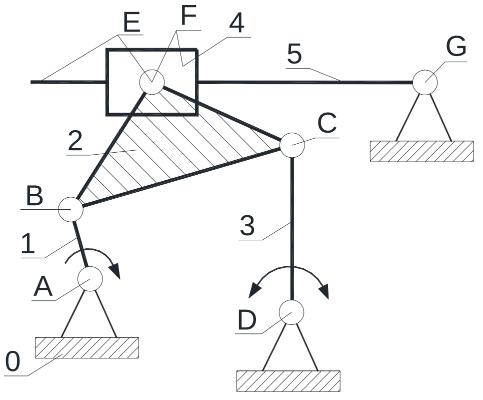
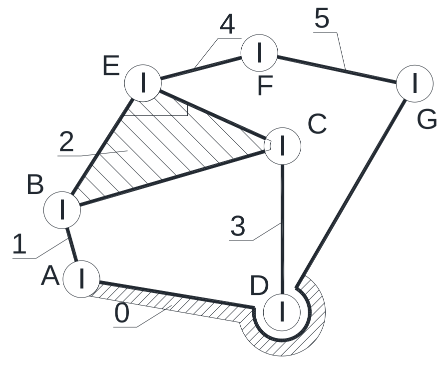
Kinematic chain notations

Kinematic scheme



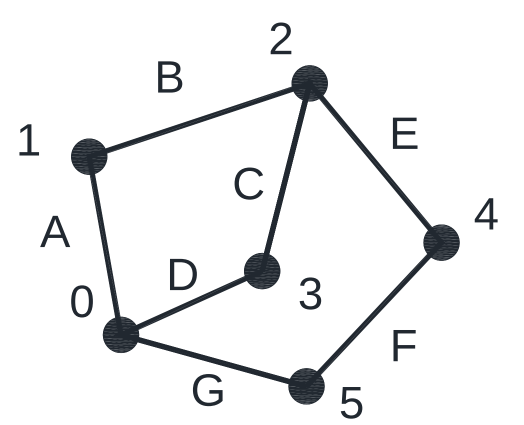
Structural scheme



Matrix of the structure

	0	1	2	3	4	5
0	-	A	-	D	-	G
1	A	-	B	-	-	-
2	-	B	-	C	E	-
3	D	-	C	-	-	-
4	-	-	E	-	-	F
5	G	-	-	-	F	-

GRAF



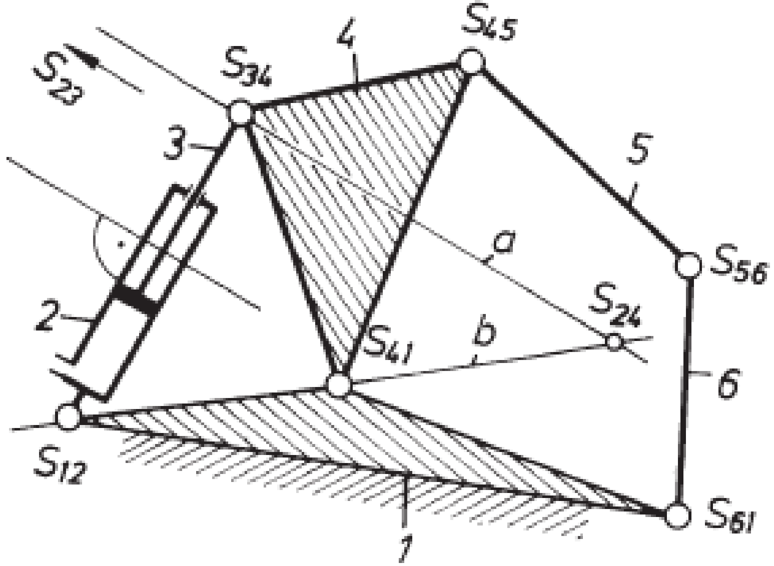
Contours

$K_1 = 0 – A – 1 – B – 2 – C – 3 – D – 0$

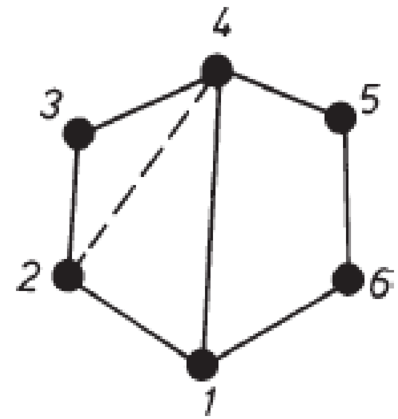
$K_2 = 0 – D – 3 – C – 2 – E – 4 – F – 5 – G - 0$

$K_{3(outer)} = 0 – A – 1 – B – 2 – E – 4 – F – 5 – G - 0$

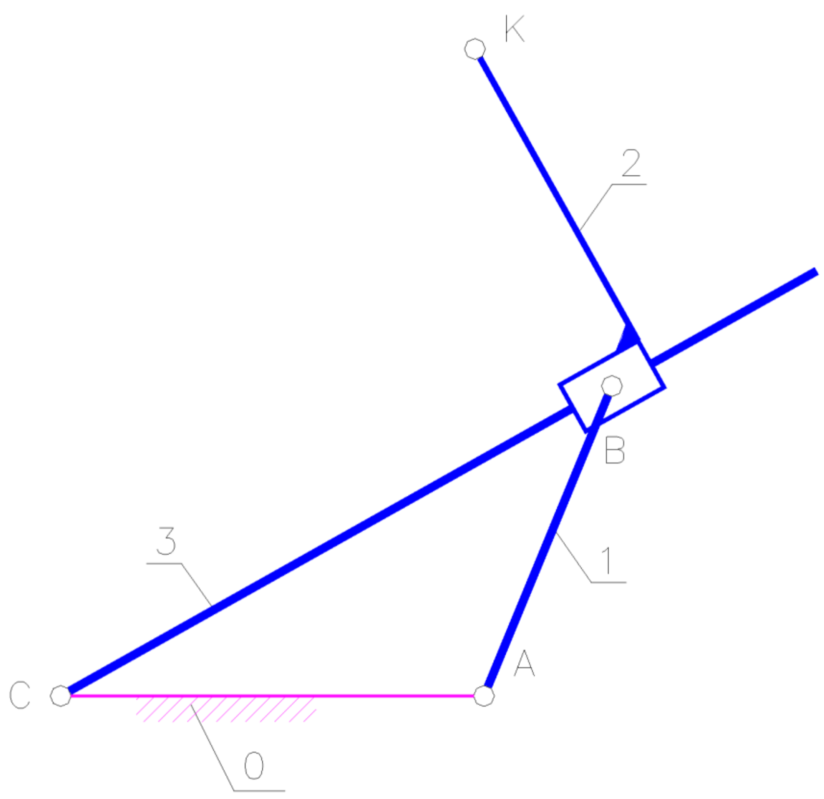
Using Graf

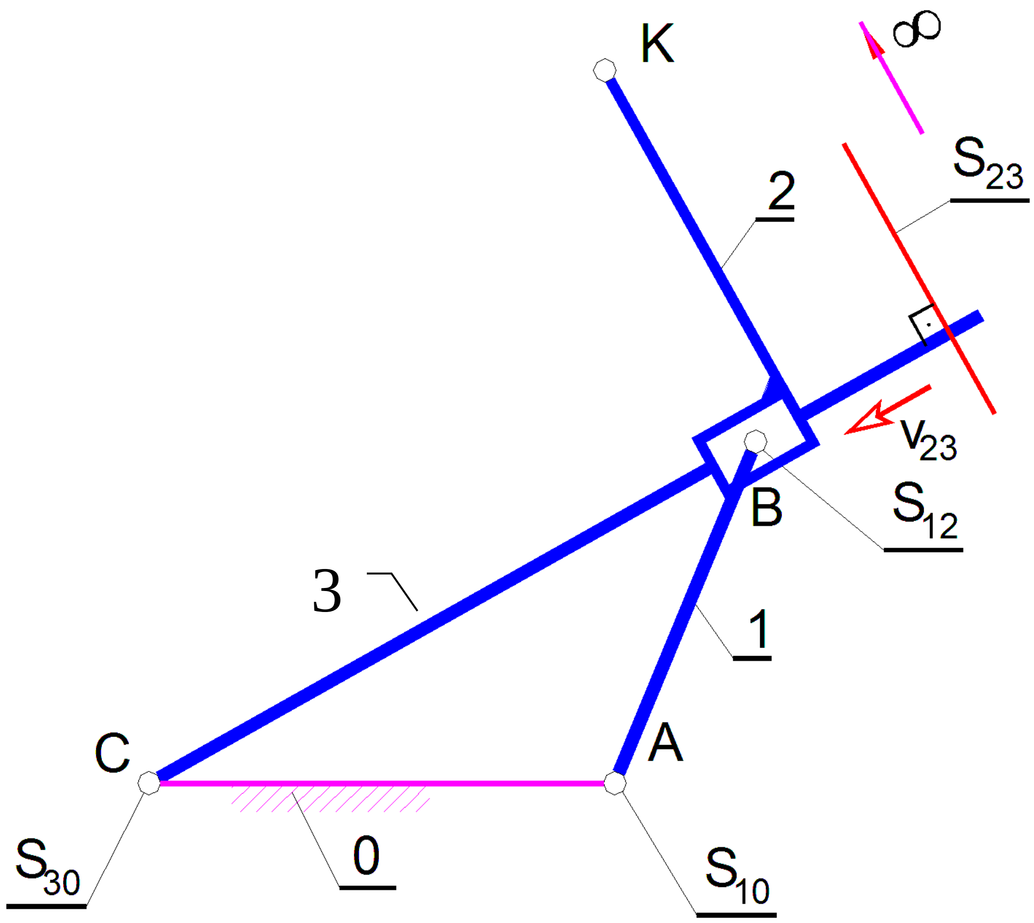


S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
	S_{23}	S_{24}	S_{25}	S_{26}
		S_{34}	S_{35}	S_{36}
			S_{45}	S_{46}
				S_{56}

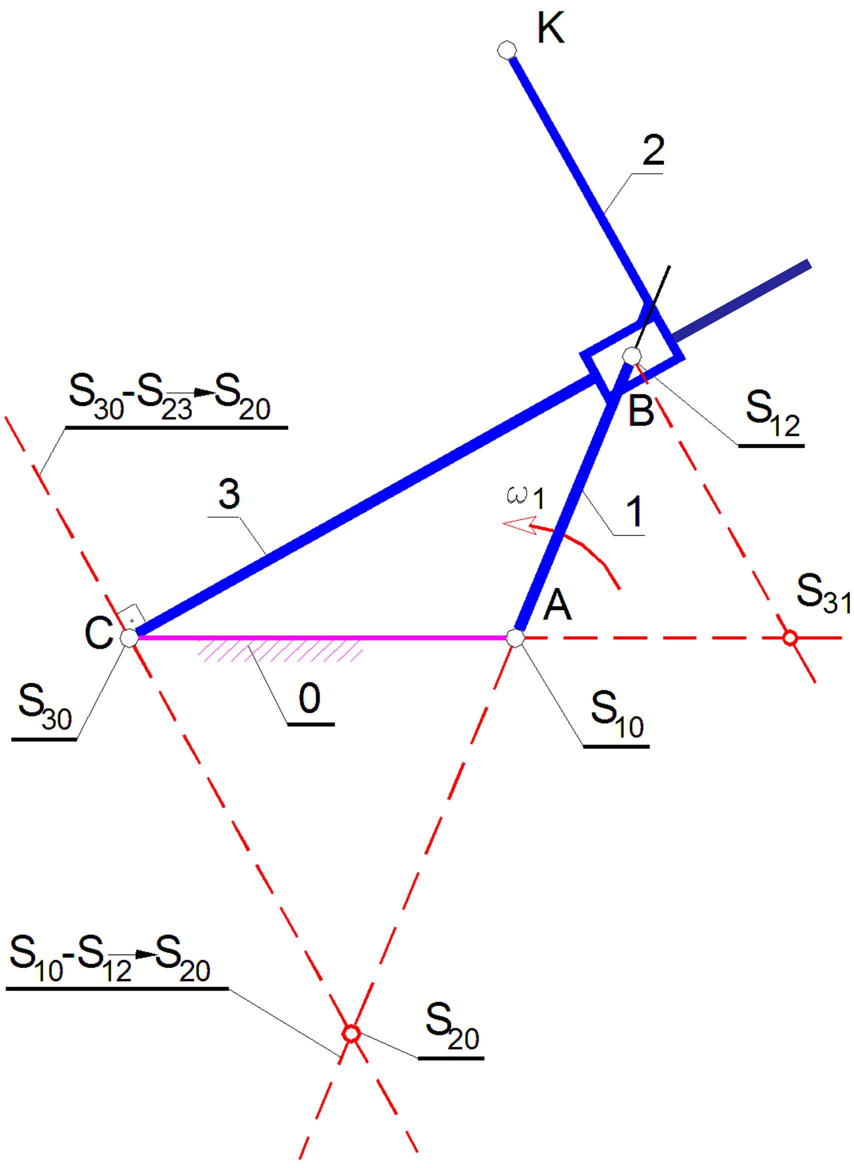


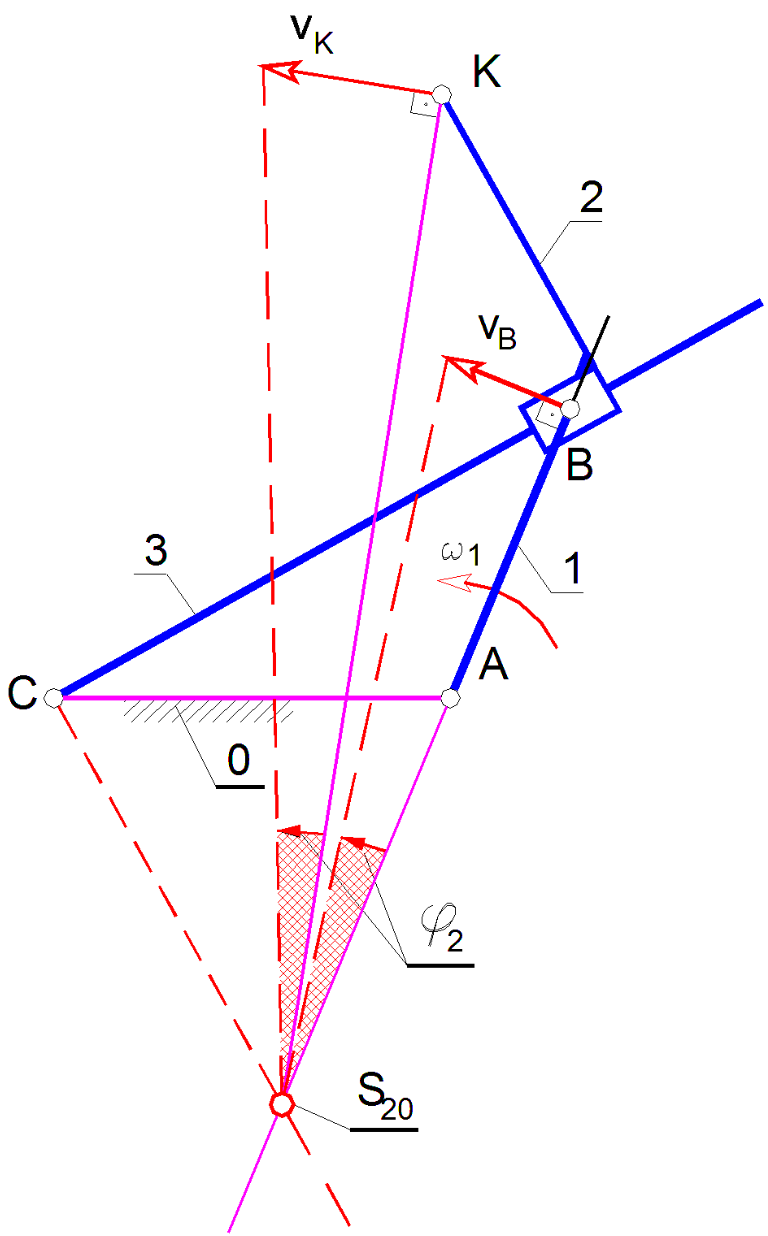
Find direction of V_K





K
∞
S23
2
v23
B
S12
3
1
C
A
S30
0
S10

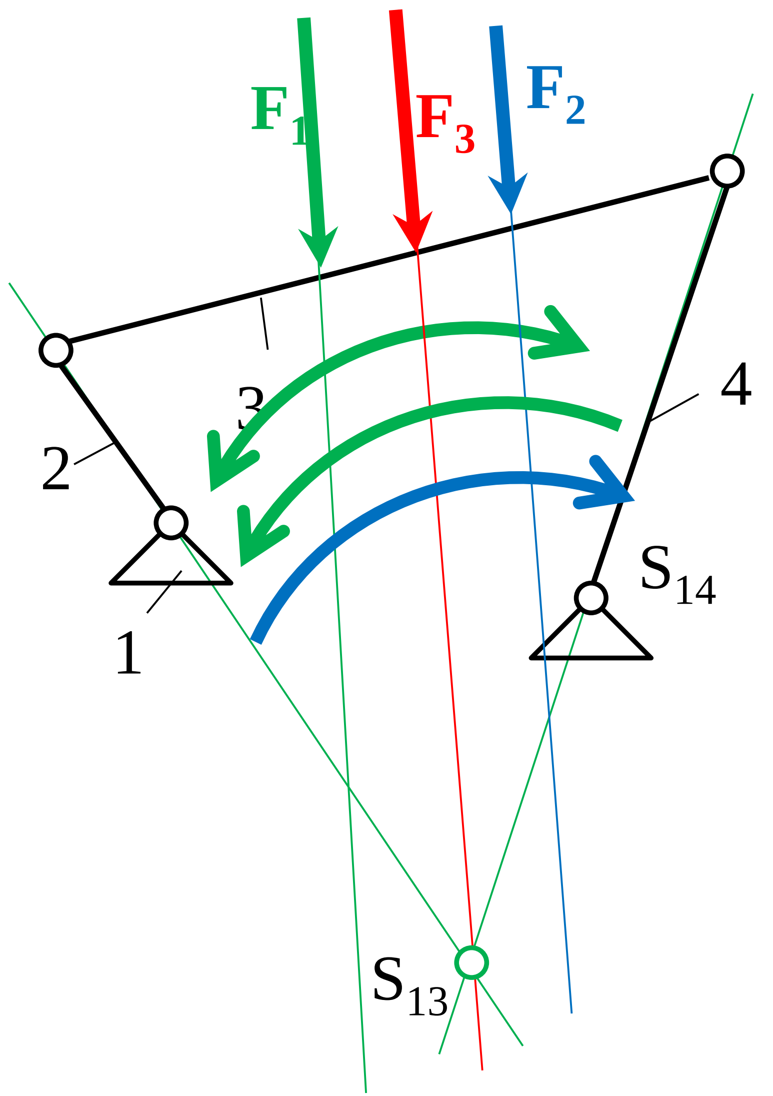

K
2
S_{30}-S_{23}→S_{20}
B
S_{12}
3
ω_1
1
A
S_{31}
C
S_{30}
0
S_{10}
S_{10}-S_{12}→S_{20}
S_{20}



$$v_B = \omega_1 \cdot AB$$

$$\frac{v_K}{KS_{20}} = \frac{v_B}{BS_{20}} = \omega_2$$

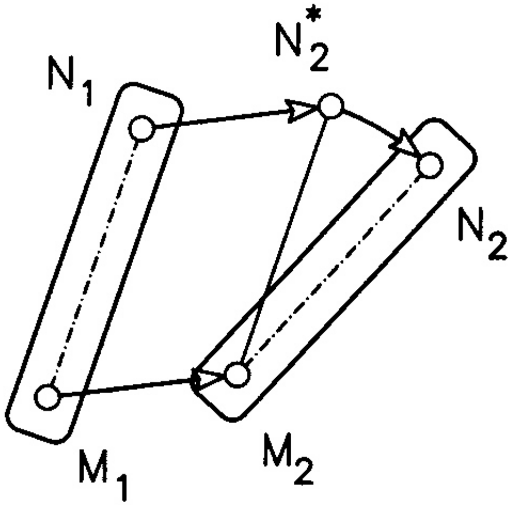
Determining the motion of the mechanism:



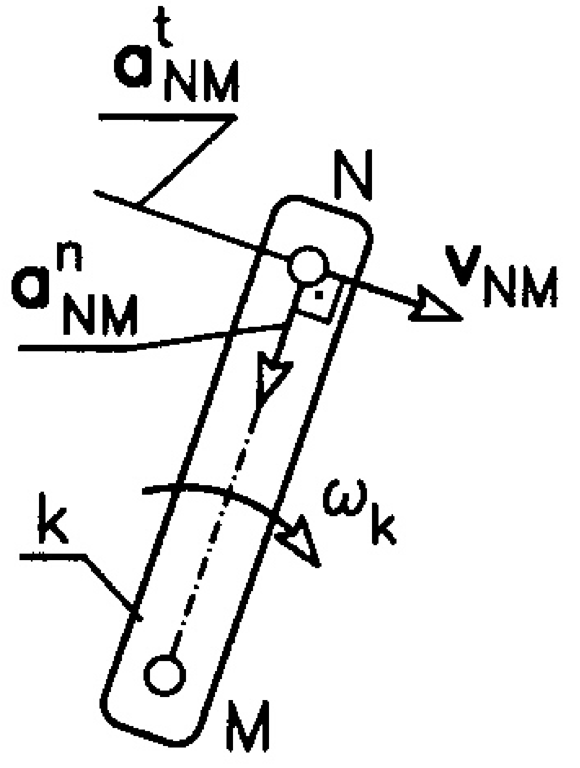
VELOCITY AND ACCELERATION

VECTOR EQUATIONS – v and a polygons

(2D) – M & N – points of one link



$$\Delta t \to 0$$



v and a vector equations (2D complex motion)

velocity:

$$\mathbf{v}_N = \mathbf{v}_M + \mathbf{v}_{NM}$$

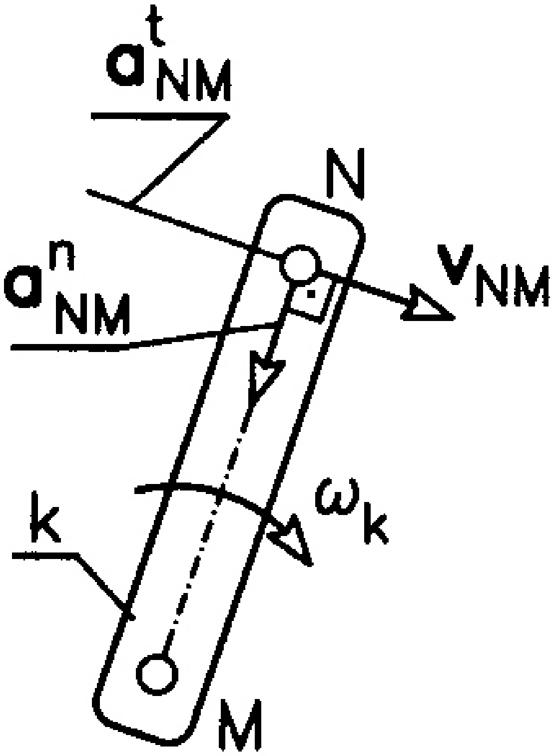
$$\mathbf{v}_{NM} = \boldsymbol{\omega}_k \times \mathbf{r}_{MN}$$

acceleration:

$$\mathbf{a}_N = \mathbf{a}_M + \mathbf{a}_{NM} = \mathbf{a}_M + \mathbf{a}_{NM}^n + \mathbf{a}_{NM}^t$$

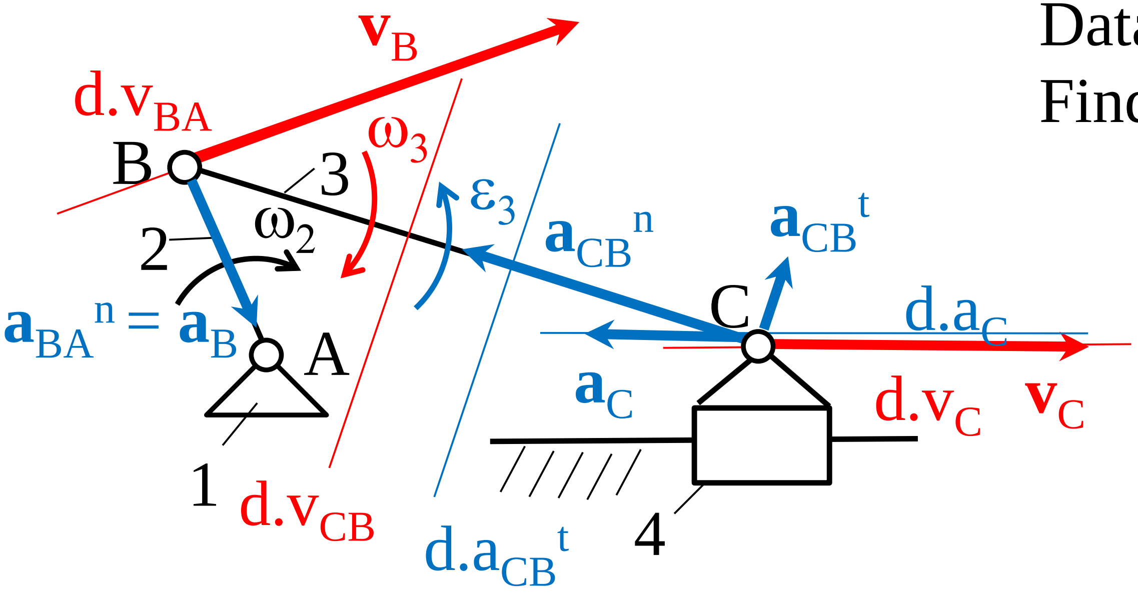
$$\mathbf{a}_{NM}^n = \boldsymbol{\omega}_k \times \left(\boldsymbol{\omega}_k \times \mathbf{r}_{MN}\right) = -\omega_k^2 \mathbf{r}_{MN}$$

$$\mathbf{a}_{NM}^t = \boldsymbol{\varepsilon}_k \times \mathbf{r}_{MN}$$

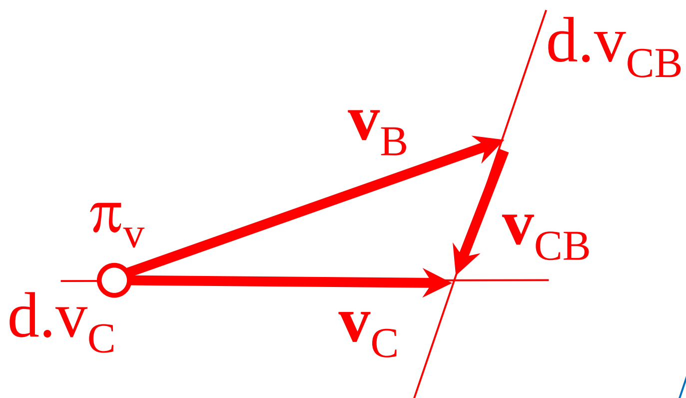


Data: $\omega_2 = \text{const}, \varepsilon_2 = 0$

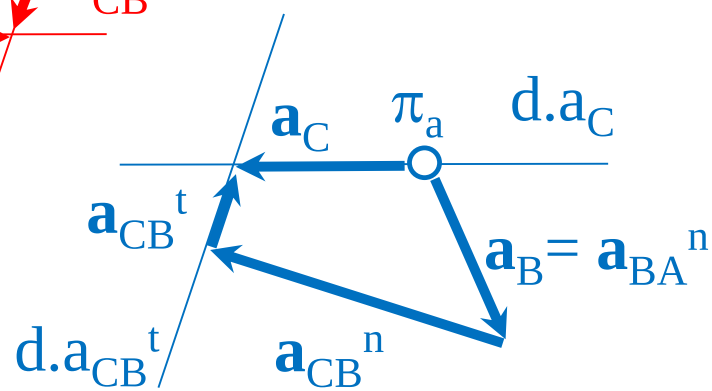
Find: $v_B, v_C, \omega_3, a_B, a_C, \varepsilon_3$



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{BA}$$
$$\mathbf{v}_A = 0$$
$$v_{BA} = \omega_2 \; BA$$
$$v_B = v_{BA} = \omega_2 \; BA$$
$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{CB}$$
$$\omega_3 = v_{CB} / CB$$

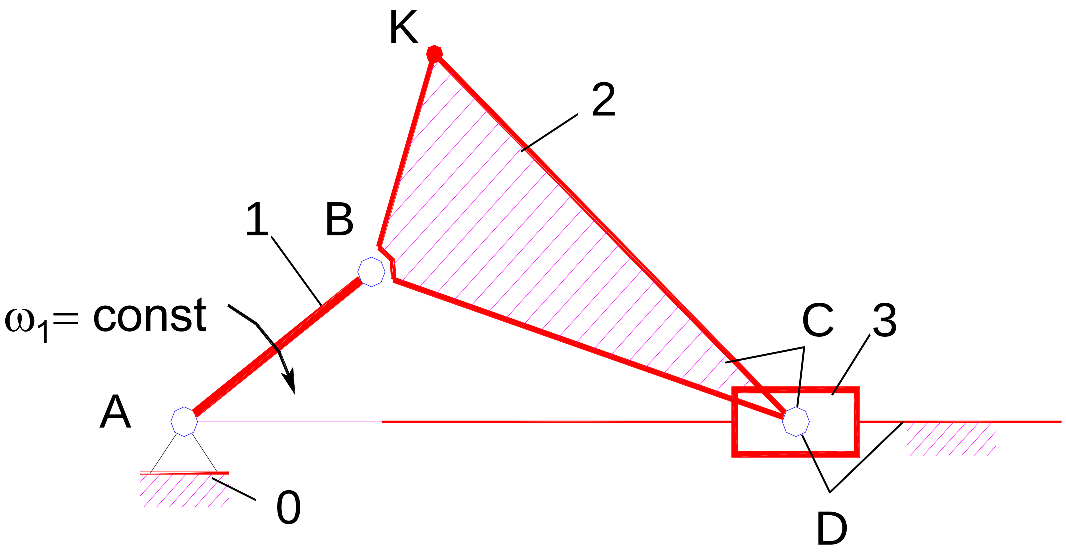


$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{BA}^n + \mathbf{a}_{BA}^t$$
$$\mathbf{a}_A = 0$$
$$a_{BA}^n = \omega_2^2 \; BA$$
$$a_{BA}^t = \varepsilon_2 \; BA = 0$$

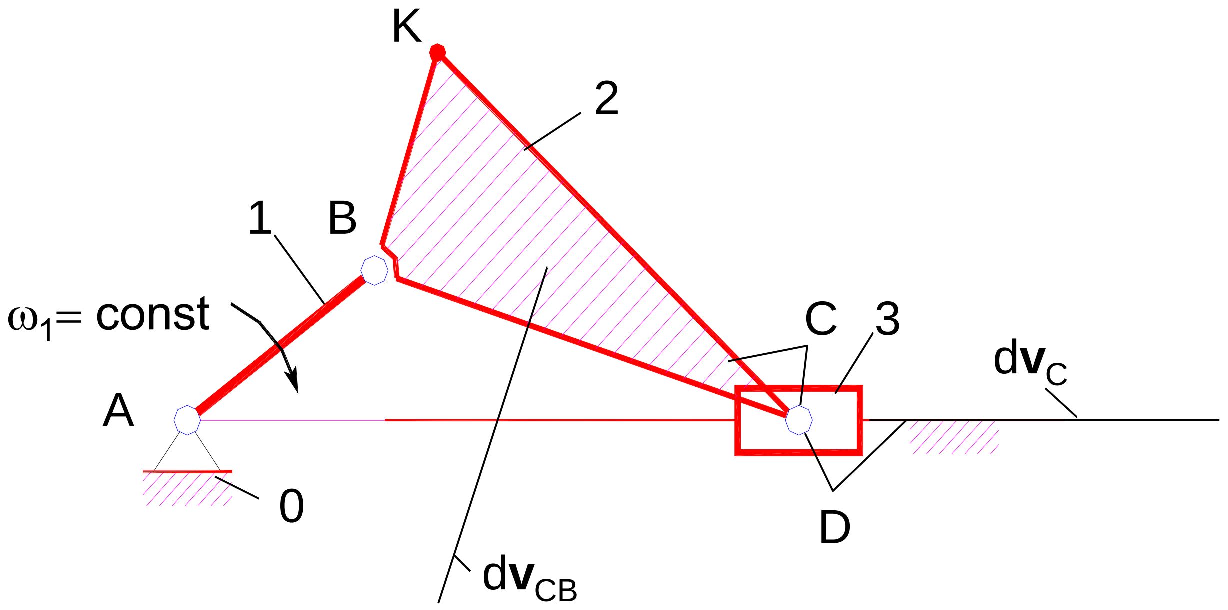


$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{CB}^n + \mathbf{a}_{CB}^t$$
$$a_{CB}^n = \omega_3^2 \; CB$$
$$a_{CB}^t = \varepsilon_3 \; CB$$
$$\varepsilon_3 = a_{CB}^t / CB$$

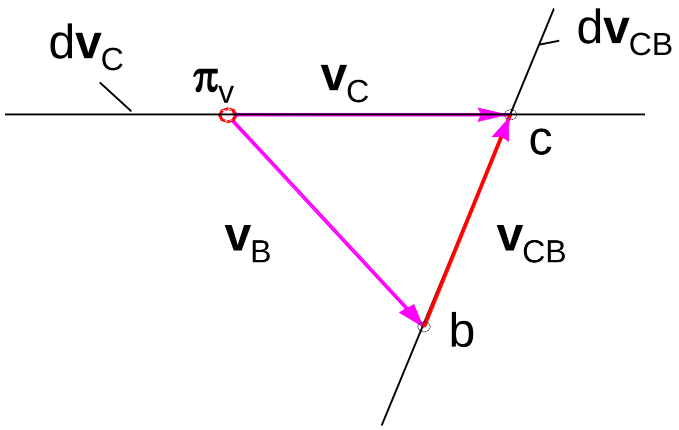
Example v and a (crank-slider)

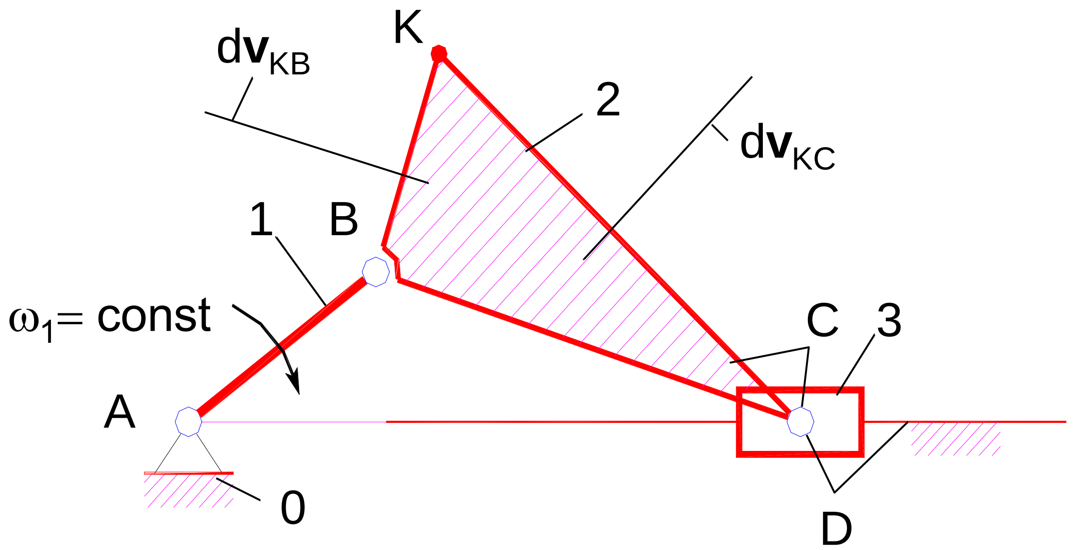


$$\mathbf{v}_B = \boldsymbol{\omega}_1 \times \mathbf{AB} \quad \Rightarrow \quad v_B = \omega_1 \cdot AB$$



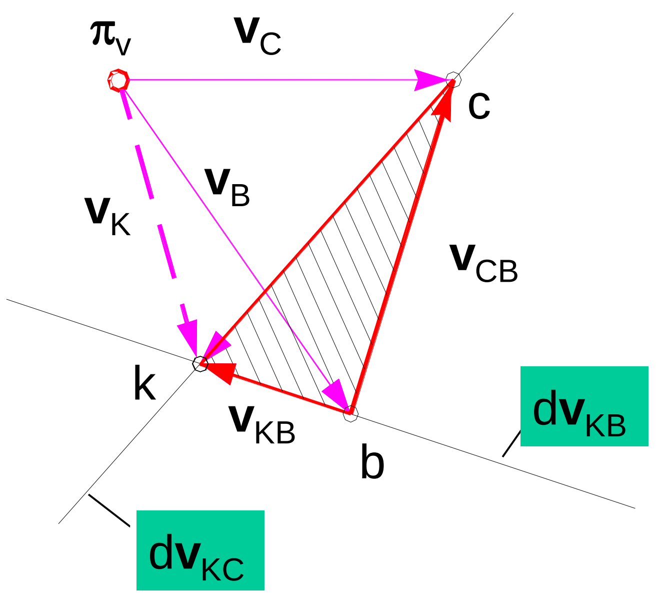
$$\underline{\mathbf{v}_C} = \underline{\underline{\mathbf{v}_B}} + \underline{\mathbf{v}_{CB}}$$

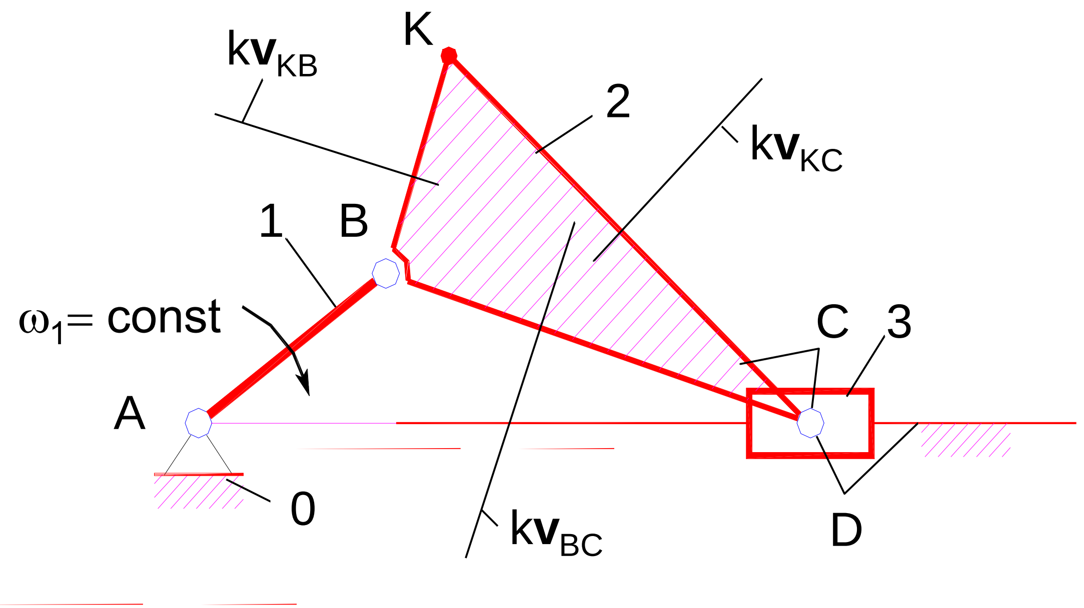




$$\begin{cases} \mathbf{v}_K = \underline{\underline{\mathbf{v}_B}} + \underline{\mathbf{v}_{KB}} \\ \mathbf{v}_K = \underline{\underline{\mathbf{v}_C}} + \underline{\mathbf{v}_{KC}} \end{cases} \Rightarrow$$

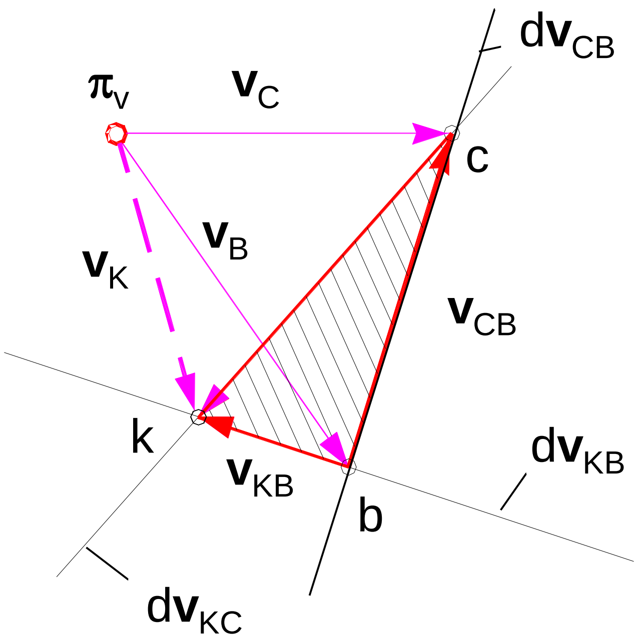
$$\Rightarrow \underline{\underline{\mathbf{v}_B}} + \underline{\mathbf{v}_{KB}} = \underline{\underline{\mathbf{v}_C}} + \underline{\mathbf{v}_{KC}}$$



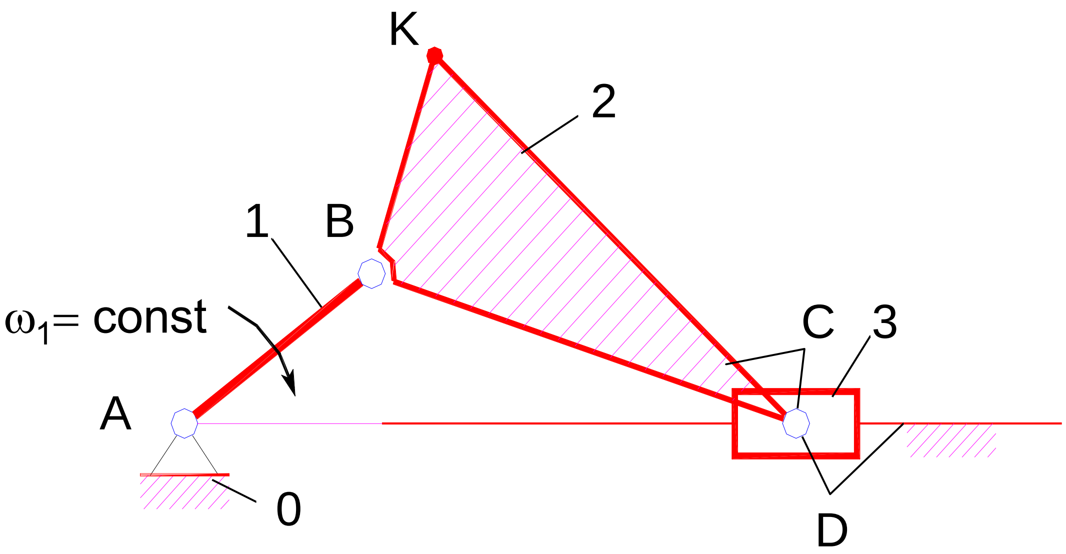


$$\Delta BCK \sim \Delta bck$$

$$\frac{BC}{bc} = \frac{BK}{bk} = \frac{KC}{kc}$$



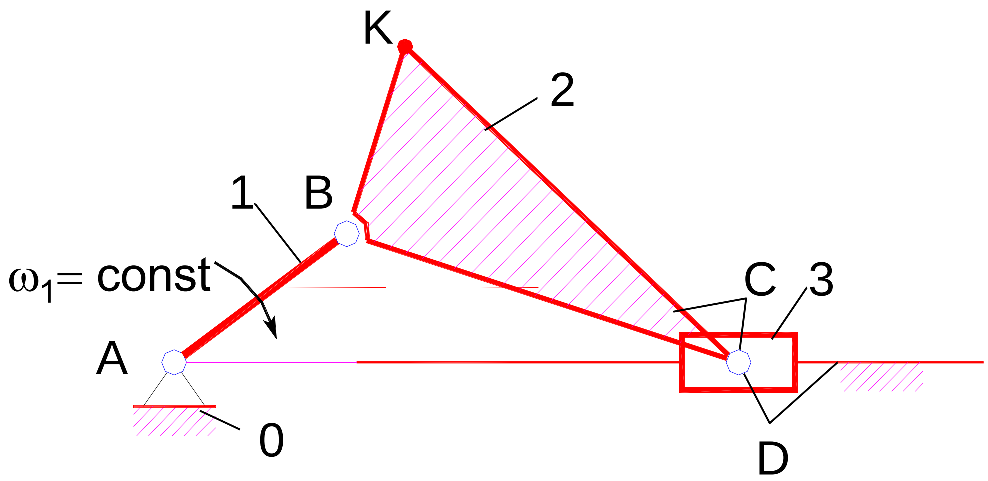
Triangles BCK and bck are similar



$$\mathbf{a}_B = \mathbf{a}_B^n + \mathbf{a}_B^t$$

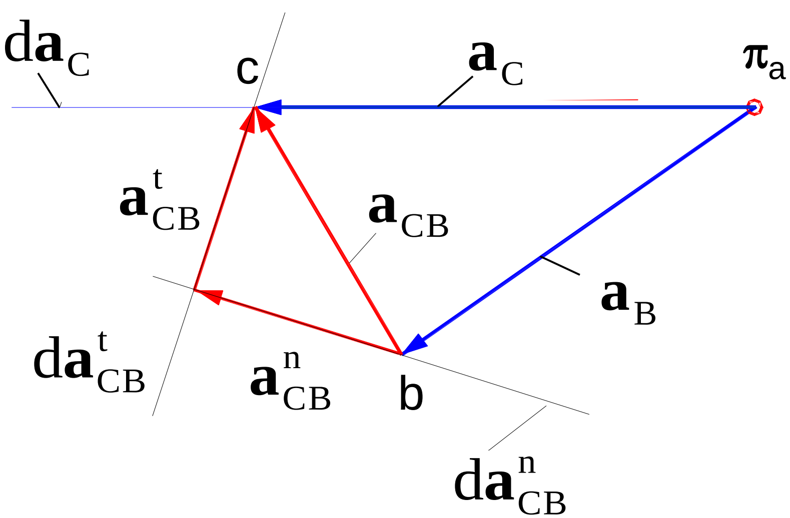
$$\mathbf{a}_B^n = \boldsymbol{\omega}_1 \times (\boldsymbol{\omega}_1 \times \mathbf{AB}) \;\Rightarrow\; a_B^n = \omega_1^2 \cdot AB = \frac{v_B^2}{AB}$$

$$\mathbf{a}_B^t = \boldsymbol{\varepsilon}_1 \times \mathbf{AB} \;\Rightarrow\; a_B^t = \varepsilon_1 \cdot AB = 0 \cdot AB = 0$$



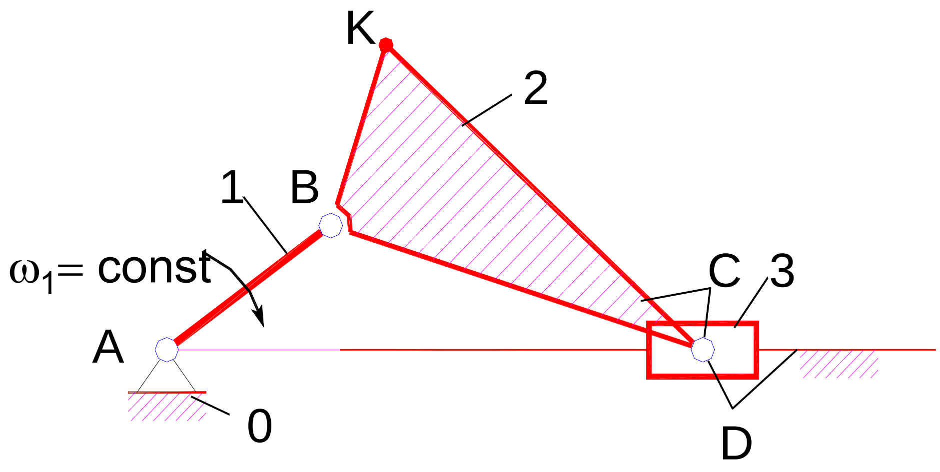
$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{CB}$$

$$\underline{\mathbf{a}_C} = \underline{\underline{\mathbf{a}_B^n}} + \underline{\mathbf{a}_B^t} + \underline{\underline{\mathbf{a}_{CB}^n}} + \underline{\mathbf{a}_{CB}^t}$$



$$a_B^n = \omega_1^2 AB = \frac{v_B^2}{AB}, \; a_B^t = 0 \text{ since } \varepsilon_1 = 0$$

$$a_{CB}^n = \frac{v_{CB}^2}{CB}$$



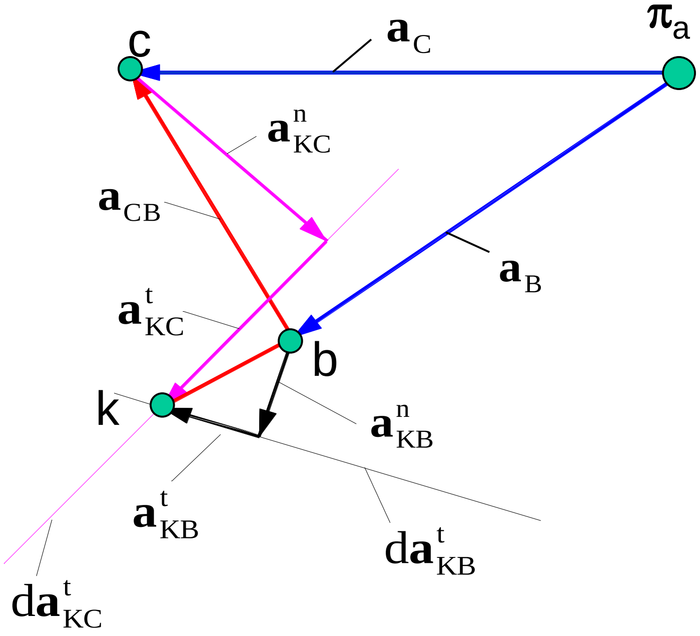
$$\begin{cases} \mathbf{a}_K = \mathbf{a}_B + \mathbf{a}_{KB} \\ \mathbf{a}_K = \mathbf{a}_C + \mathbf{a}_{KC} \end{cases} \Rightarrow$$

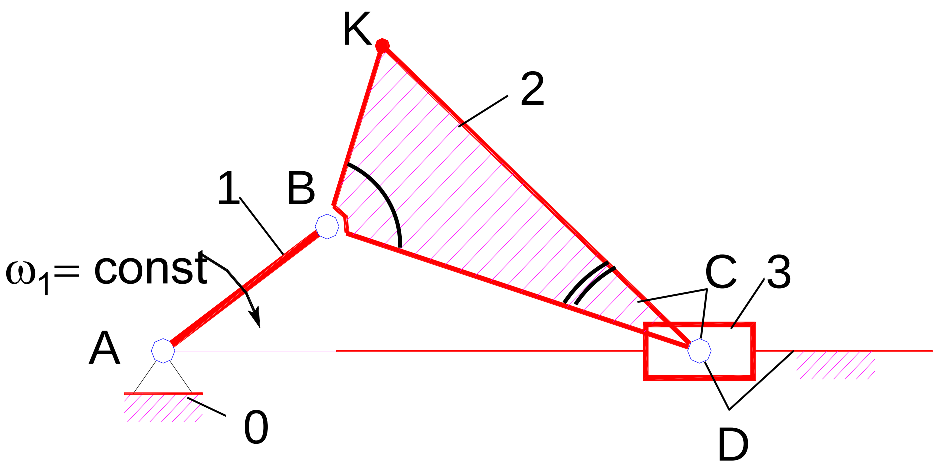
$$\Rightarrow \mathbf{a}_B + \mathbf{a}_{KB} = \mathbf{a}_C + \mathbf{a}_{KC}$$

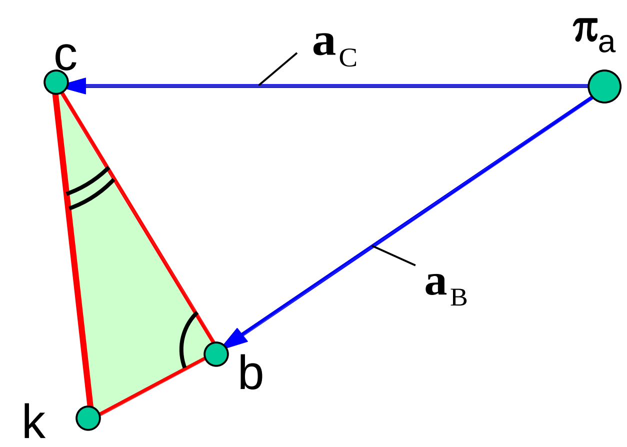
$$\underline{\underline{a_B}} + \underline{\underline{\mathbf{a}^n_{KB}}} + \underline{\mathbf{a}^t_{KB}} =$$

$$= \underline{\underline{a_C}} + \underline{\underline{\mathbf{a}^n_{KC}}} + \underline{\mathbf{a}^t_{KC}}$$

$$a^n_{KB} = \frac{v^2_{KB}}{KB}, \qquad a^n_{KC} = \frac{v^2_{KC}}{KC}$$





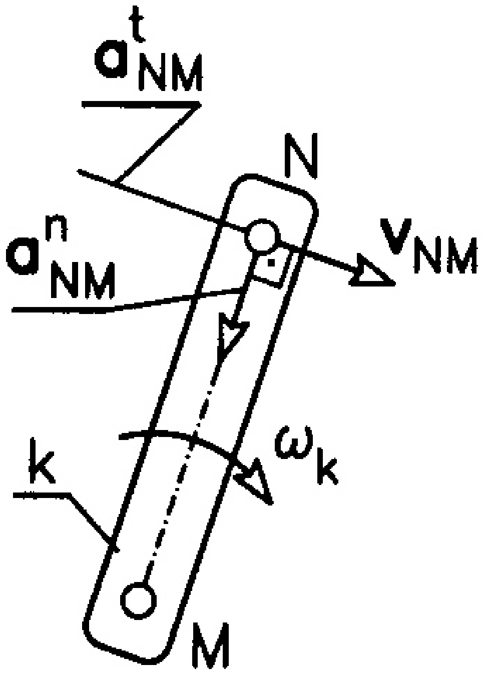


similarity (link and its accel polygon)

$\Delta BCK \sim \Delta bck$

VELOCITY AND ACCELARATION – extension

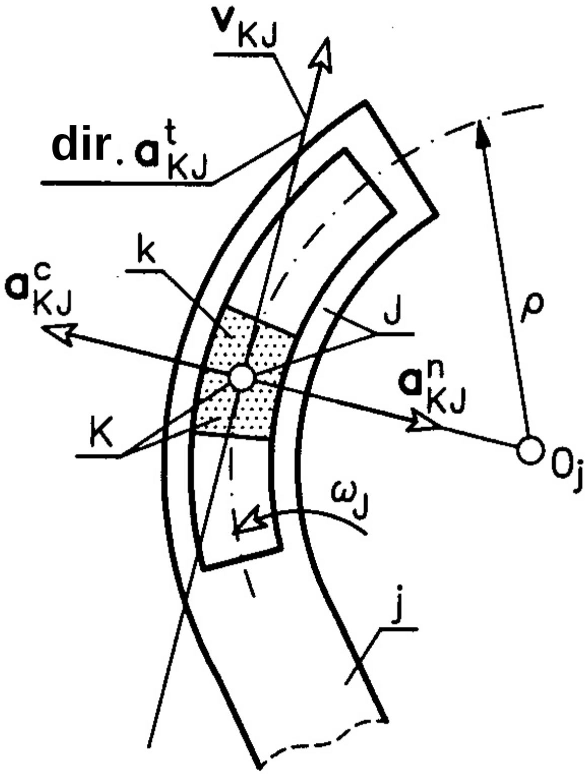
In last 2D example: M and N – points of one link



$$\mathbf{v}_N = \mathbf{v}_M + \mathbf{v}_{NM}$$

$$\mathbf{a}_N = \mathbf{a}_M + \mathbf{a}_{NM} = \mathbf{a}_M + \mathbf{a}_{NM}^n + \mathbf{a}_{NM}^t$$

(2D) – points J and K belong to j and k



$$\mathbf{v}_K = \mathbf{v}_J + \mathbf{v}_{KJ}$$

$$\mathbf{v}_{KJ} = {}^j\boldsymbol{\omega}_k \times \boldsymbol{\rho}$$

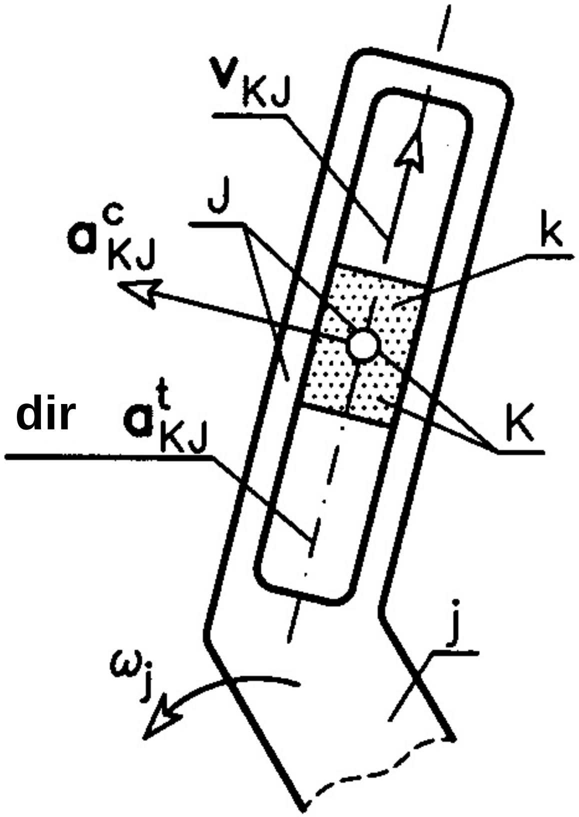
ρ - radius of curvature

$$\mathbf{a}_K = \mathbf{a}_J + \mathbf{a}_{KJ} = \mathbf{a}_J + \mathbf{a}^n_{KJ} + \mathbf{a}^t_{KJ} + \mathbf{a}^C_{KJ}$$

$$\mathbf{a}^n_{KJ} = {}^j\boldsymbol{\omega}_k \times \left({}^j\boldsymbol{\omega}_k \times \boldsymbol{\rho}\right) = -{}^j\omega_k^2 \boldsymbol{\rho}$$

$$\mathbf{a}^t_{KJ} = {}^j\boldsymbol{\varepsilon}_k \times \boldsymbol{\rho}$$

$$\mathbf{a}^C_{KJ} = 2\boldsymbol{\omega}_j \times \mathbf{v}_{KJ}$$



Since

$$\mathbf{a}_{KJ}^{n} = {}^{j}\boldsymbol{\omega}_{k} \times \left({}^{j}\boldsymbol{\omega}_{k} \times \boldsymbol{\rho}\right) = -\,{}^{j}\omega_{k}^{2}\boldsymbol{\rho} = 0$$

we have

$$\mathbf{a}_{K} = \mathbf{a}_{J} + \mathbf{a}_{KJ} = \mathbf{a}_{J} + \mathbf{a}_{KJ}^{t} + \mathbf{a}_{KJ}^{C}$$