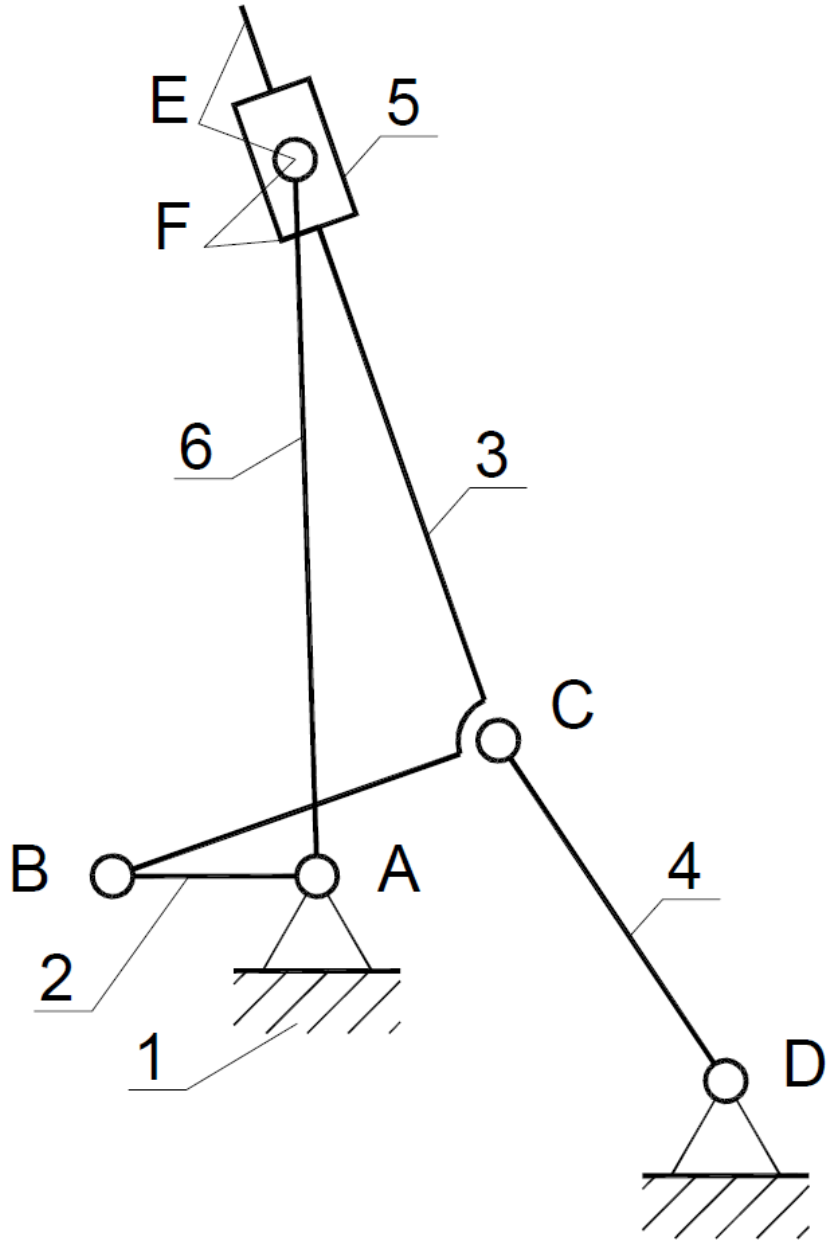
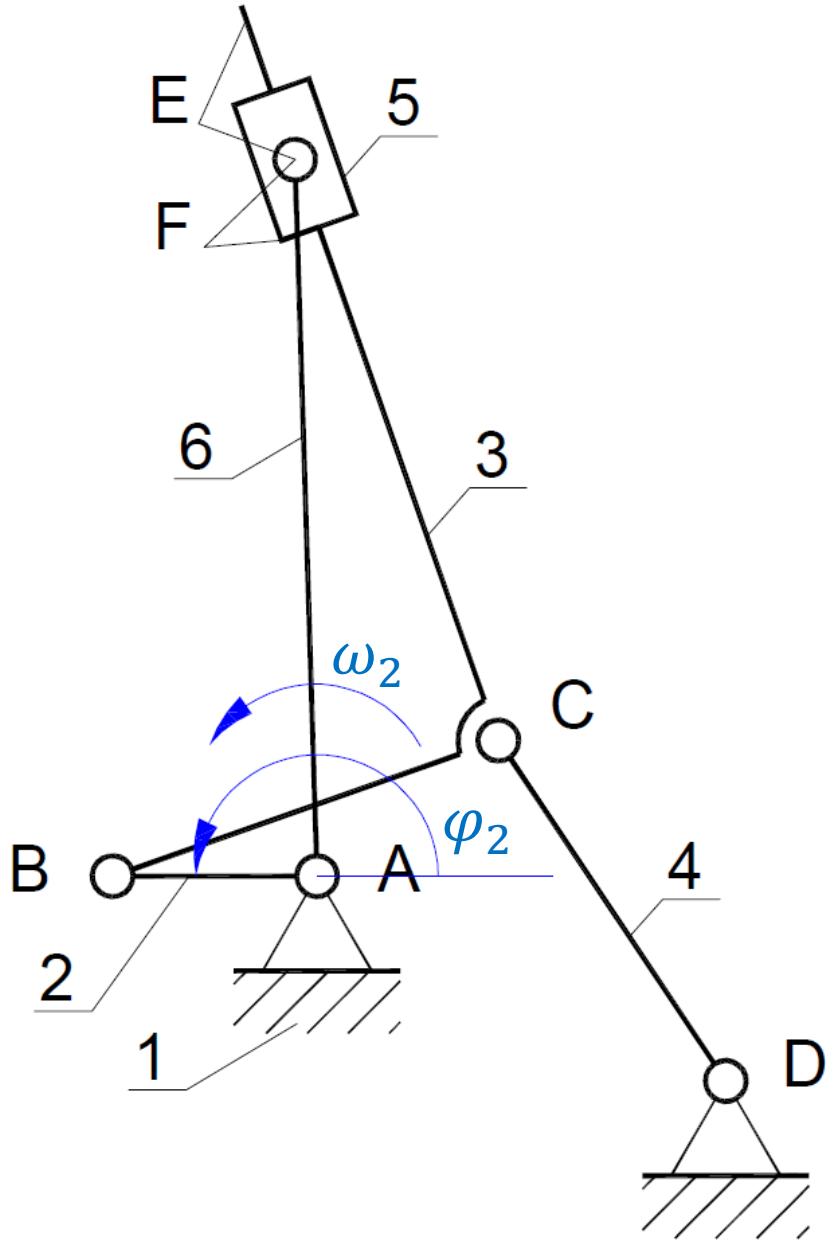
Example: kinematic scheme



Dimensions



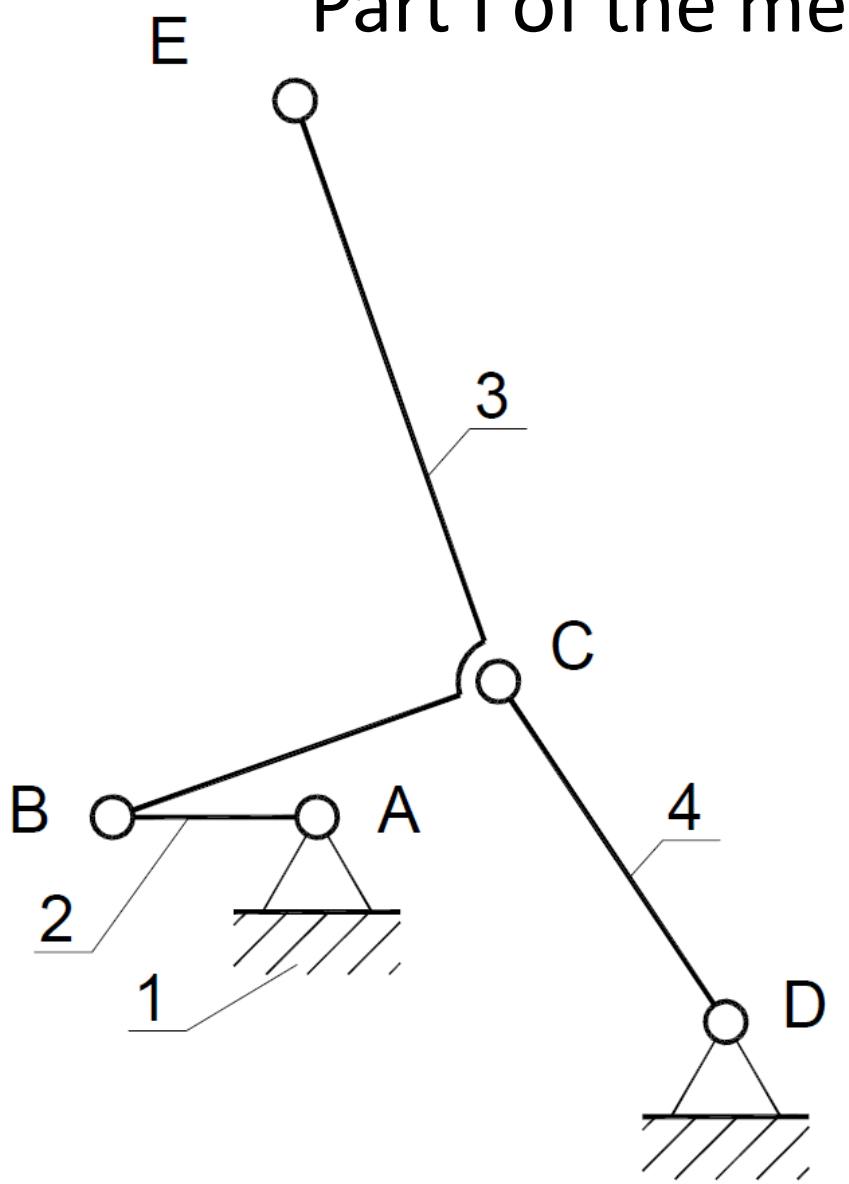
$x_A = y_A = 0$
$x_D = 0.2[m]$
$y_D = -0.1\ [m]$
$|AC| = 0.1\ [m]$
$|BC| = 0.2\ [m]$
$|CD| = 0.2\ [m]$
$|AF| = 0.35\ [m]$
$\angle BCE = 90°$

Analysis based on:

$\varphi_2 = 180^o \quad \omega_2 = 2\left[\frac{rad}{s}\right]$

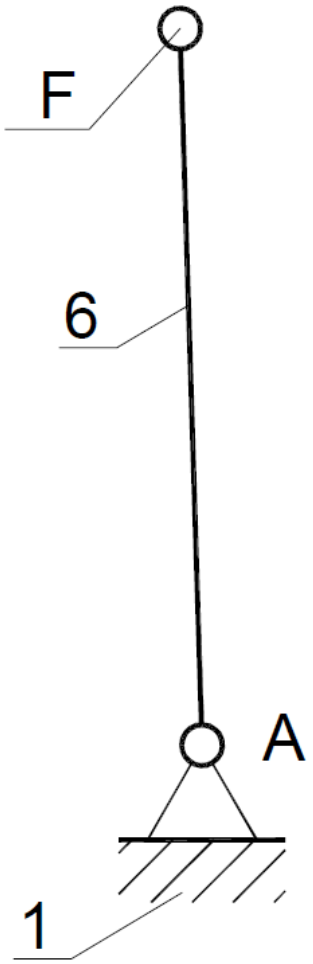
Graphical method (using polygons and vector equations)

Part I of the mechanism

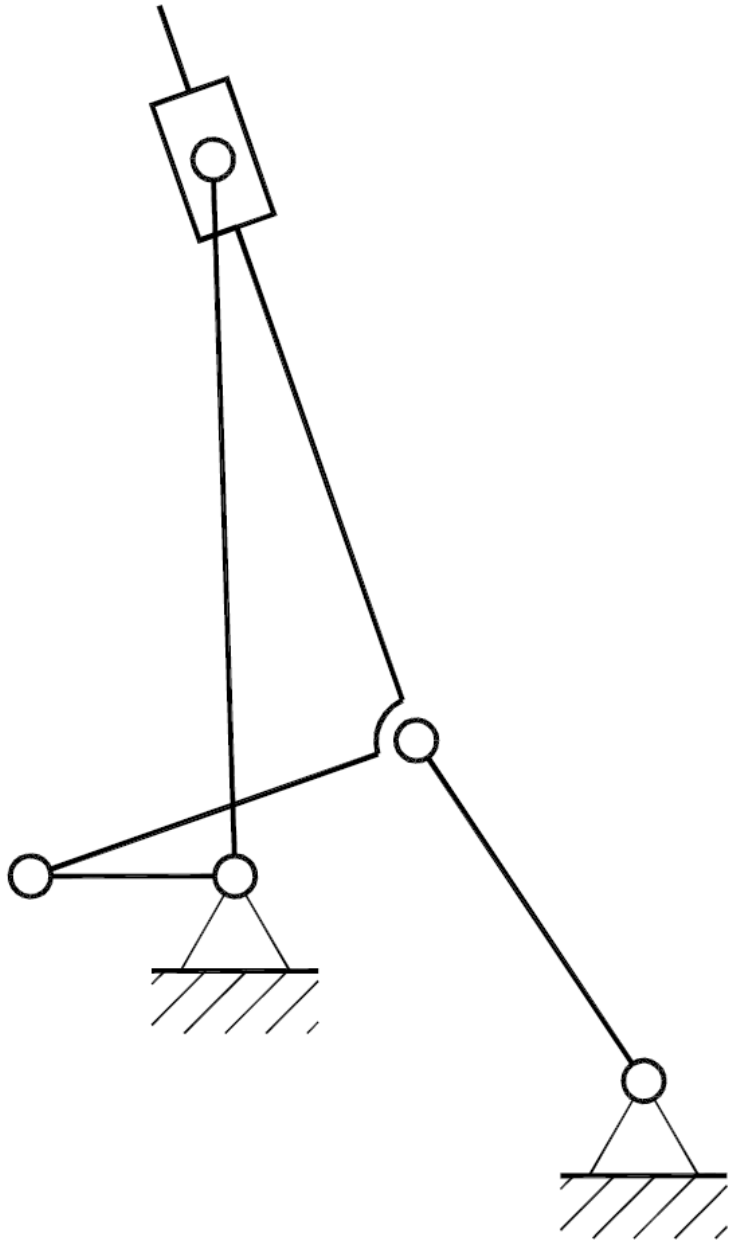


Graphical method (using polygons and vectora equations)

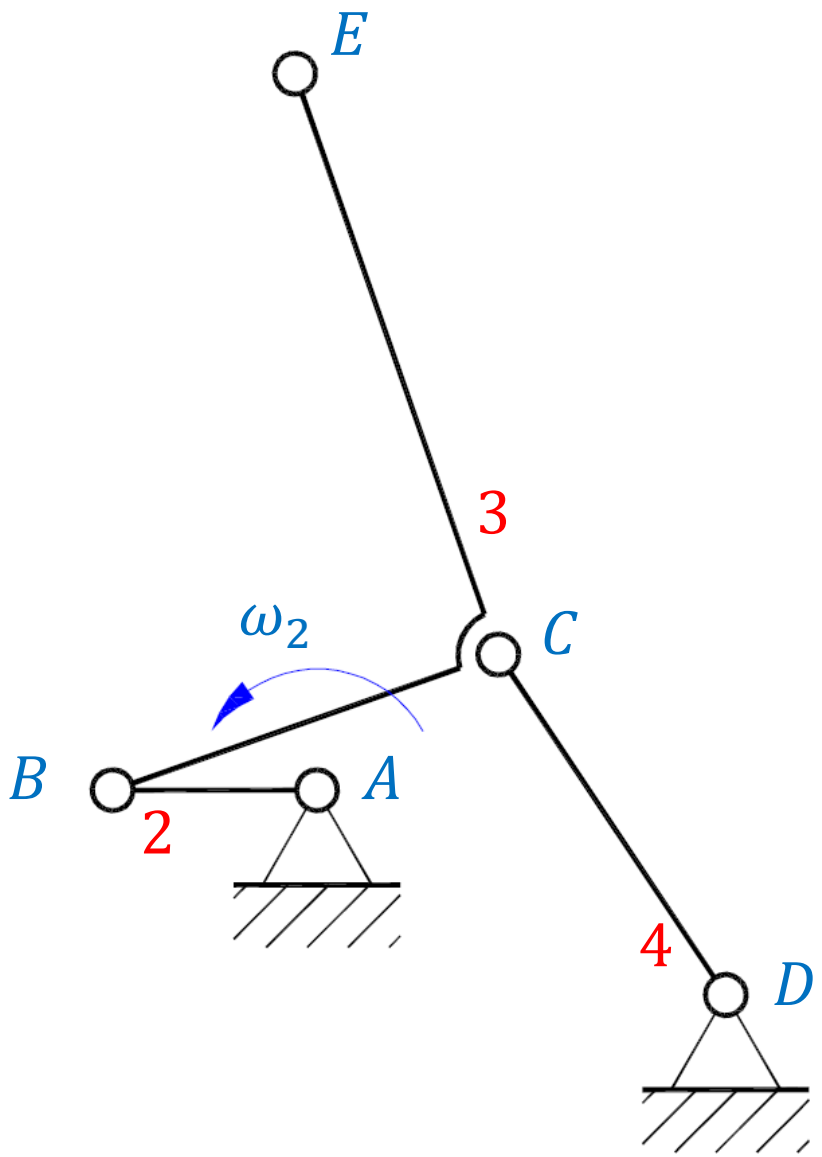
Part II of the mechanism



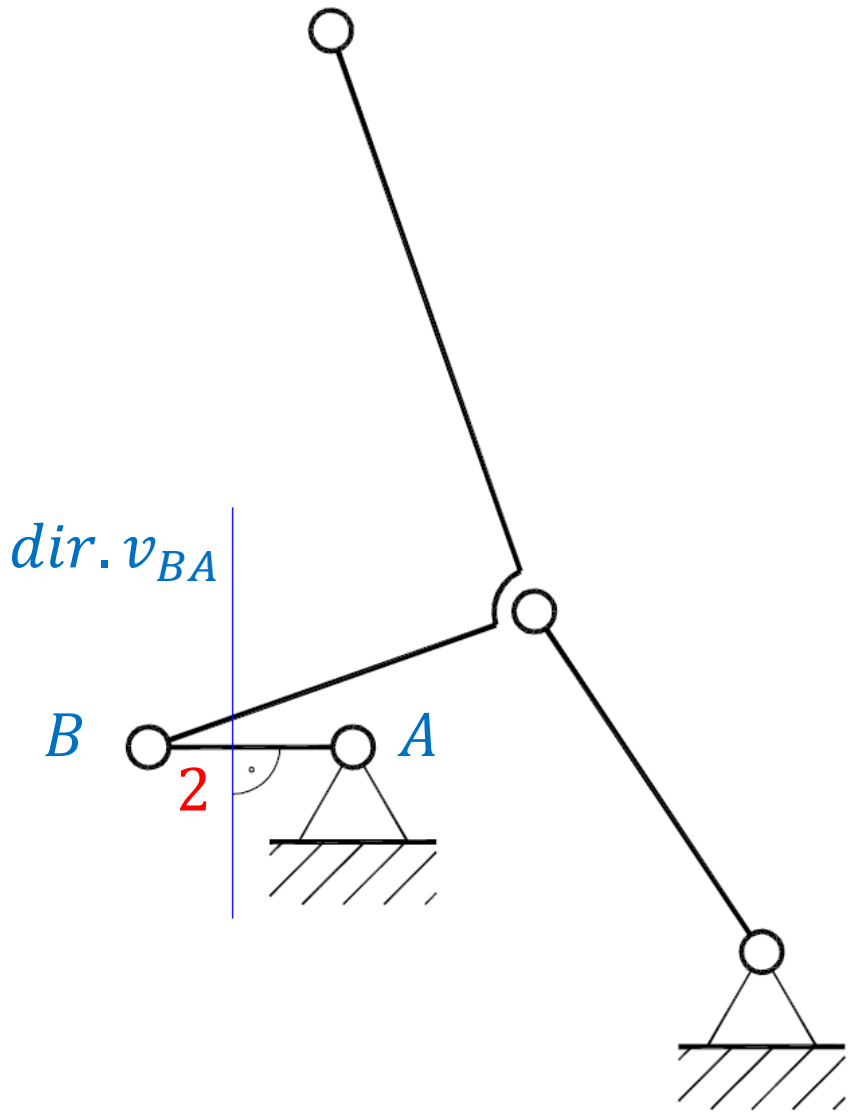
Slider is the connecting link



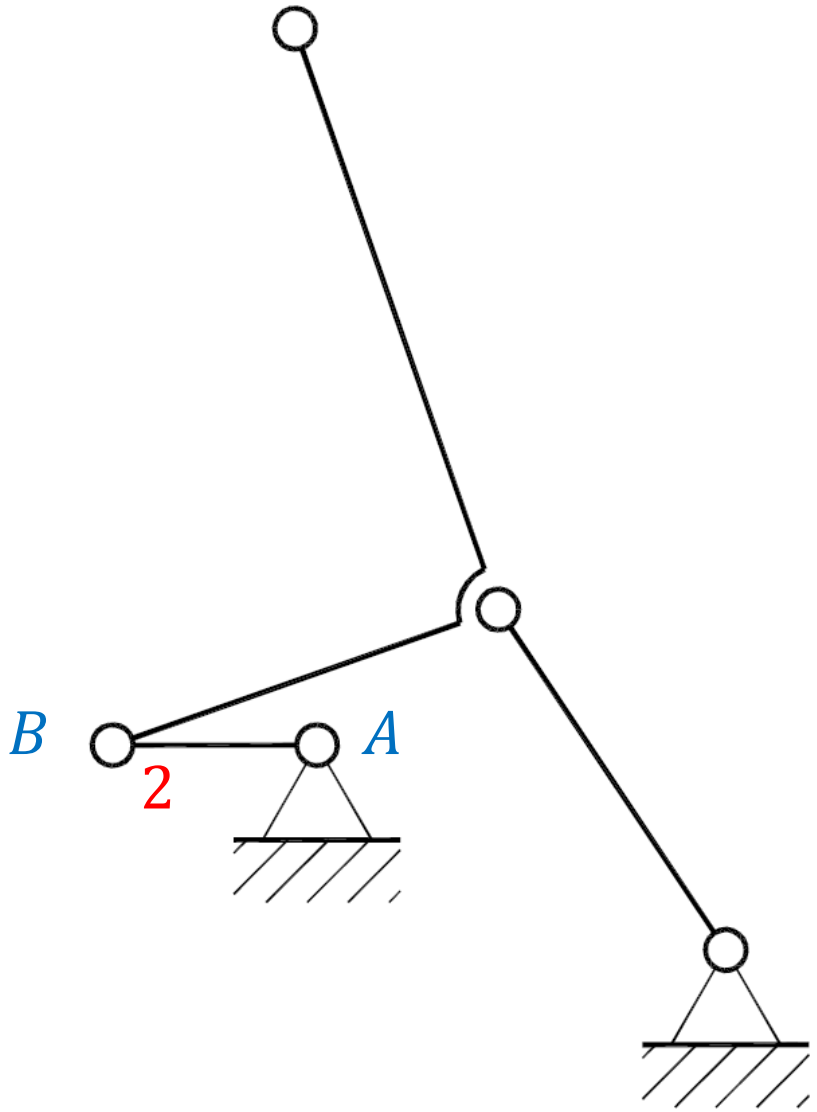
STEP I - Velocity



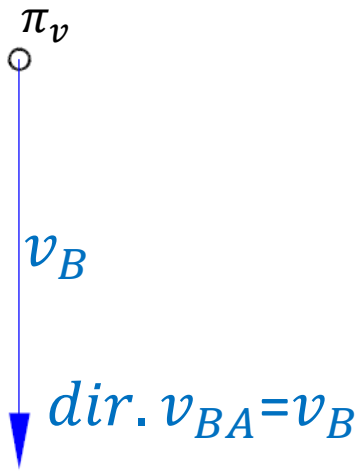
$v_{BA}=\omega_2 \cdot \mathrm{AB}$



π_v

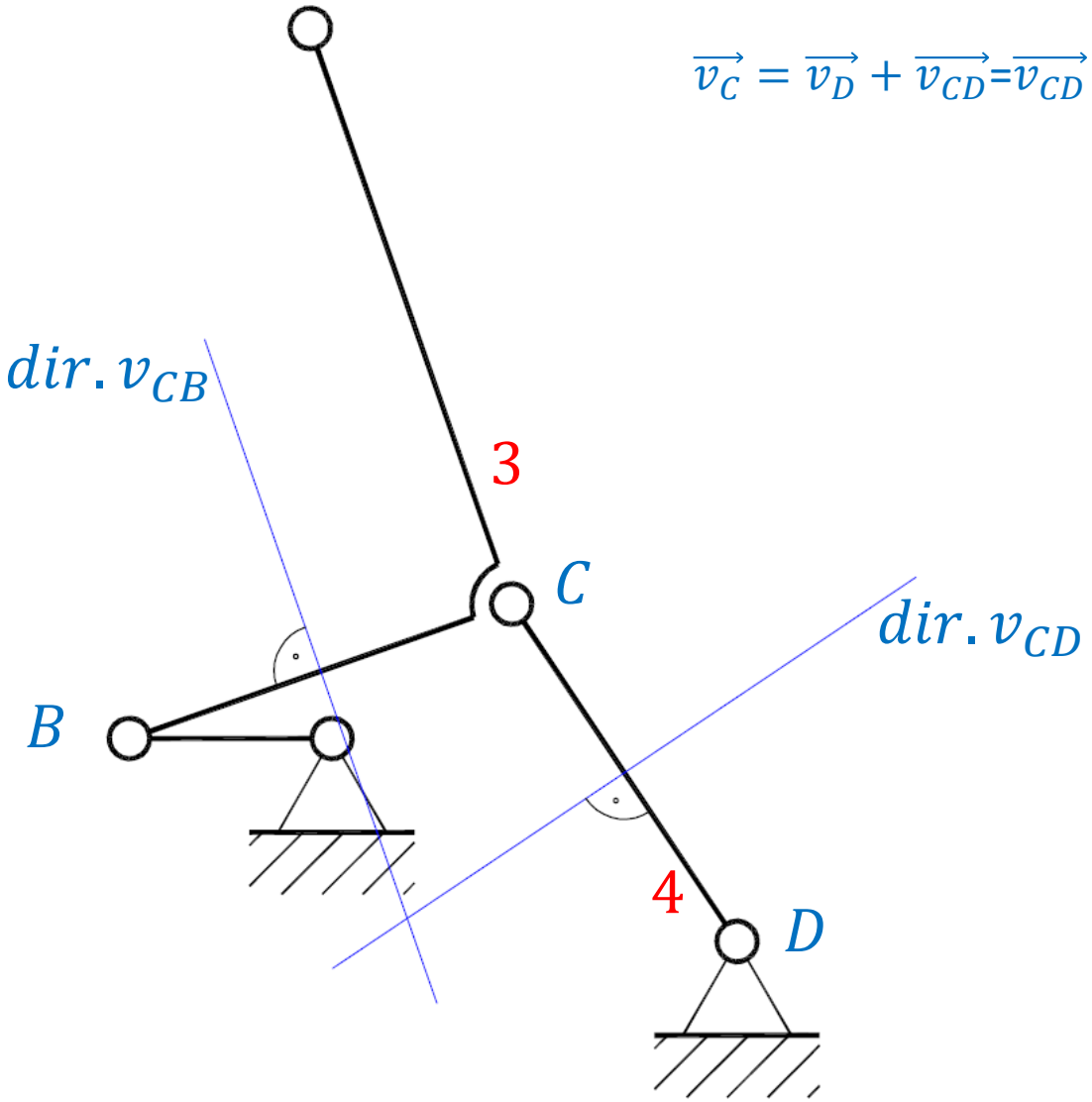


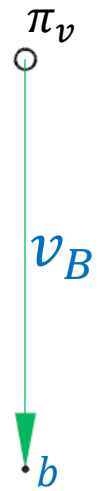
$\overrightarrow{v_B} = \overrightarrow{v_A} + \overrightarrow{v_{BA}} = \omega_2 \cdot \mathrm{AB} = 0{,}2\left[\frac{m}{s}\right]$

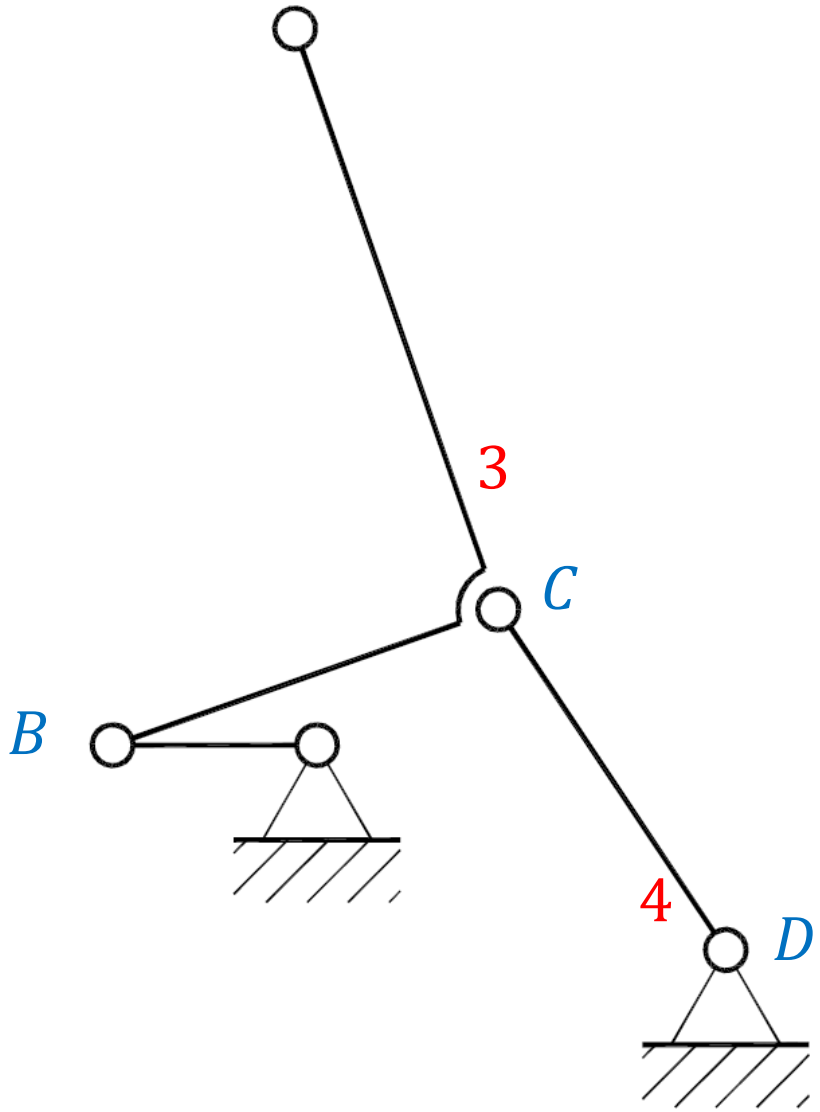


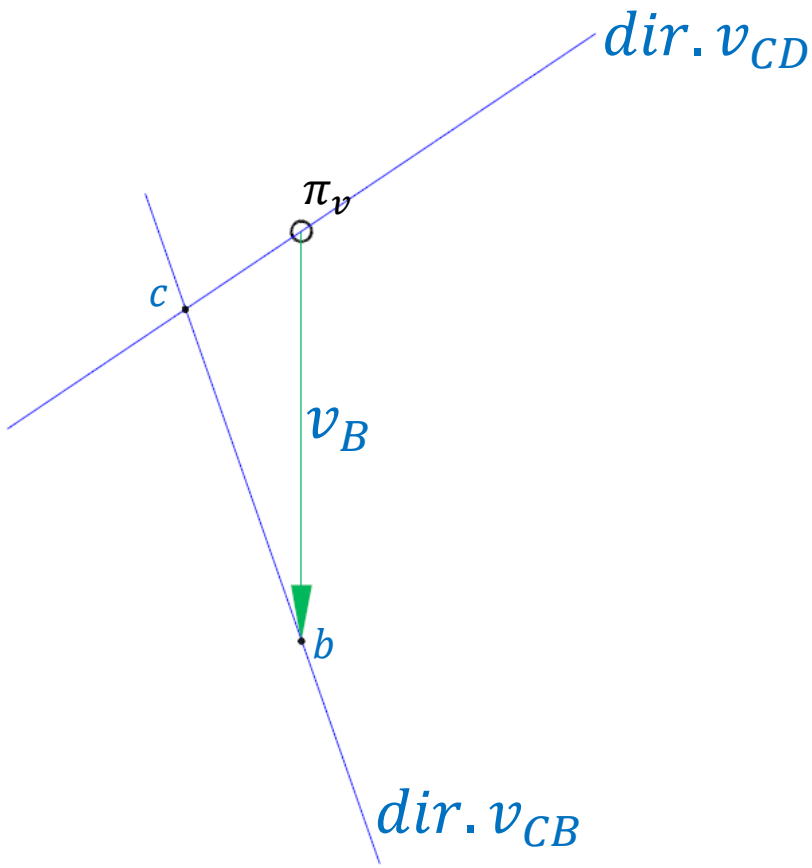
$$\overrightarrow{v_C} = \overrightarrow{v_B} + \overrightarrow{v_{CB}}$$

$$\overrightarrow{v_C} = \overrightarrow{v_D} + \overrightarrow{v_{CD}} = \overrightarrow{v_{CD}}$$

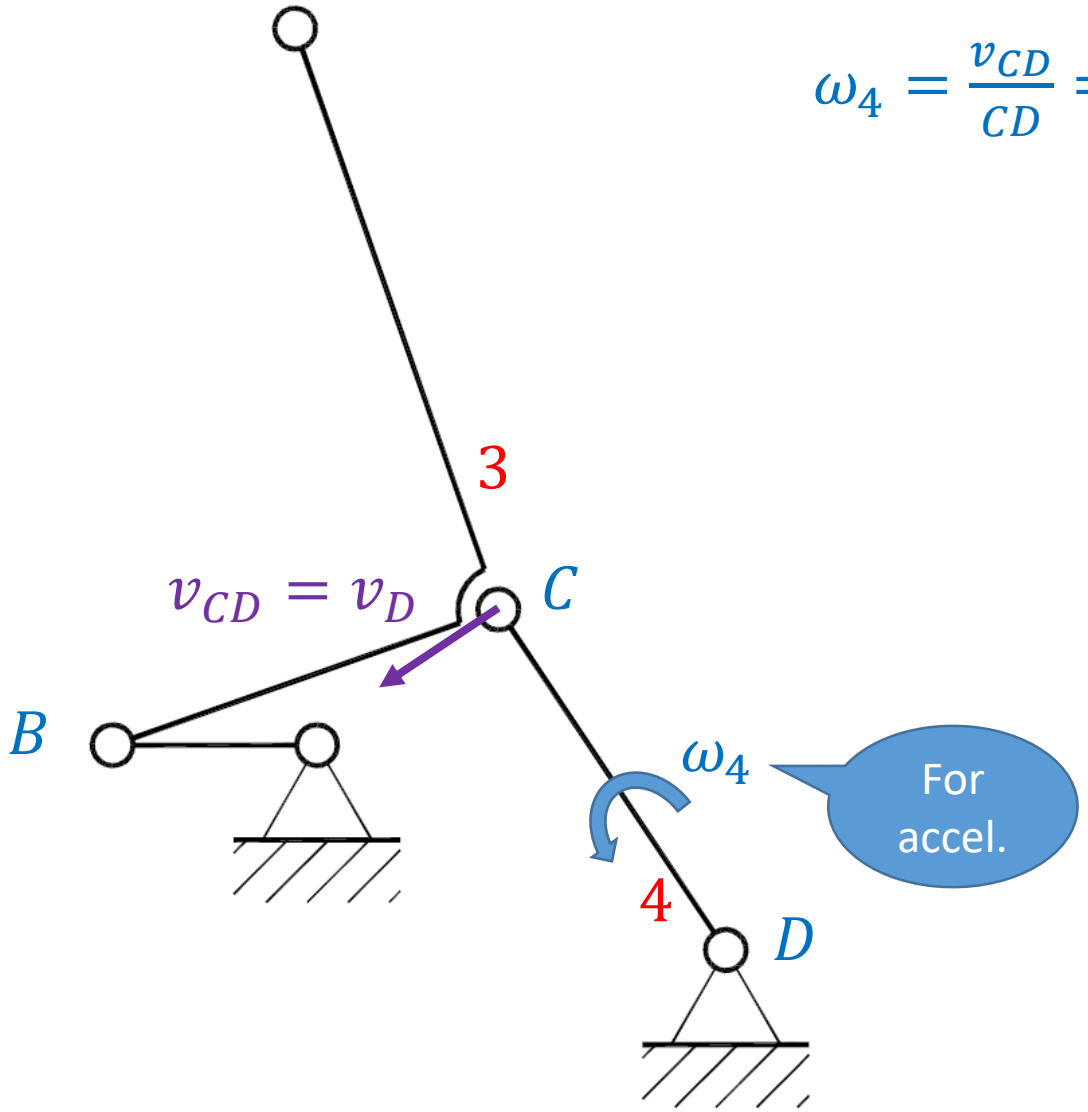


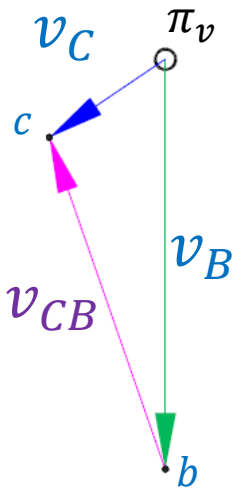



3
C
B
4
D

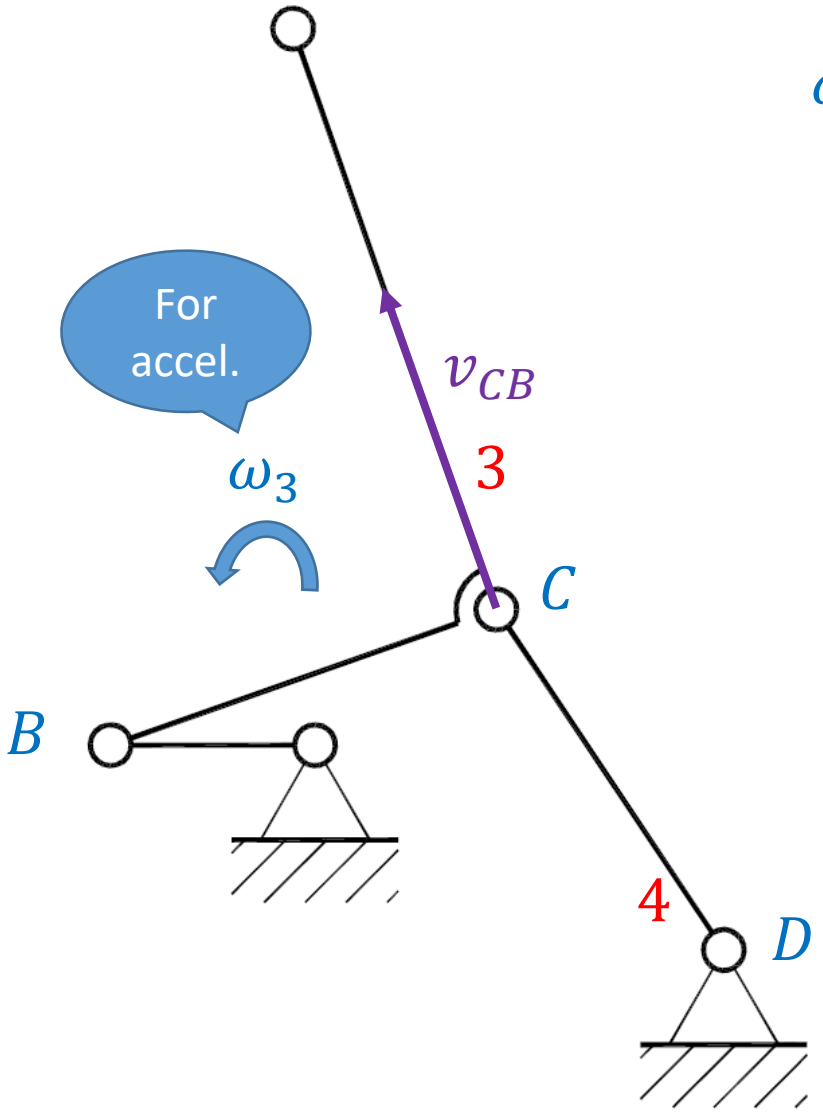

$dir.v_{CD}$
π_v
c
v_B
b
$dir.v_{CB}$

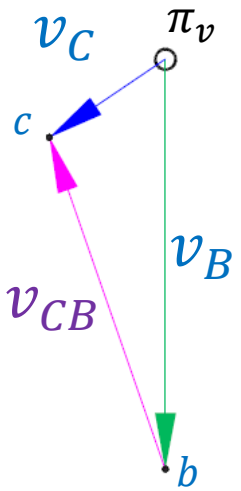
$$\omega_4 = \frac{v_{CD}}{CD} = \frac{0{,}06836\left[\frac{m}{s}\right]}{0{,}2[m]} = 0{,}3418\left[\frac{rad}{s}\right]$$





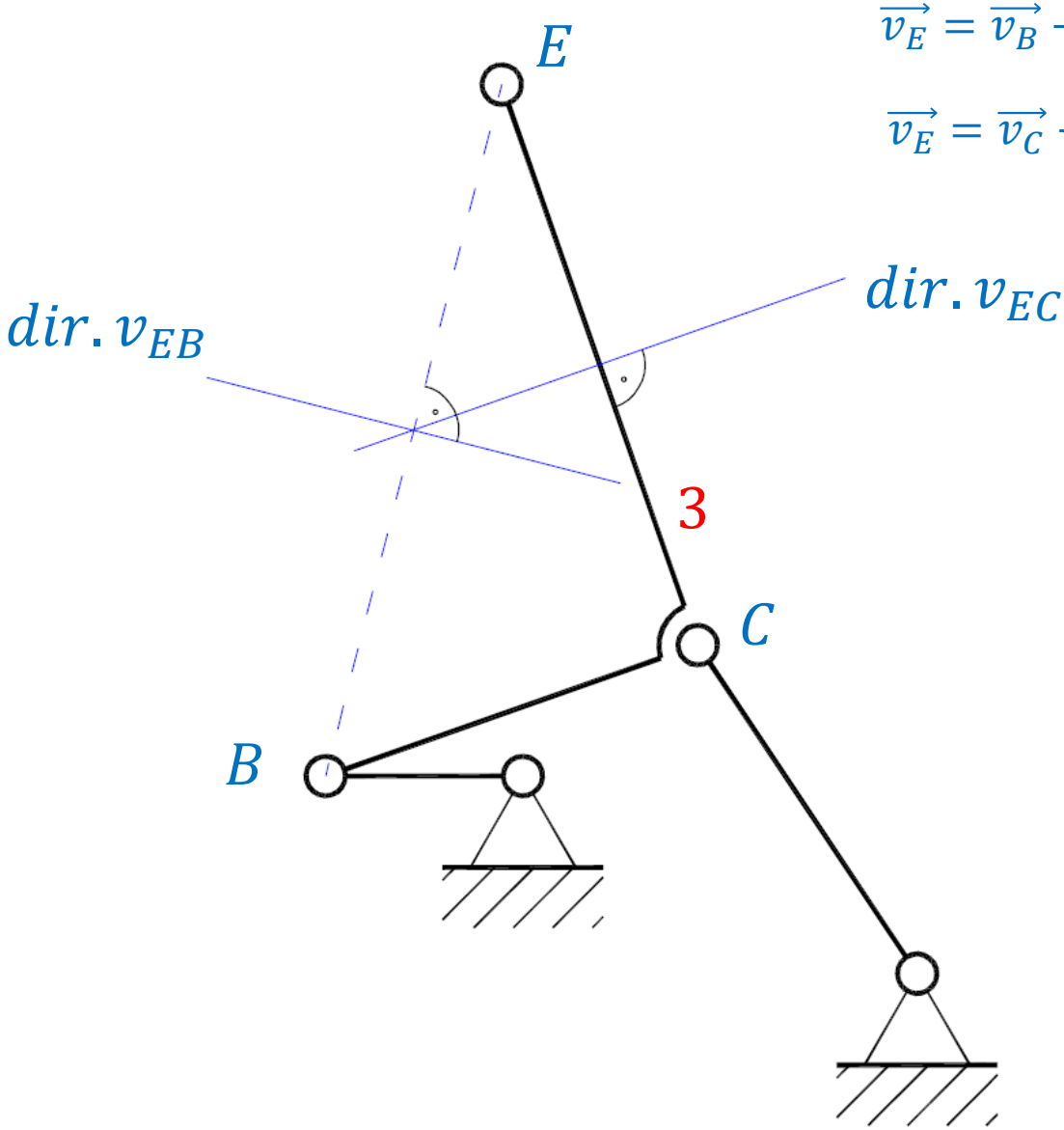
$$\omega_3 = \frac{v_{CB}}{CB} = \frac{0{,}17164\left[\frac{m}{s}\right]}{0{,}2[m]} = 0{,}8582\left[\frac{rad}{s}\right]$$

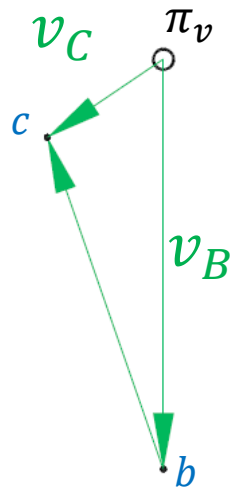


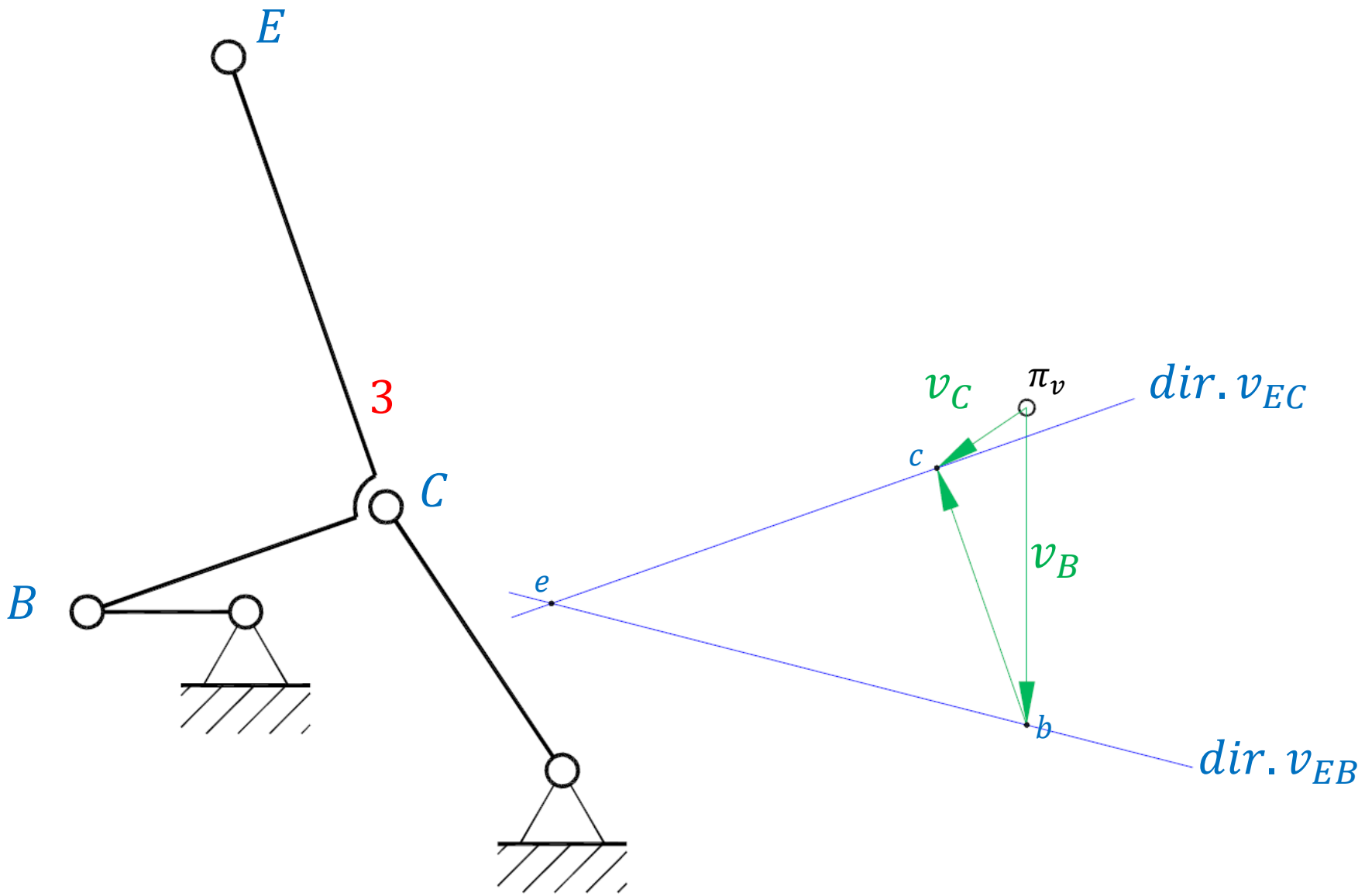


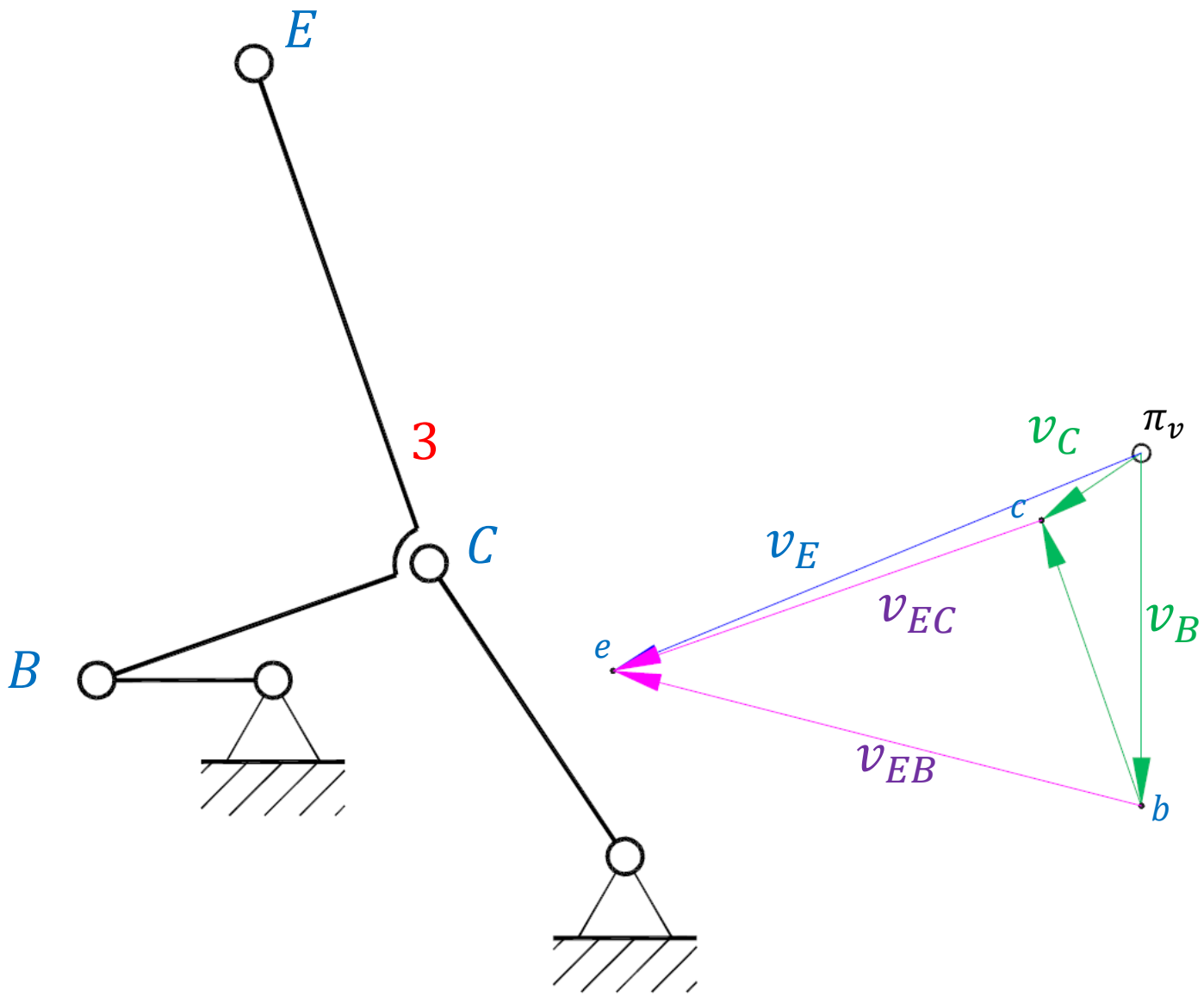
$$\overrightarrow{v_E} = \overrightarrow{v_B} + \overrightarrow{v_{EB}}$$

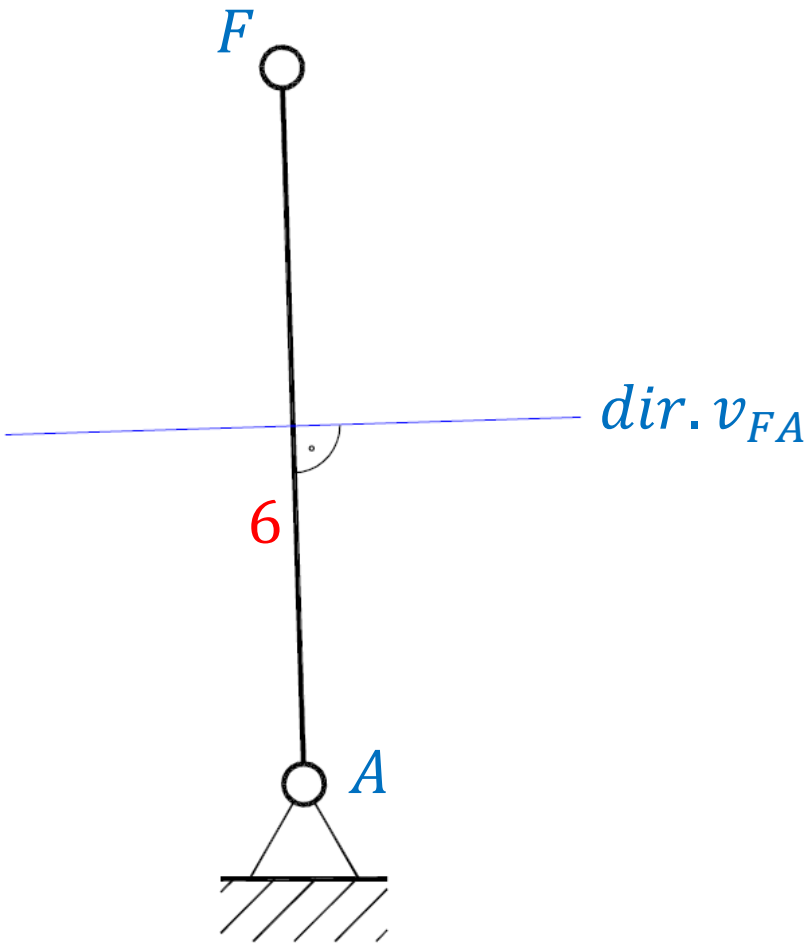
$$\overrightarrow{v_E} = \overrightarrow{v_C} + \overrightarrow{v_{EC}}$$

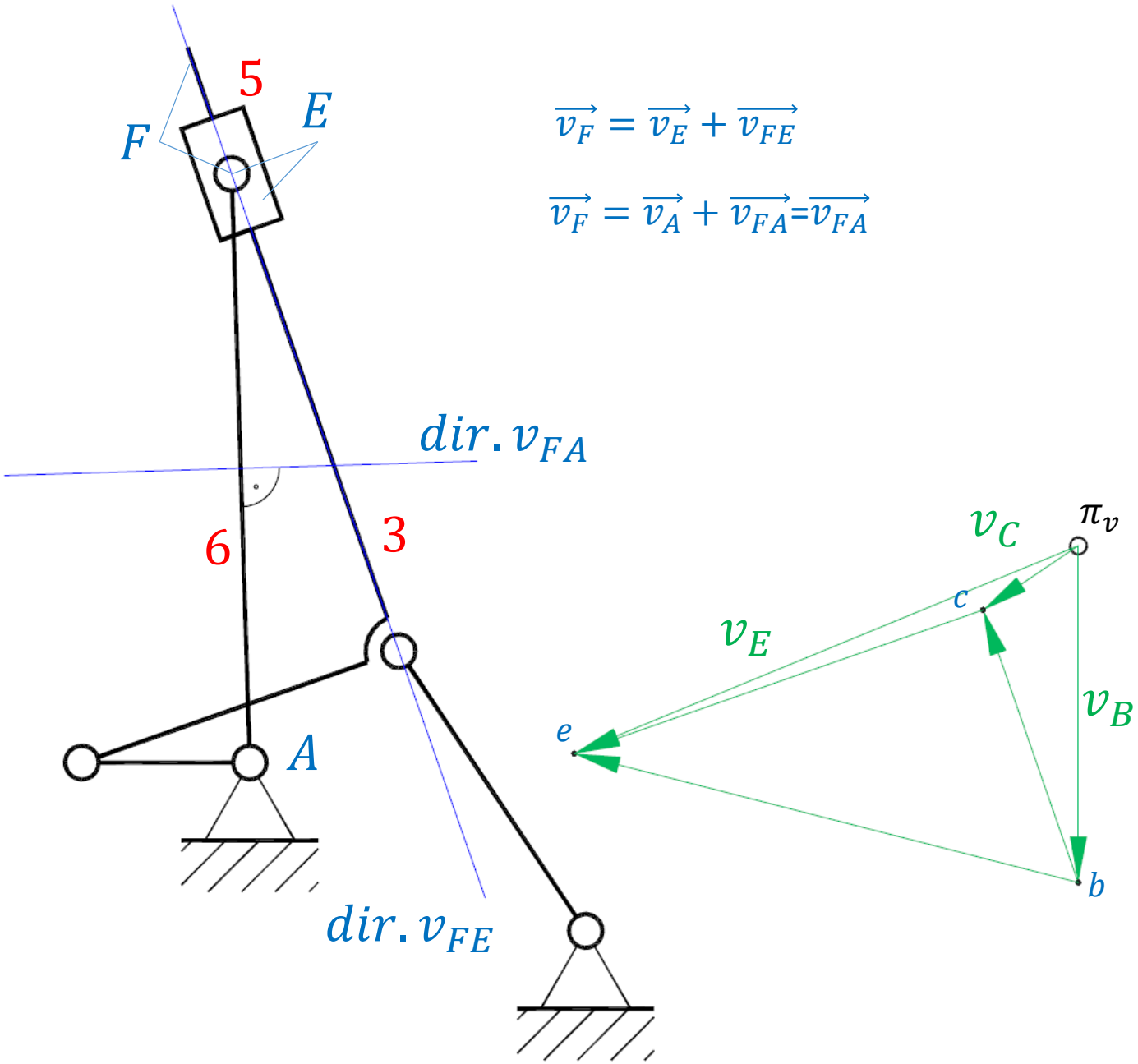





E
3
C
B
v_C
π_v
$dir.v_{EC}$
c
v_B
e
b
$dir.v_{EB}$

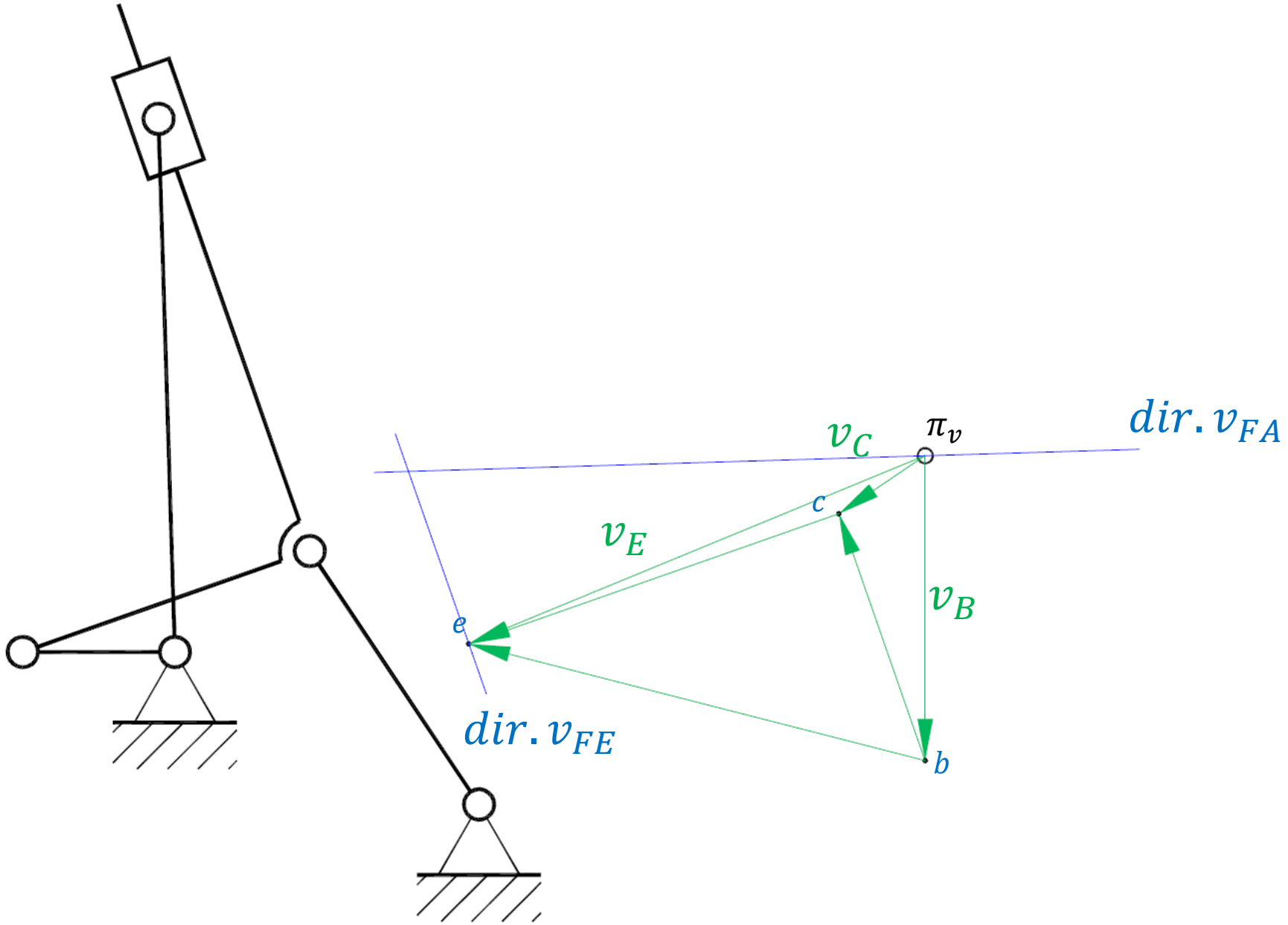

E
3
C
B
v_C
π_v
c
v_E
v_{EC}
v_B
e
v_{EB}
b


F
dir. v_{FA}
6
A



$$\overrightarrow{v_F} = \overrightarrow{v_E} + \overrightarrow{v_{FE}}$$

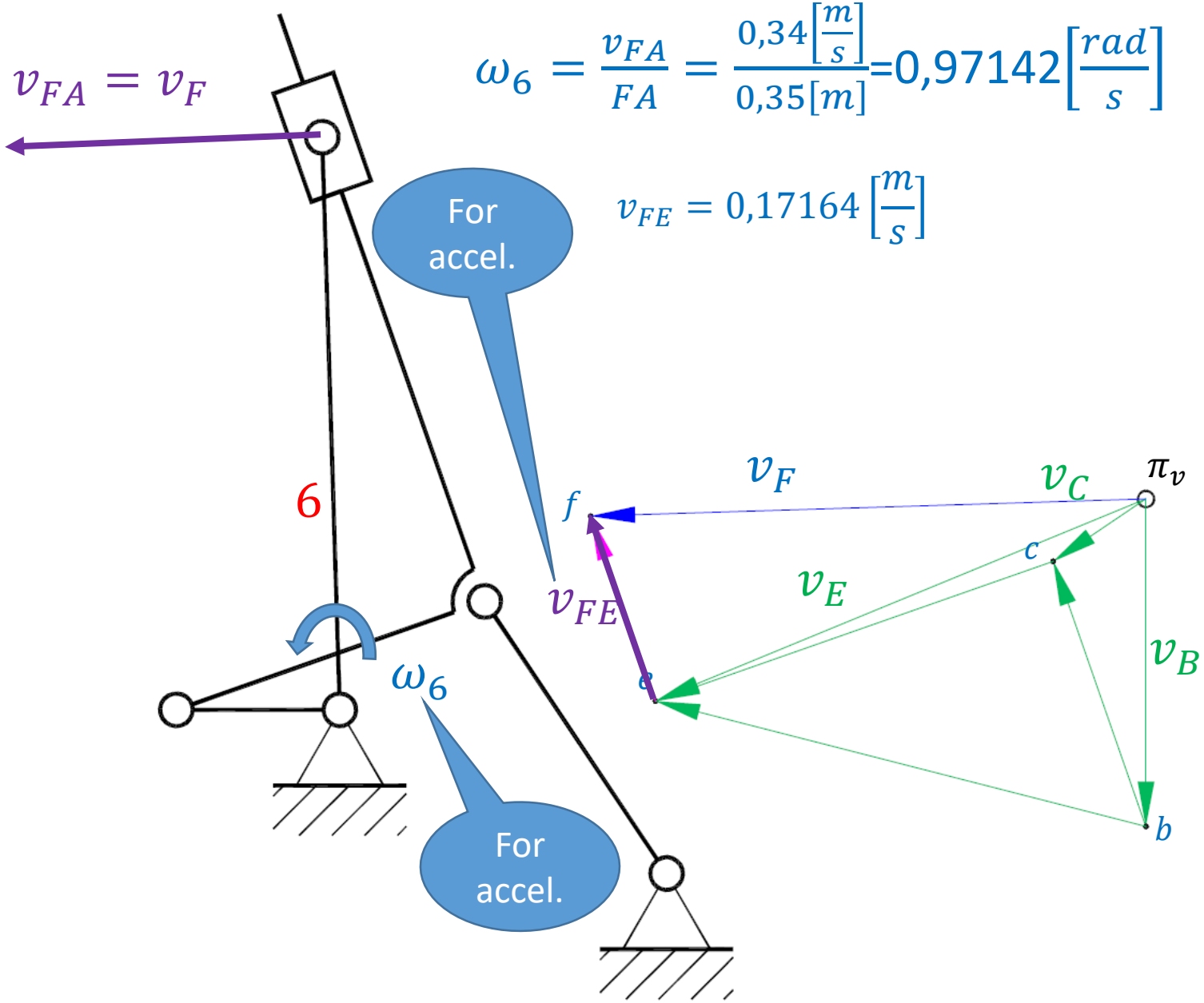
$$\overrightarrow{v_F} = \overrightarrow{v_A} + \overrightarrow{v_{FA}} = \overrightarrow{v_{FA}}$$

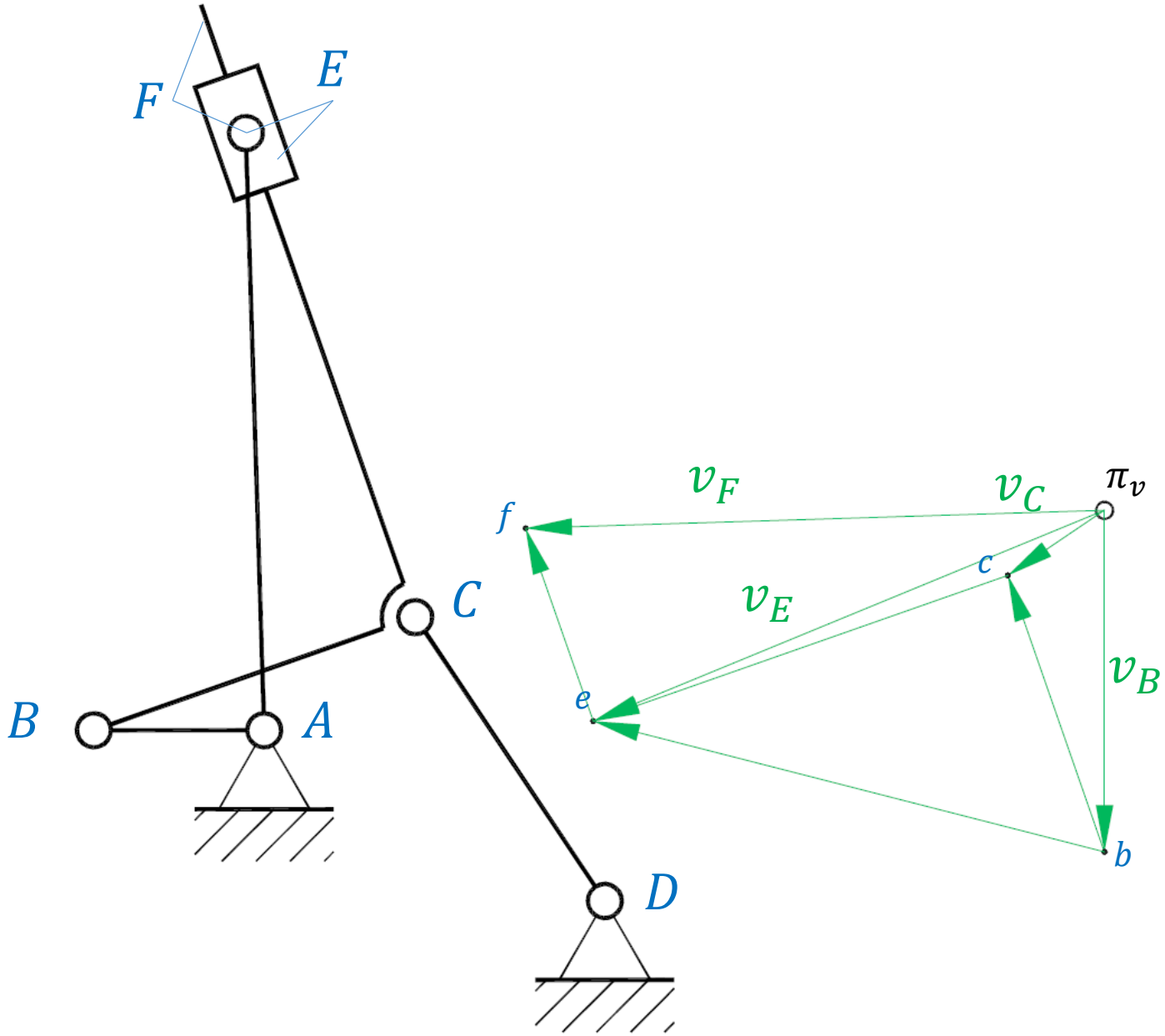

v_C
π_v
$dir.v_{FA}$
v_E
c
v_B
e
$dir.v_{FE}$
b

$$v_{FA} = v_F$$

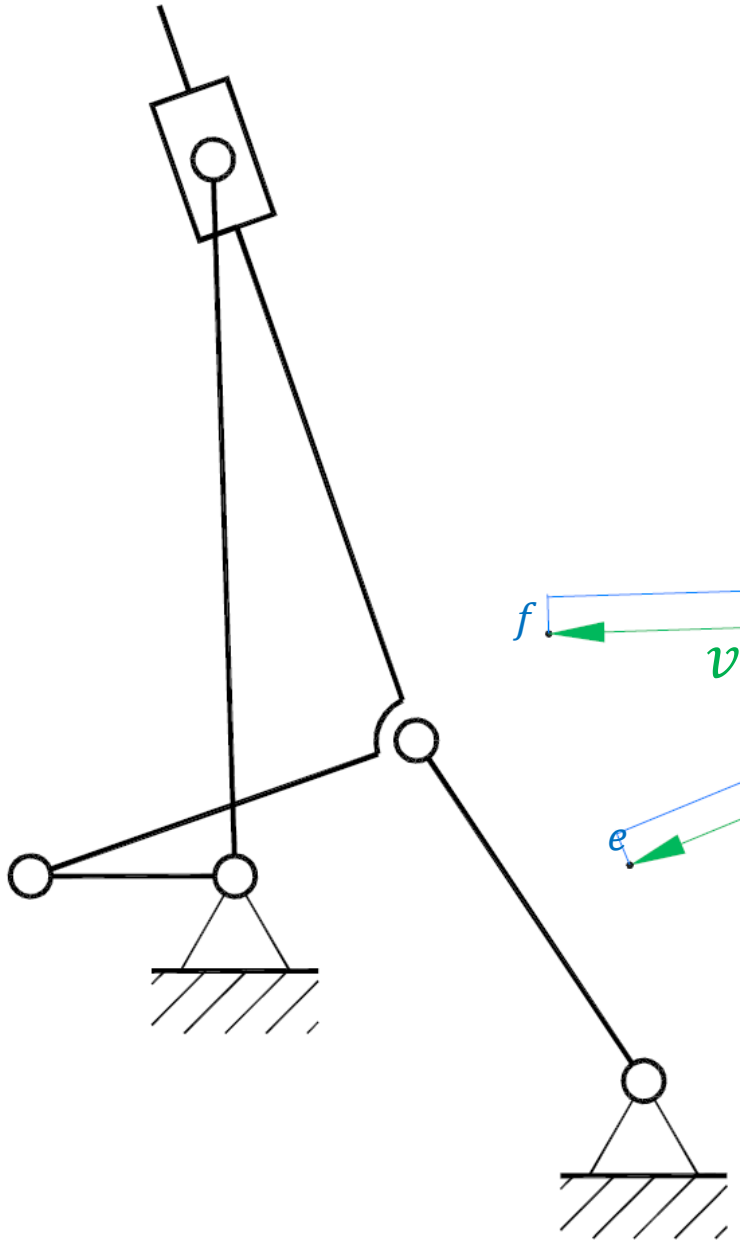
$$\omega_6 = \frac{v_{FA}}{FA} = \frac{0{,}34\left[\frac{m}{s}\right]}{0{,}35[m]} = 0{,}97142\left[\frac{rad}{s}\right]$$

$$v_{FE} = 0{,}17164\left[\frac{m}{s}\right]$$

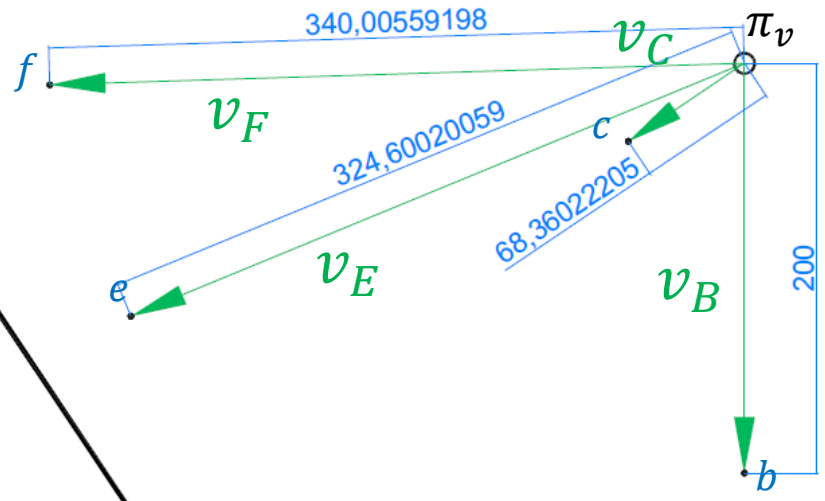



F
E
C
B
A
D
v_F
v_C
π_v
f
c
v_E
v_B
e
b

Velocity scale



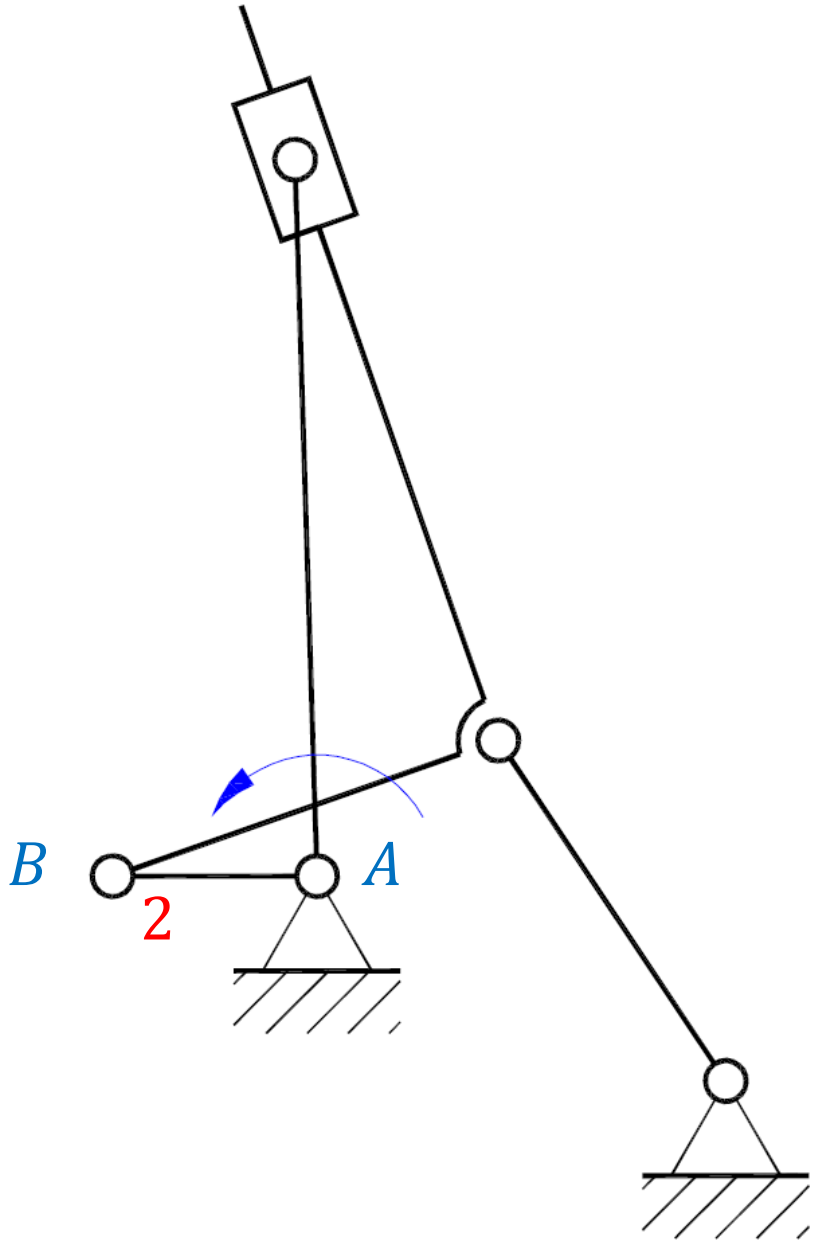
	polygon	
v_B	0,2	
v_C	0,06836	
v_E	0,3246	
v_F	0,34	



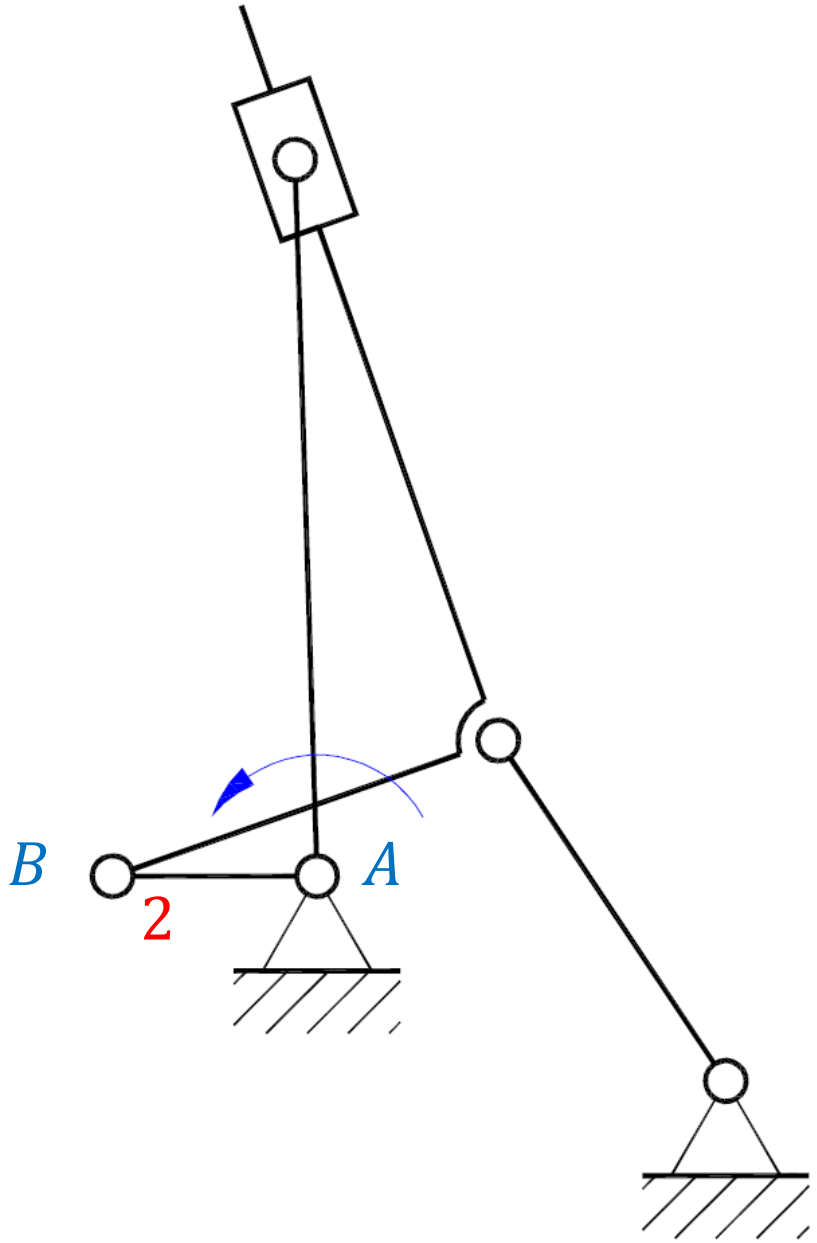
$$100\ [mm]$$

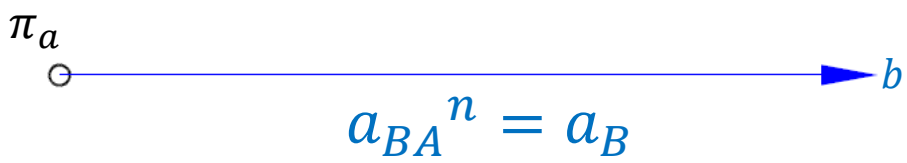
$$0{,}1\left[\frac{m}{s}\right]$$

STEP II - Acceleration



$$\overrightarrow{a_B} = \overrightarrow{v_A} + \overrightarrow{a_{BA}} = \overrightarrow{a_{BA}^{n}} + \overrightarrow{a_{BA}^{t}} = \overrightarrow{a_{BA}^{n}} = \omega_2^2 \cdot \mathrm{AB} = 0{,}4\left[\frac{m}{s^2}\right]$$





$100\ [mm]$

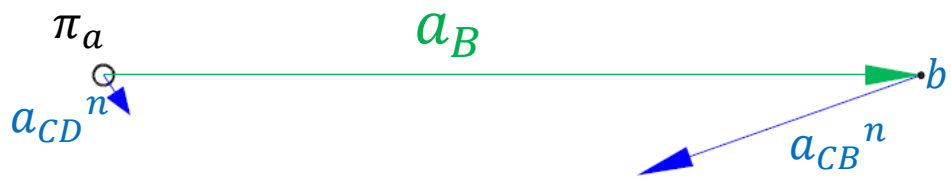
$0{,}1\ \left[\frac{m}{s^2}\right]$

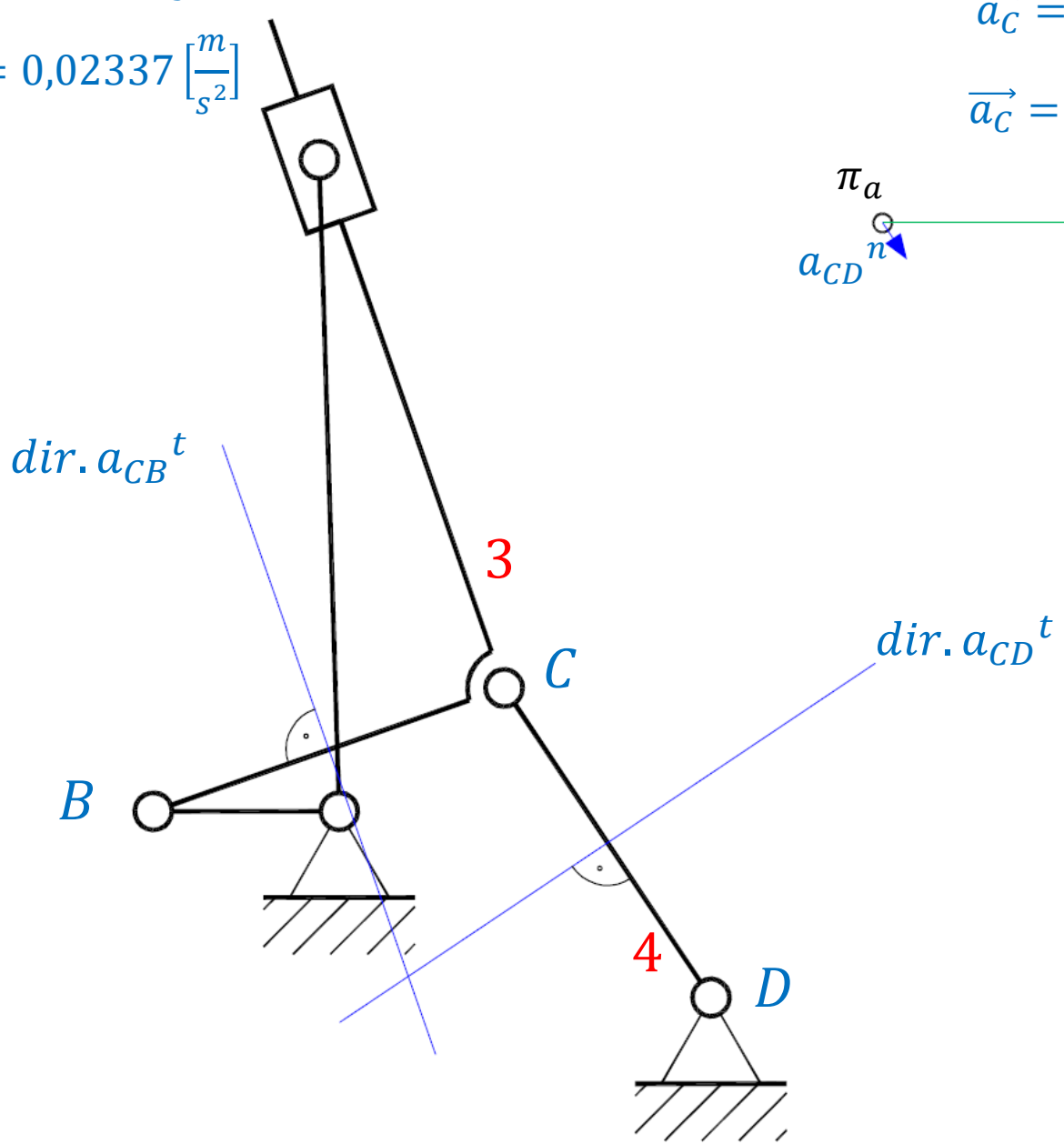
$$a_{CB}{}^{n} = \omega_3{}^2 CB = 0{,}1473 \left[\frac{m}{s^2}\right]$$

$$a_{CD}{}^{n} = \omega_4{}^2 CD = 0{,}02337 \left[\frac{m}{s^2}\right]$$

$$\overrightarrow{a_C} = \overrightarrow{a_B} + \overrightarrow{a_{CB}{}^{n}} + \overrightarrow{a_{CB}{}^{t}}$$

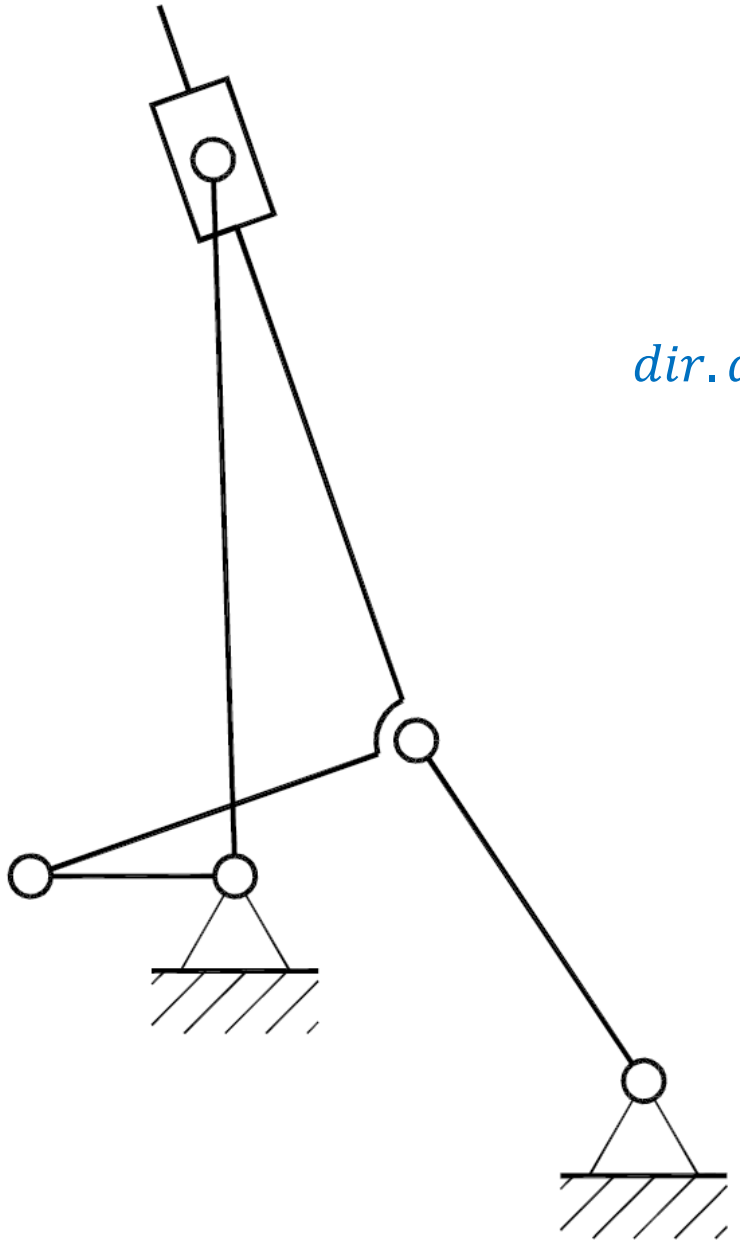
$$\overrightarrow{a_C} = \overrightarrow{a_D} + \overrightarrow{a_{CD}{}^{n}} + \overrightarrow{a_{CD}{}^{t}} = \overrightarrow{a_{CD}{}^{n}} + \overrightarrow{a_{CD}{}^{t}}$$

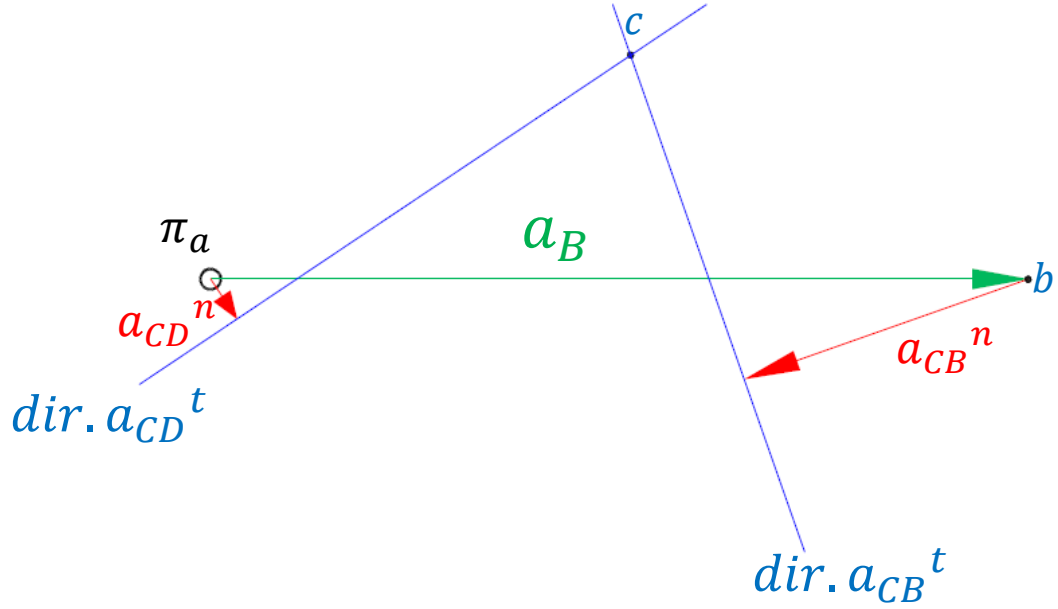




$100\ [mm]$

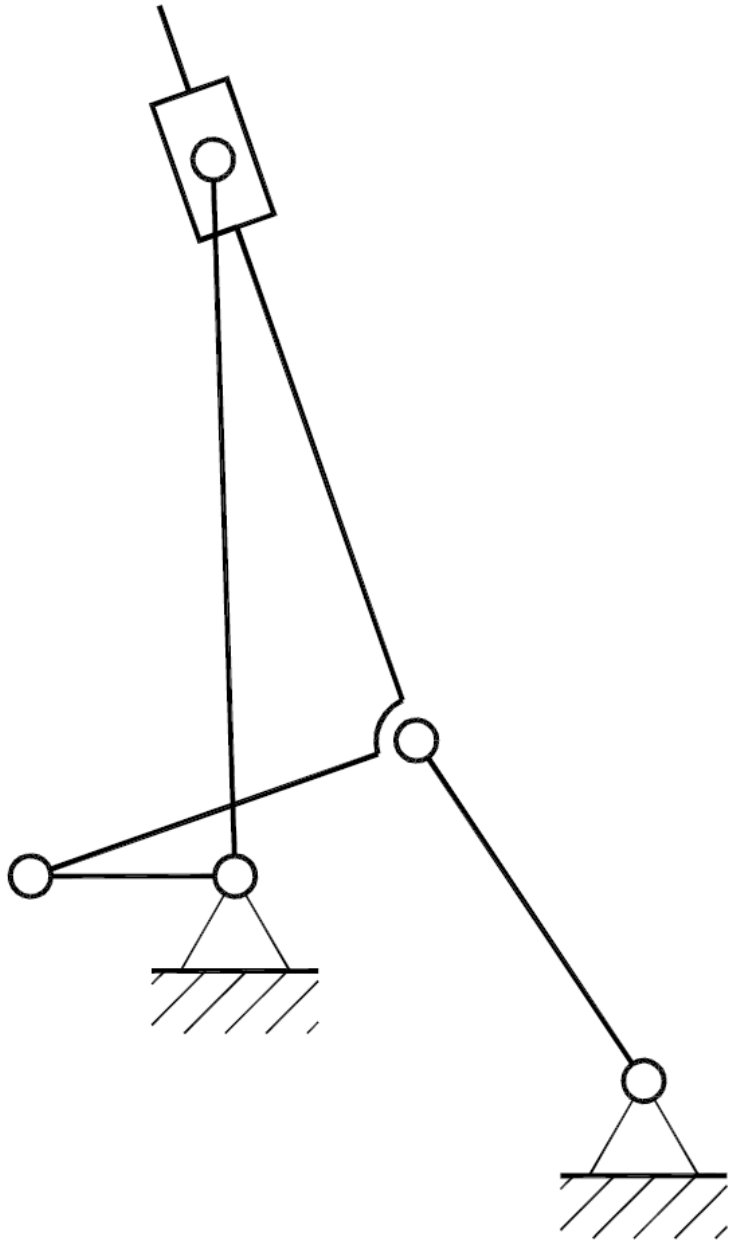
$0{,}1 \left[\frac{m}{s^2}\right]$

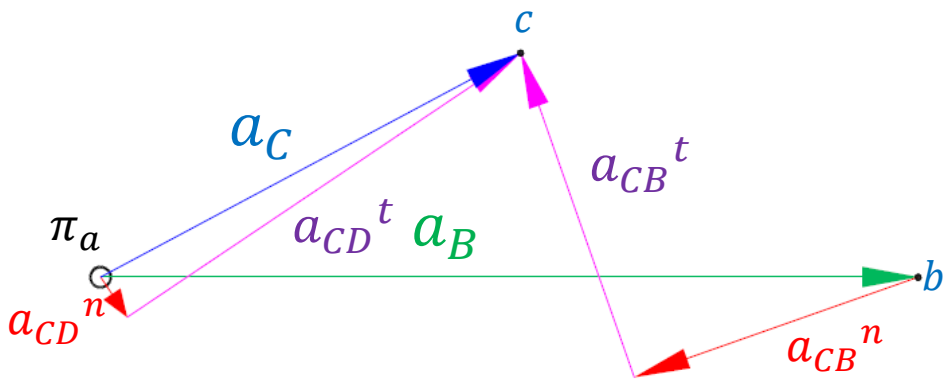




$100\ [mm]$

$0{,}1\ \left[\frac{m}{s^2}\right]$





$$100\ [mm]$$

$$0{,}1\left[\frac{m}{s^2}\right]$$

$a_{EB}{}^{n} = \omega_3{}^2 BC = 0{,}26591 \left[\frac{m}{s^2}\right]$

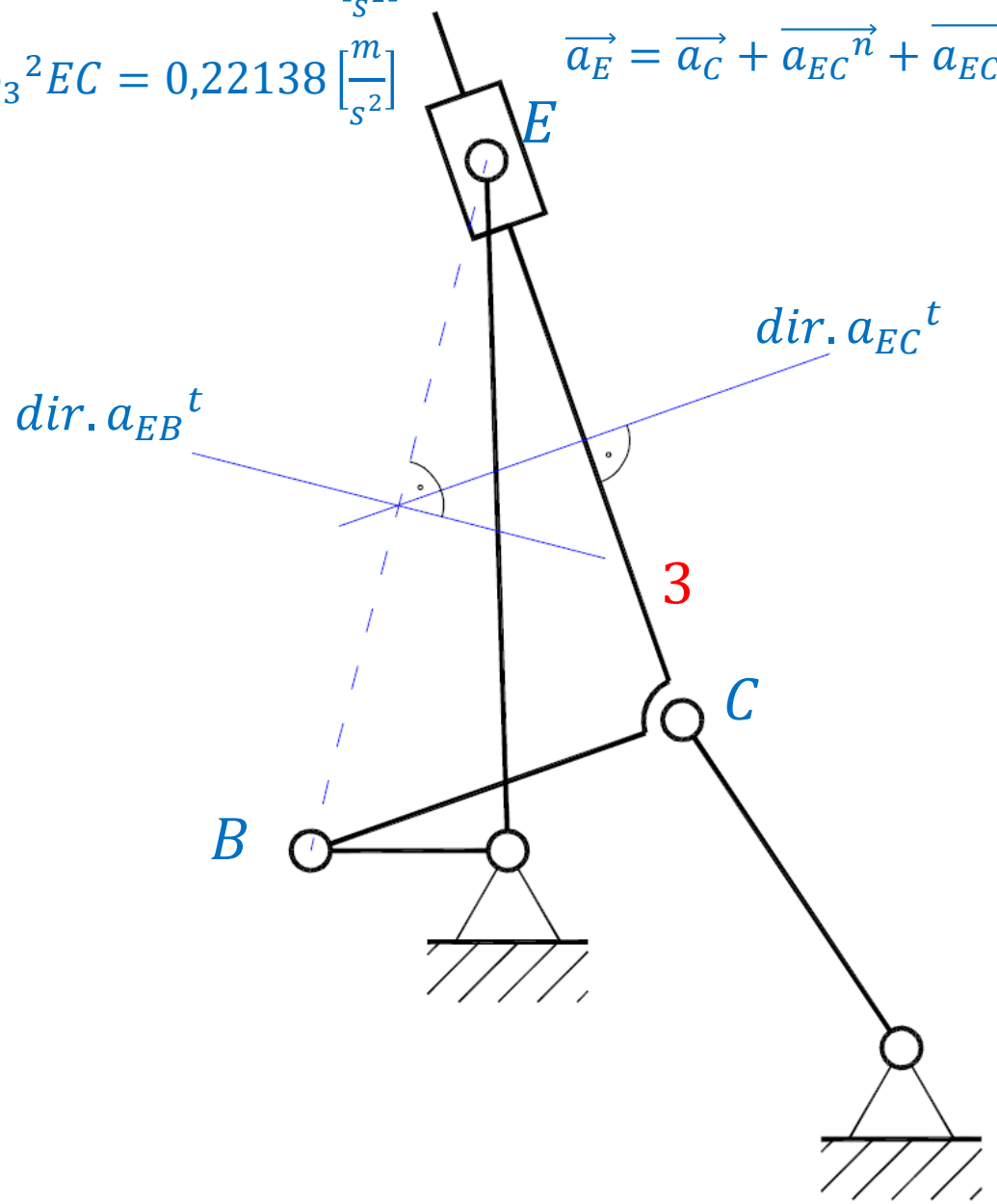
$a_{EC}{}^{n} = \omega_3{}^2 EC = 0{,}22138 \left[\frac{m}{s^2}\right]$

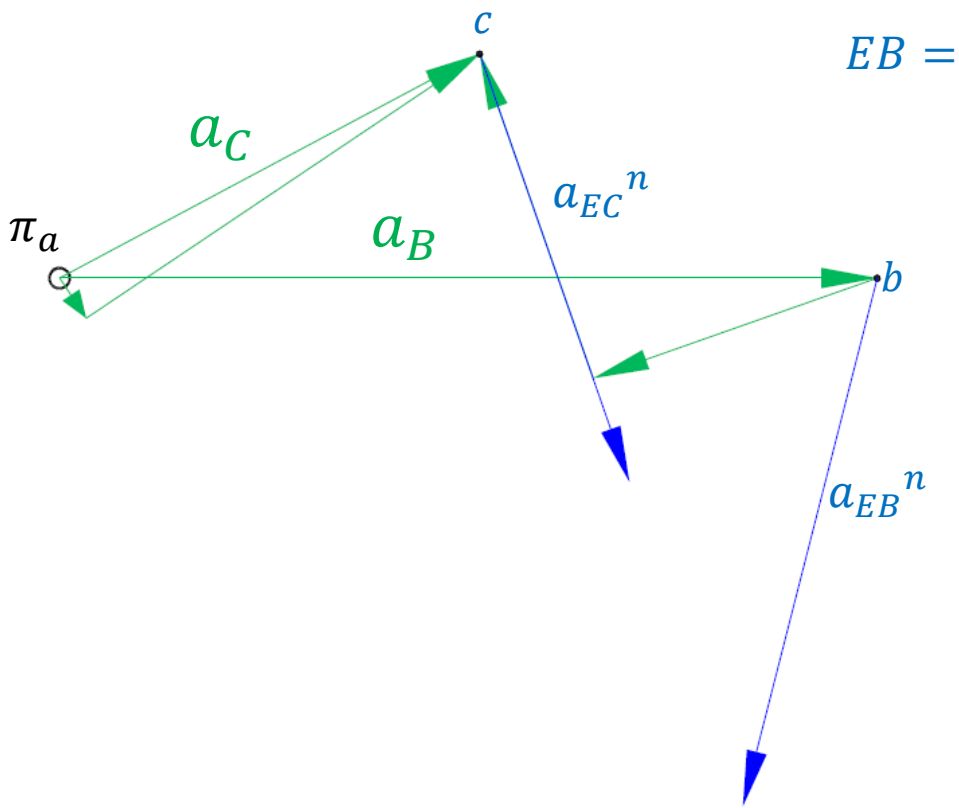
$\overrightarrow{a_E} = \overrightarrow{a_B} + \overrightarrow{a_{EB}{}^{n}} + \overrightarrow{a_{EB}{}^{t}}$

$\overrightarrow{a_E} = \overrightarrow{a_C} + \overrightarrow{a_{EC}{}^{n}} + \overrightarrow{a_{EC}{}^{t}}$

$EC = 0{,}30058[m]$

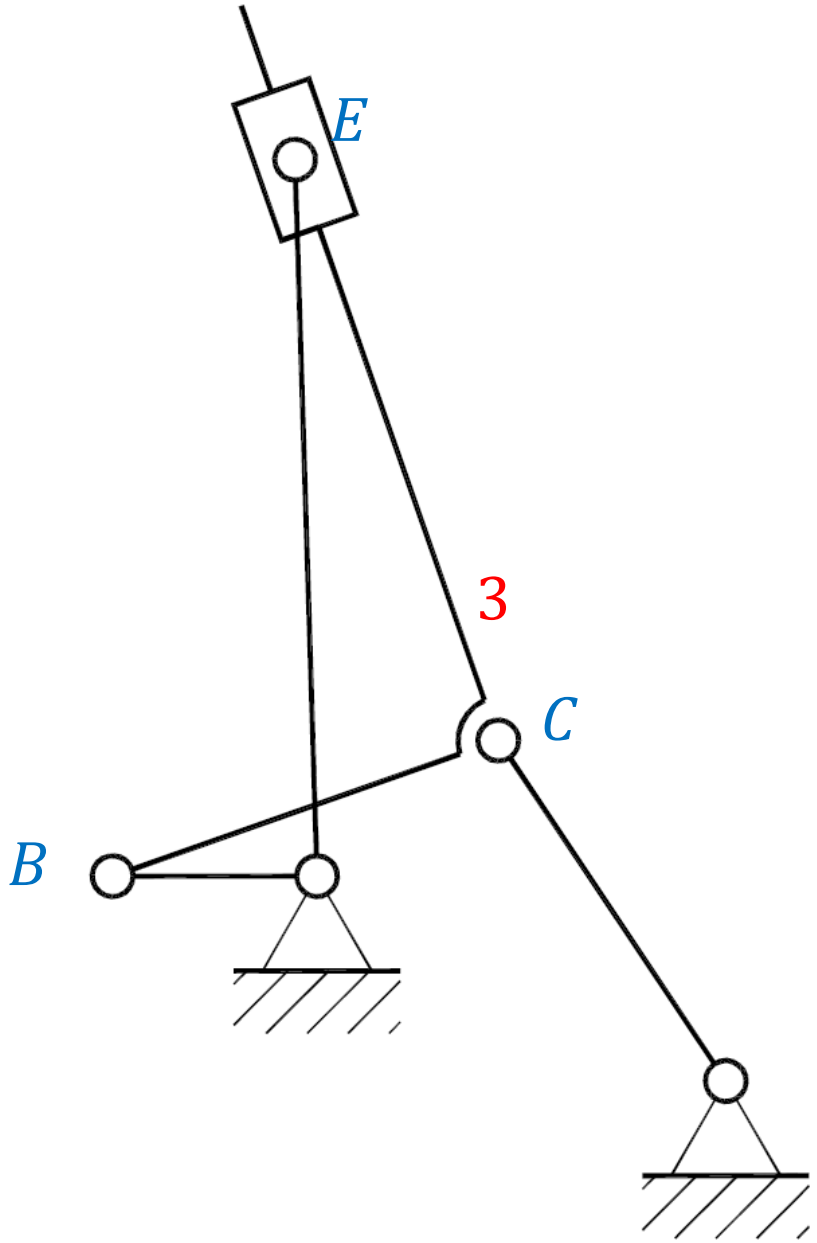
$EB = 0{,}36104[m]$

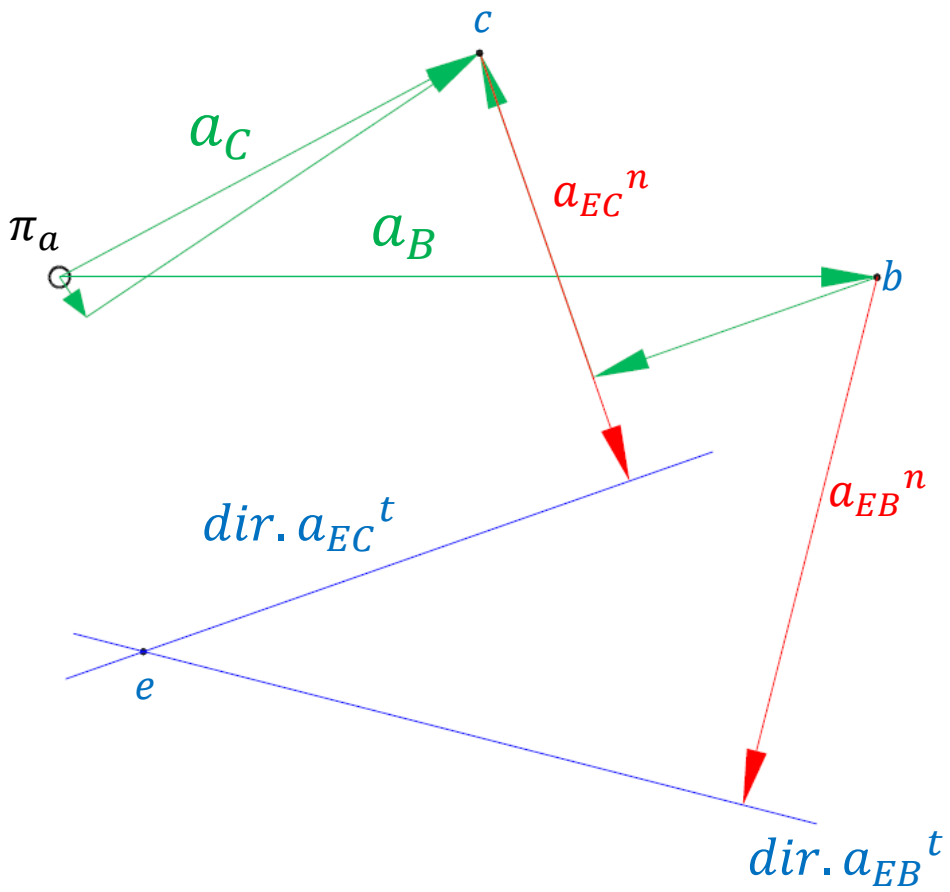




$100\ [mm]$

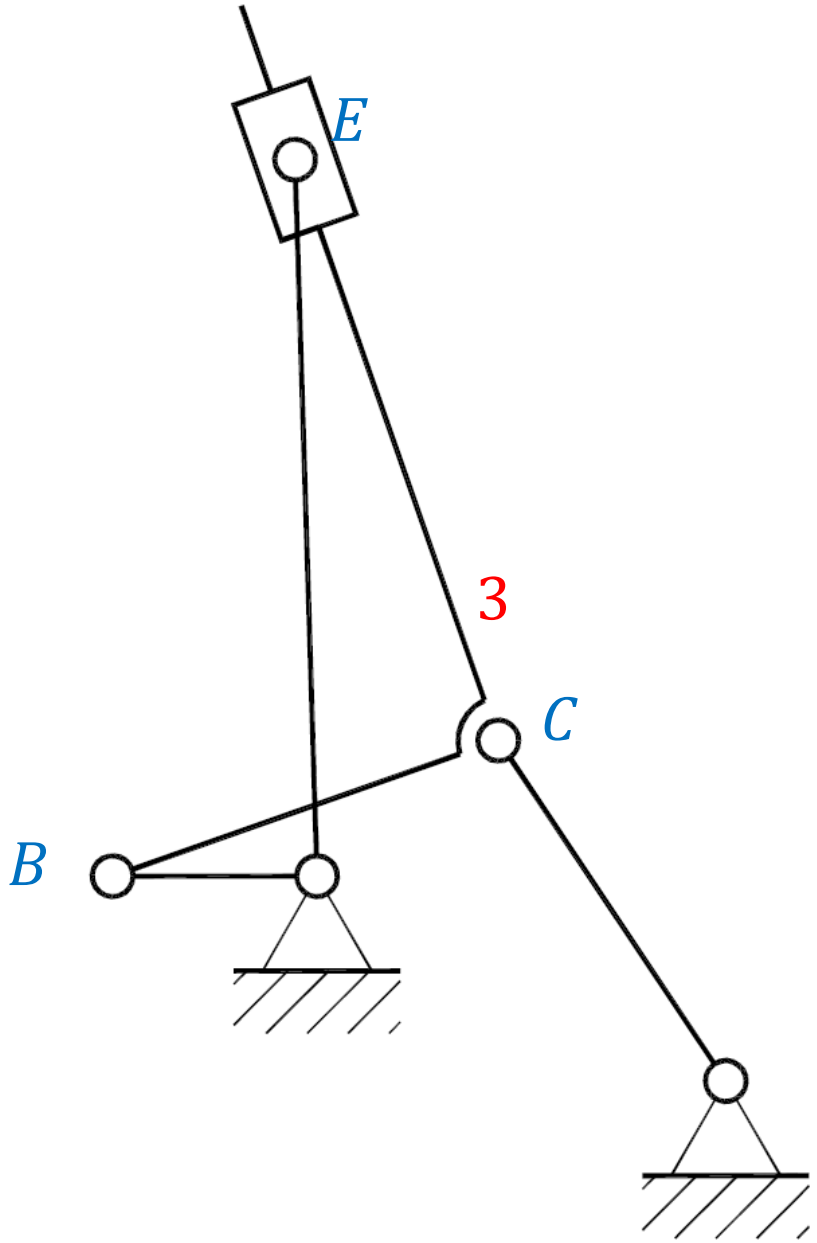
$0{,}1 \left[\frac{m}{s^2}\right]$

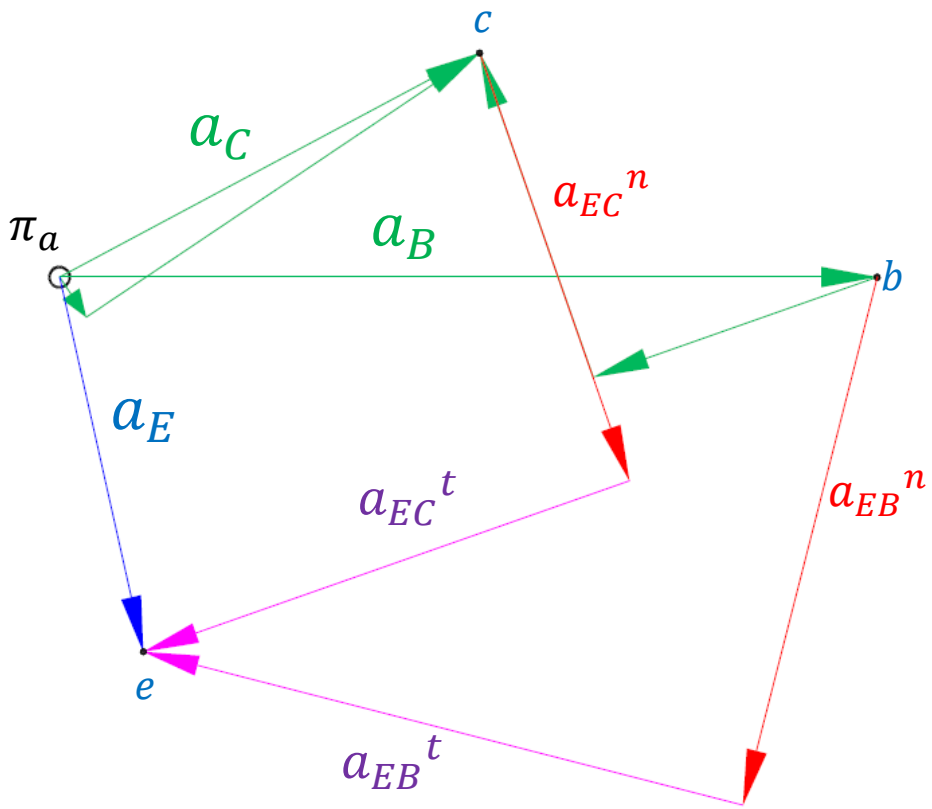




$100\ [mm]$

$0{,}1\left[\frac{m}{s^2}\right]$





$100\ [mm]$

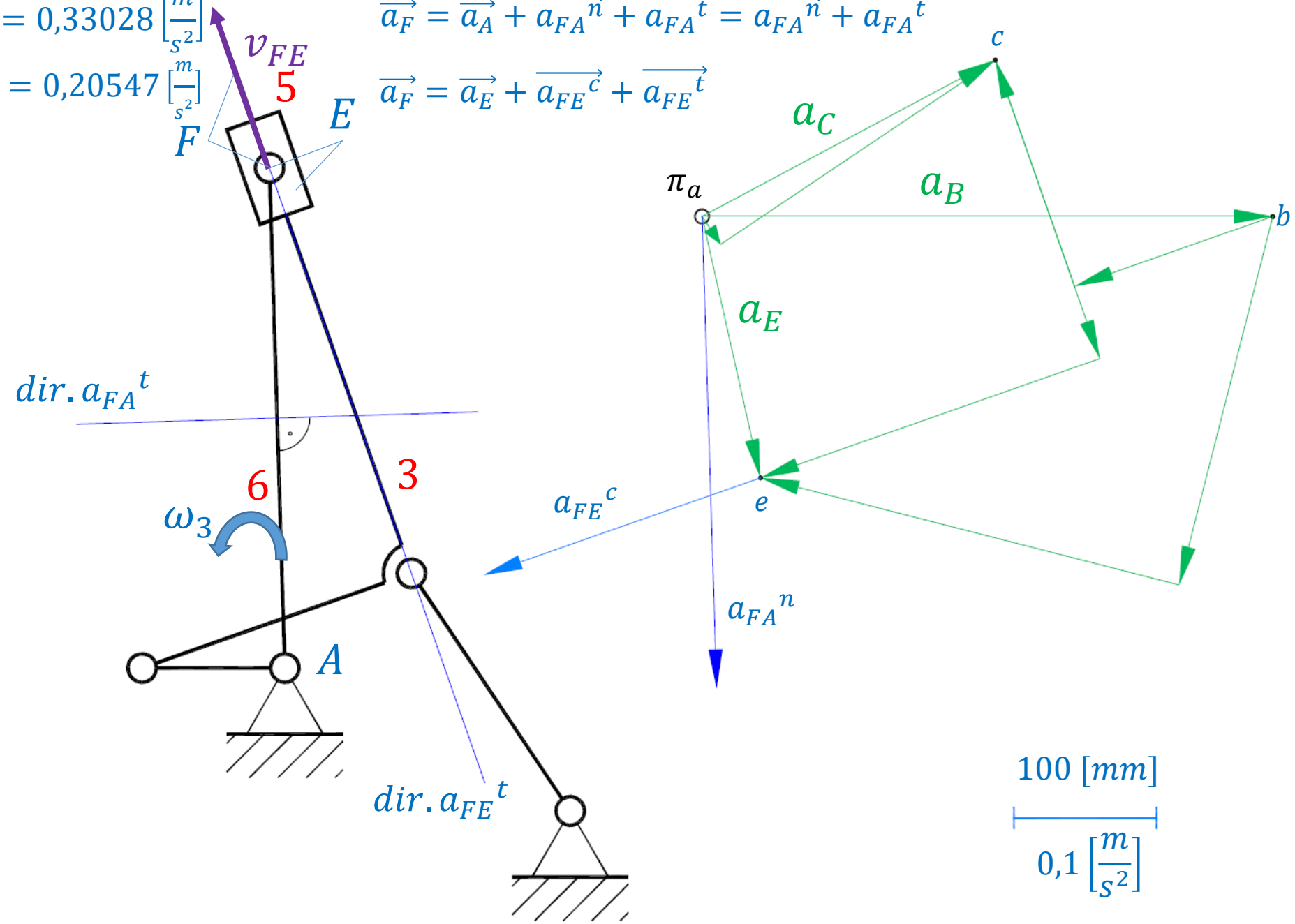
$0{,}1\ \left[\frac{m}{s^2}\right]$

$a_{FA}{}^{n} = \omega_6{}^2 FA = 0{,}33028 \left[\frac{m}{s^2}\right]$

$a_{FE}{}^{c} = 2\omega_3 v_{FE} = 0{,}20547 \left[\frac{m}{s^2}\right]$

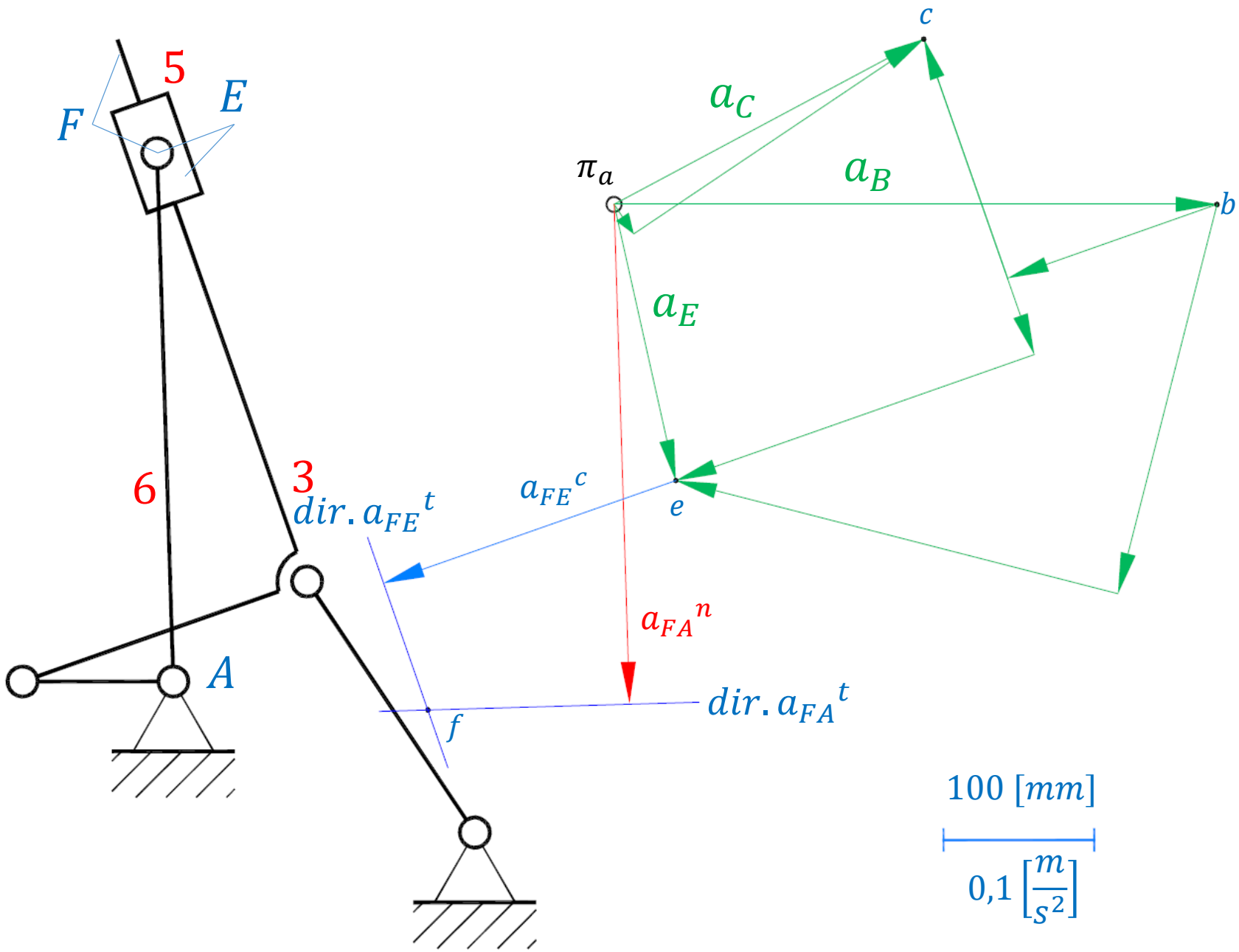
$\overrightarrow{a_F} = \overrightarrow{a_A} + \overrightarrow{a_{FA}{}^n} + \overrightarrow{a_{FA}{}^t} = \overrightarrow{a_{FA}{}^n} + \overrightarrow{a_{FA}{}^t}$

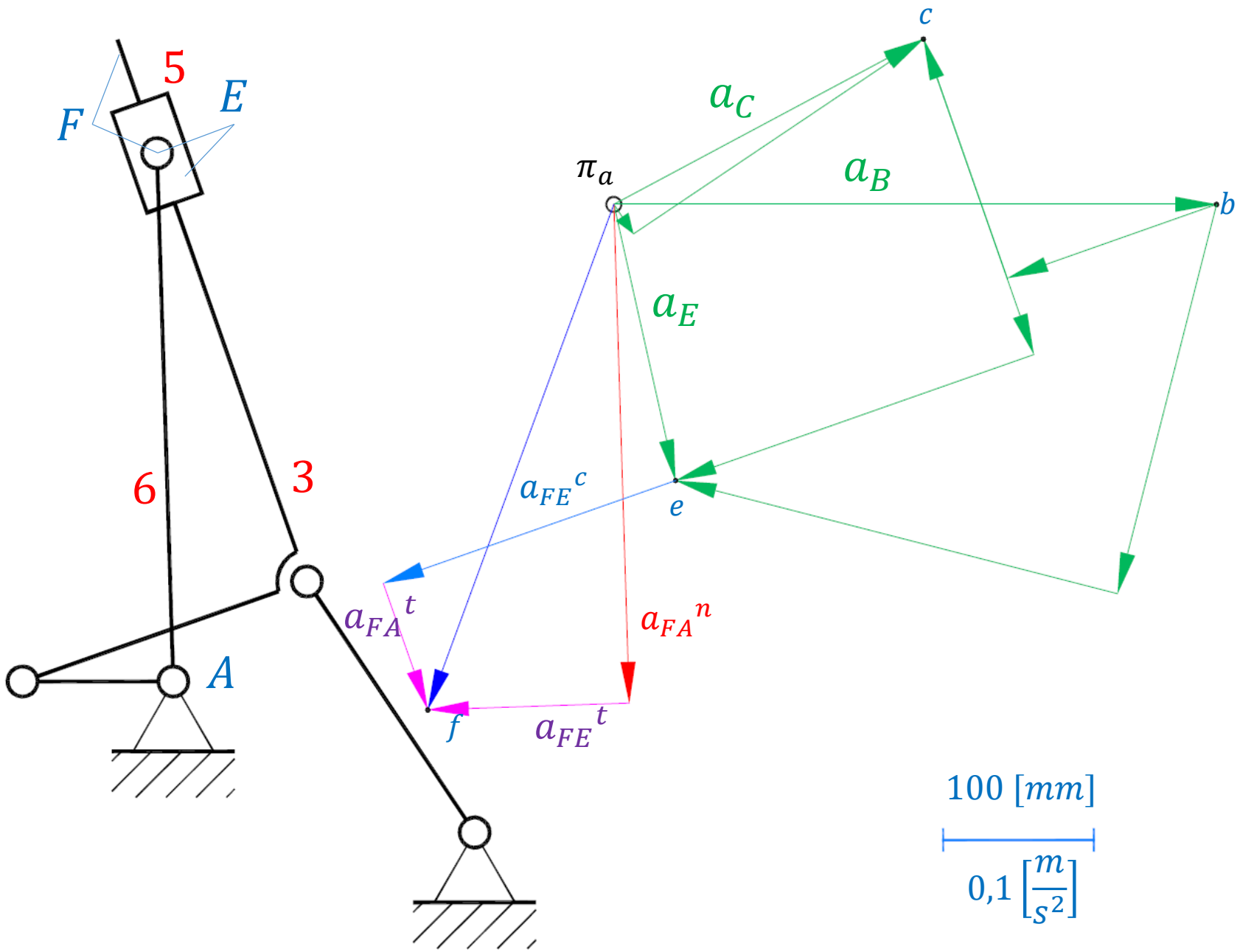
$\overrightarrow{a_F} = \overrightarrow{a_E} + \overrightarrow{a_{FE}{}^c} + \overrightarrow{a_{FE}{}^t}$

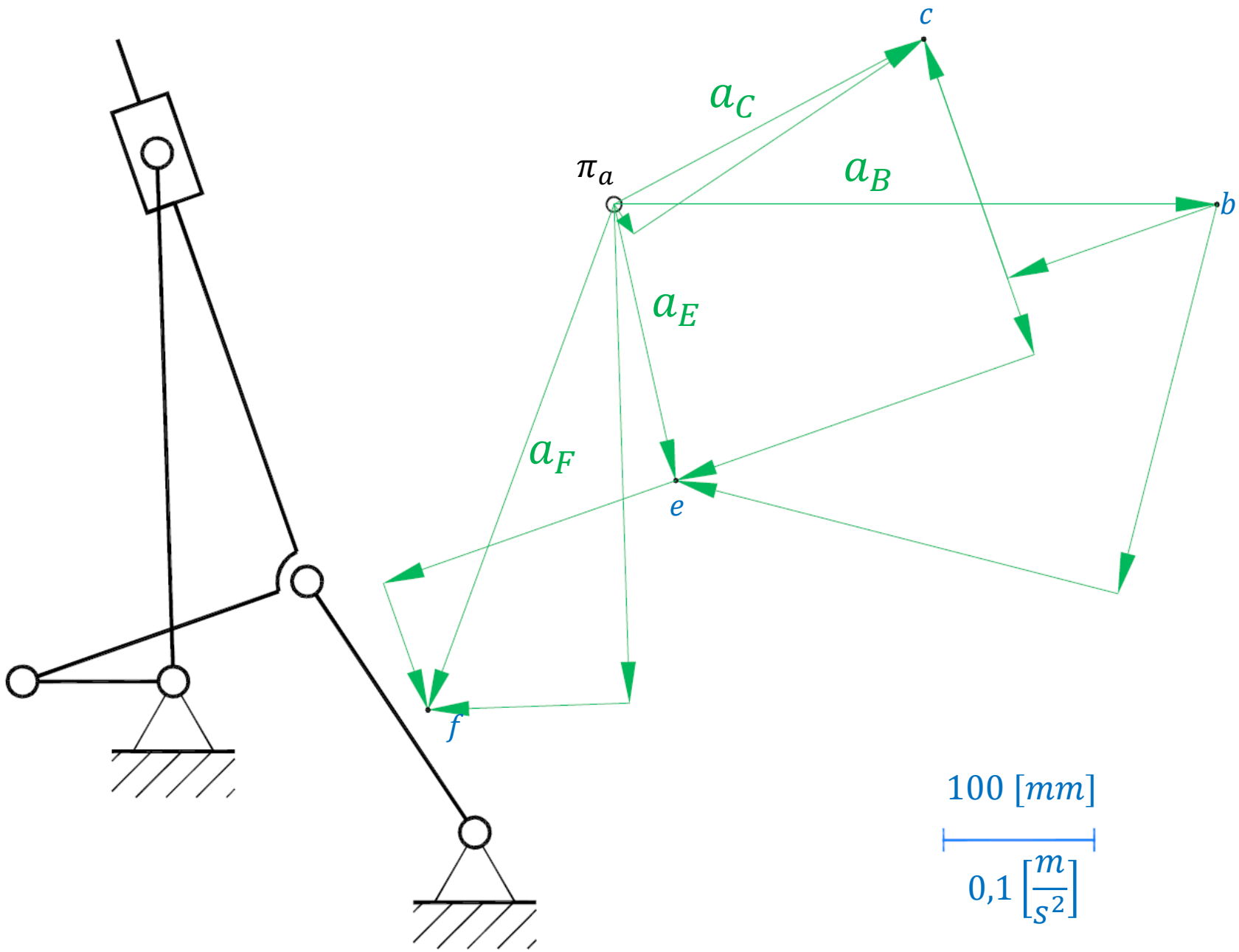


100 [mm]

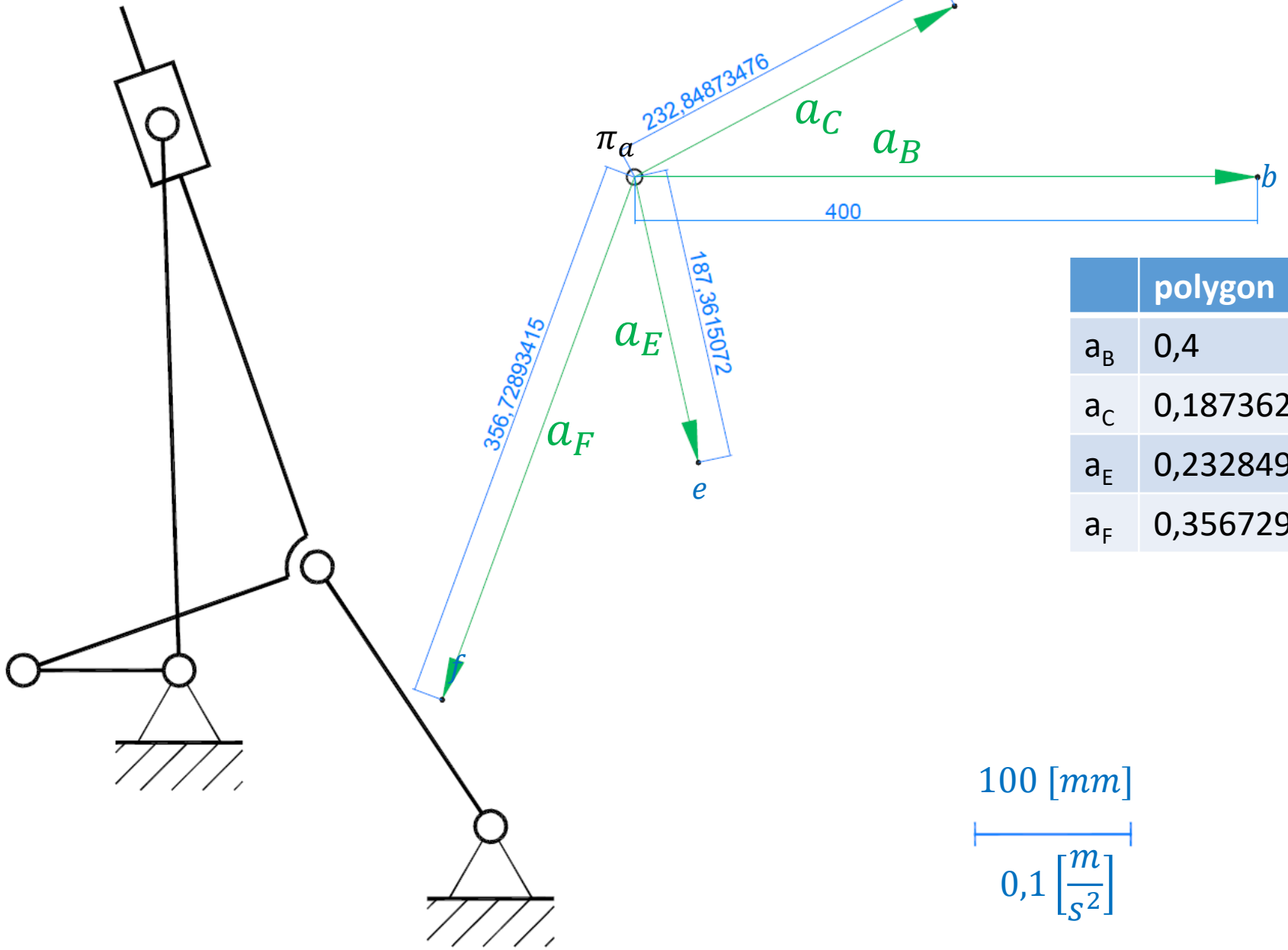
$0{,}1 \left[\frac{m}{s^2}\right]$


5
F
E
6
3
A
$dir.a_{FE}{}^{t}$
$a_{FE}{}^{c}$
$a_{FA}{}^{n}$
$dir.a_{FA}{}^{t}$
f
e
π_a
c
b
a_C
a_B
a_E
100 [mm]
$0{,}1\left[\frac{m}{s^2}\right]$


5
F
E
6
3
A
c
a_C
π_a
a_B
b
a_E
$a_{FE}{}^c$
e
$a_{FA}{}^t$
$a_{FA}{}^n$
f
$a_{FE}{}^t$
100 [mm]
0,1 $\left[\frac{m}{s^2}\right]$


c
a_C
π_a
a_B
b
a_E
a_F
e
f
100 [mm]
0,1 $\left[\frac{m}{s^2}\right]$

Acceleration scale



	polygon	
a_B	0,4	
a_C	0,187362	
a_E	0,232849	
a_F	0,356729	

$$\frac{100\ [mm]}{0{,}1\ \left[\frac{m}{s^2}\right]}$$

Comparison after numerical method (SAM)

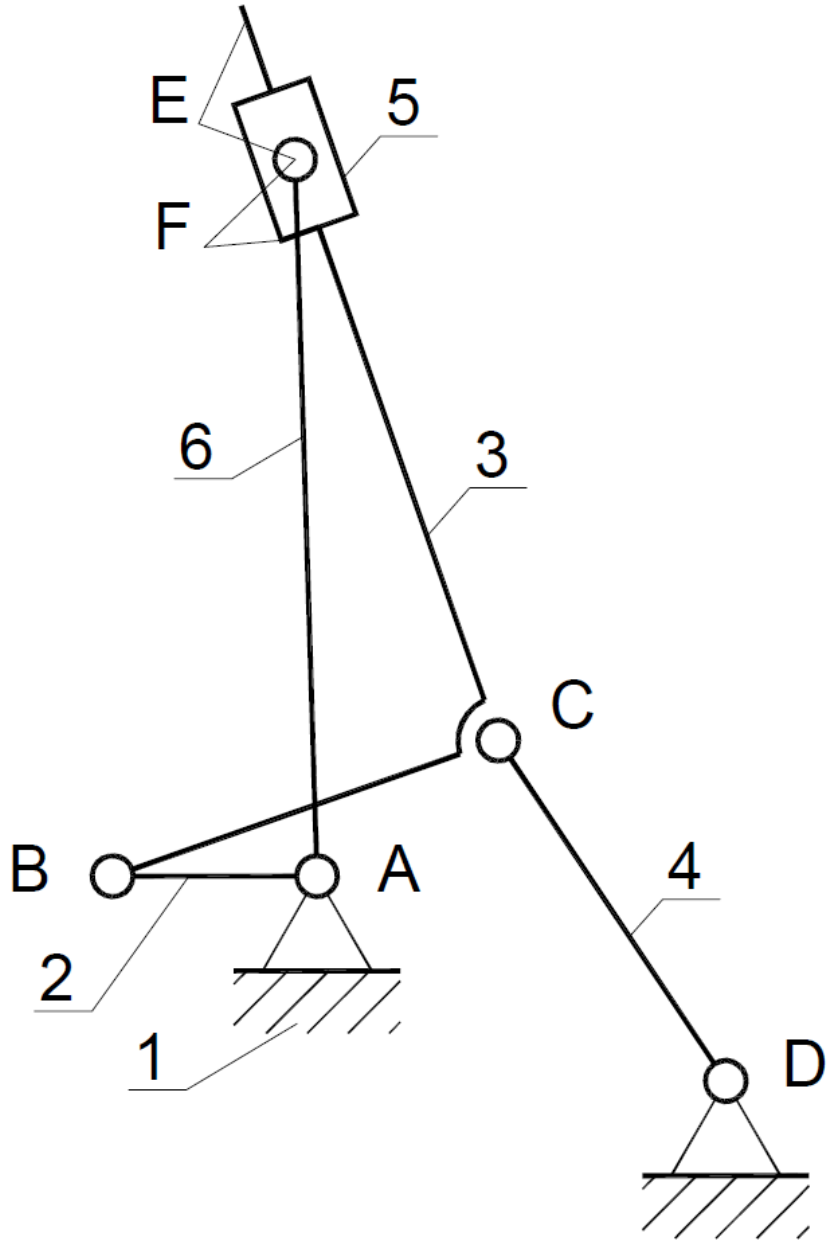
	polygon	SAM
v_B	0,2	0.20000
v_C	0,06836	0.06836
v_E	0,3246	0.32460
v_F	0,34	0.34001

	polygon	SAM
a_B	0,4	0.40000
a_C	0,187362	0.18736
a_E	0,232849	0.23285
a_F	0,356729	0.35639

SAM velocity example file

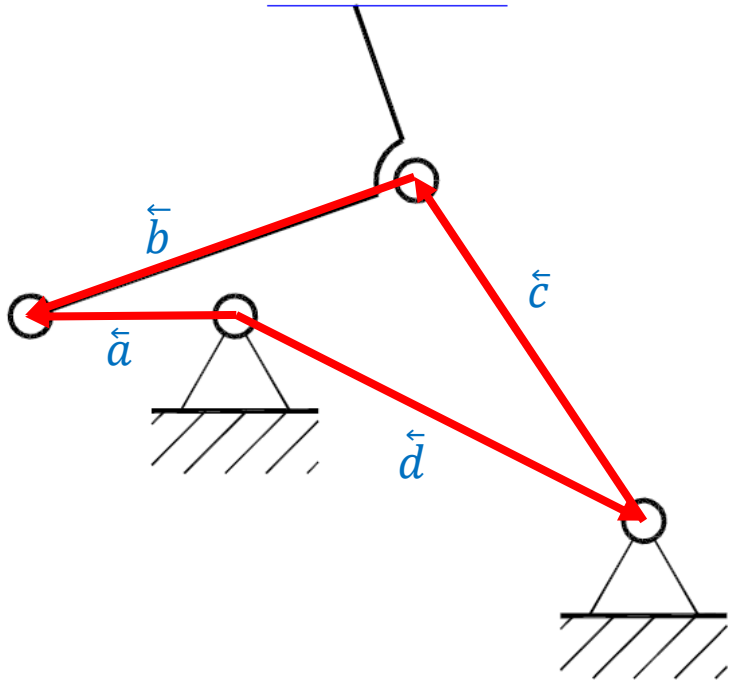
SAM acceleration example file

Analitical method (using vectors and loops)



First loop

$$\vec{a}\text{-}\vec{b}\text{-}\vec{c}\text{-}\vec{d}=0$$



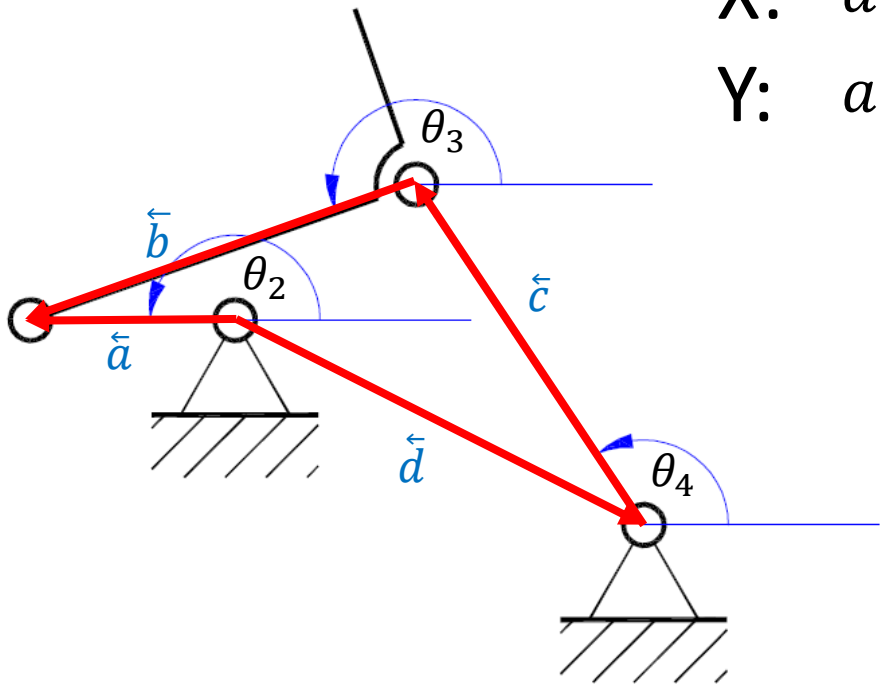
First loop

$$\vec{a}\text{-}\vec{b}\text{-}\vec{c}\text{-}\vec{d}=0$$

Projections

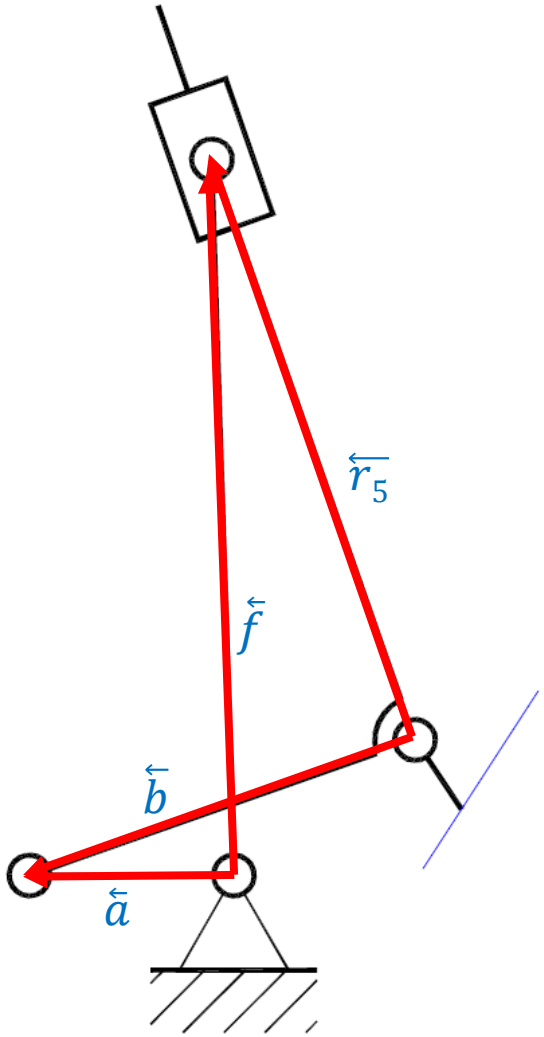
X: $a\cos\theta_2 - b\cos\theta_3 + c\cos\theta_4 \text{-}(x_D\text{-}x_A) = 0$

Y: $a\sin\theta_2 - b\sin\theta_3 + c\sin\theta_4 \text{-}(y_D\text{-}y_A) = 0$



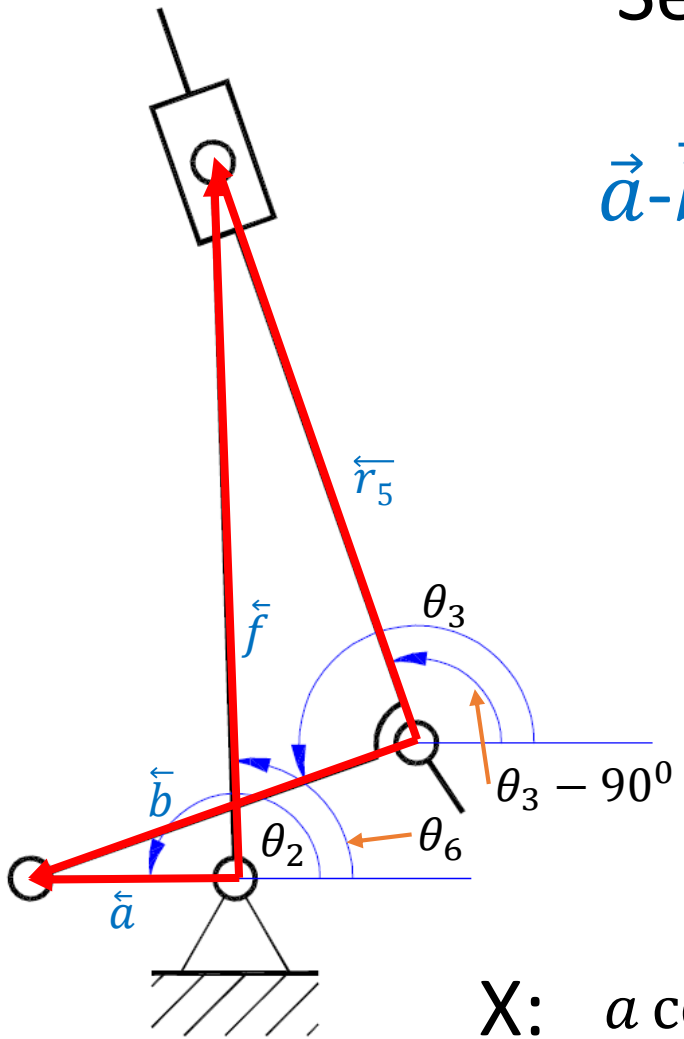
Second loop

$\vec{a}$-$\vec{b}$+$\vec{e}$-$\vec{f}$=0



Second loop

$\vec{a}$-$\vec{b}$+$\vec{e}$-$\vec{f}$=0

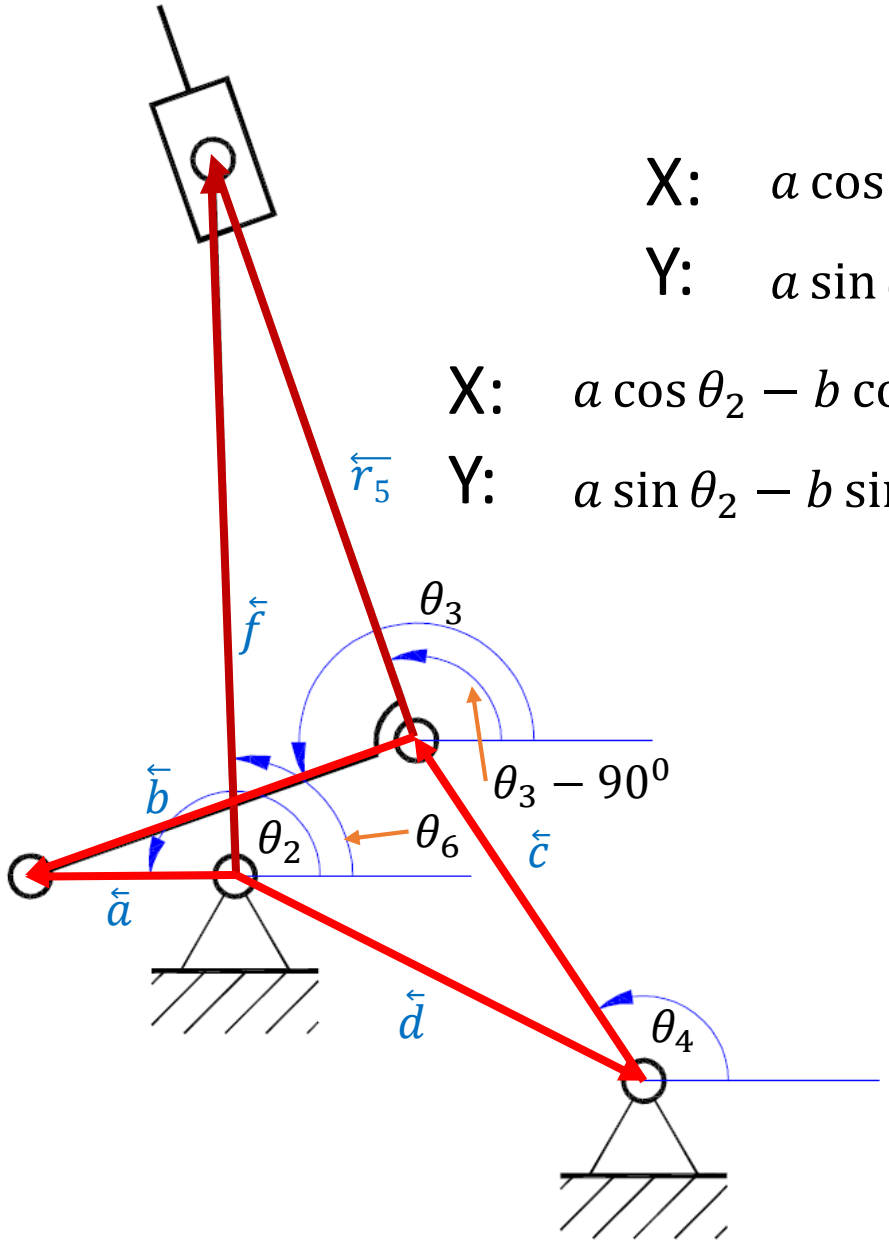


Projections

X: $a\cos\theta_2 - b\cos\theta_3 + r_5\cos(\theta_3 - 90^o) - f\cos\theta_6 = 0$

Y: $a\sin\theta_2 - b\sin\theta_3 + r_5\sin(\theta_3 - 90^o) - f\sin\theta_6 = 0$

Projections of all loops



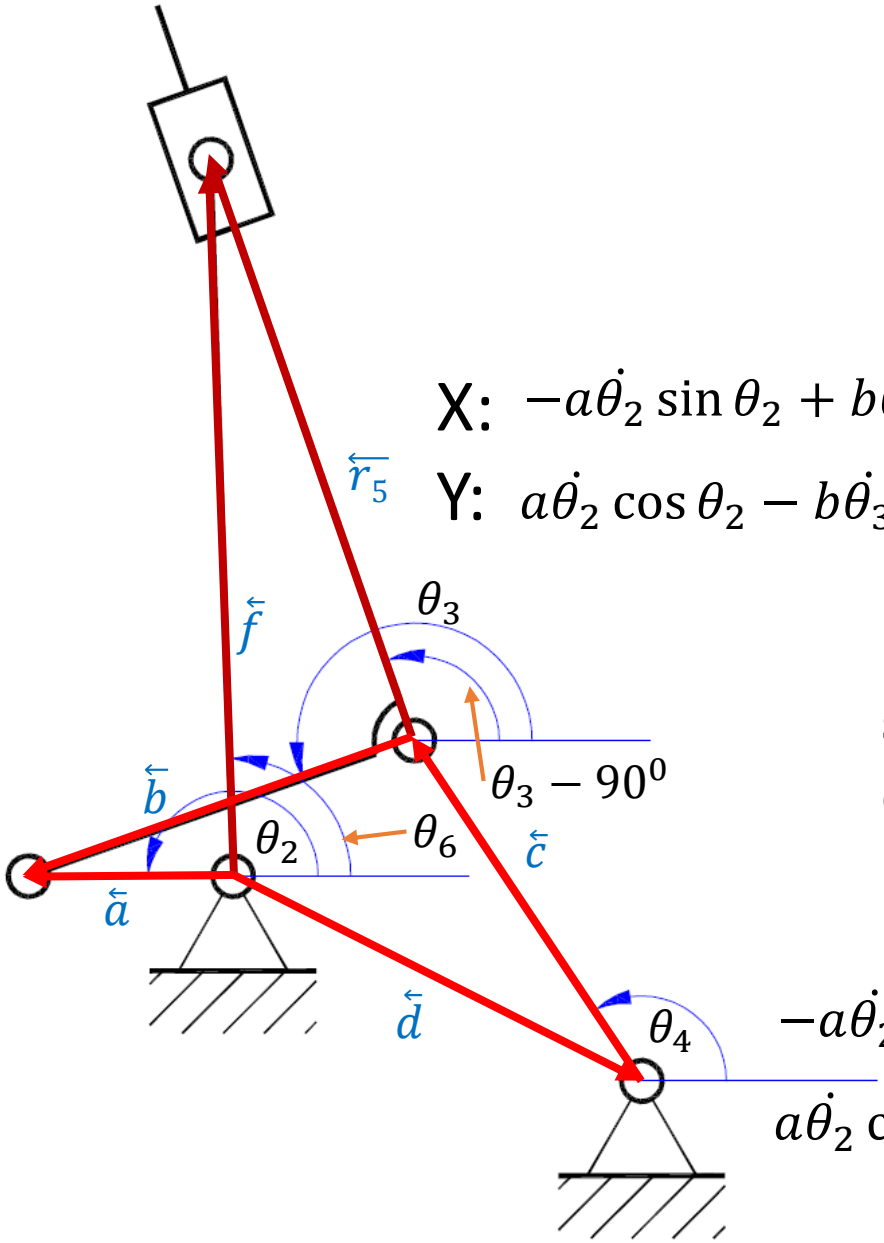
X: $a\cos\theta_2 - b\cos\theta_3 + c\cos\theta_4 - (x_D - x_A) = 0$

Y: $a\sin\theta_2 - b\sin\theta_3 + c\sin\theta_4 - (y_D - y_A) = 0$

X: $a\cos\theta_2 - b\cos\theta_3 + r_5\cos(\theta_3 - 90^o) - f\cos\theta_6 = 0$

Y: $a\sin\theta_2 - b\sin\theta_3 + r_5\sin(\theta_3 - 90^o) - f\sin\theta_6 = 0$

Projections of all loops : velocities



X: $-a\dot{\theta}_2 \sin\theta_2 + b\dot{\theta}_3 \sin\theta_3 - c\dot{\theta}_4 \sin\theta_4 = 0$

Y: $a\dot{\theta}_2 \cos\theta_2 - b\dot{\theta}_3 \cos\theta_3 + c\dot{\theta}_4 \cos\theta_4 = 0$

X: $-a\dot{\theta}_2 \sin\theta_2 + b\dot{\theta}_3 \sin\theta_3 - r_5\dot{\theta}_3 \sin(\theta_3 - 90^o) + \dot{r_5} \cos(\theta_3 - 90^o) + f\dot{\theta}_6 \sin\theta_6 = 0$

Y: $a\dot{\theta}_2 \cos\theta_2 - b\dot{\theta}_3 \cos\theta_3 + r_5\dot{\theta}_3 \cos(\theta_3 - 90^o) + \dot{r_5} \sin(\theta_3 - 90^o) - f\dot{\theta}_6 \cos\theta_6 = 0$

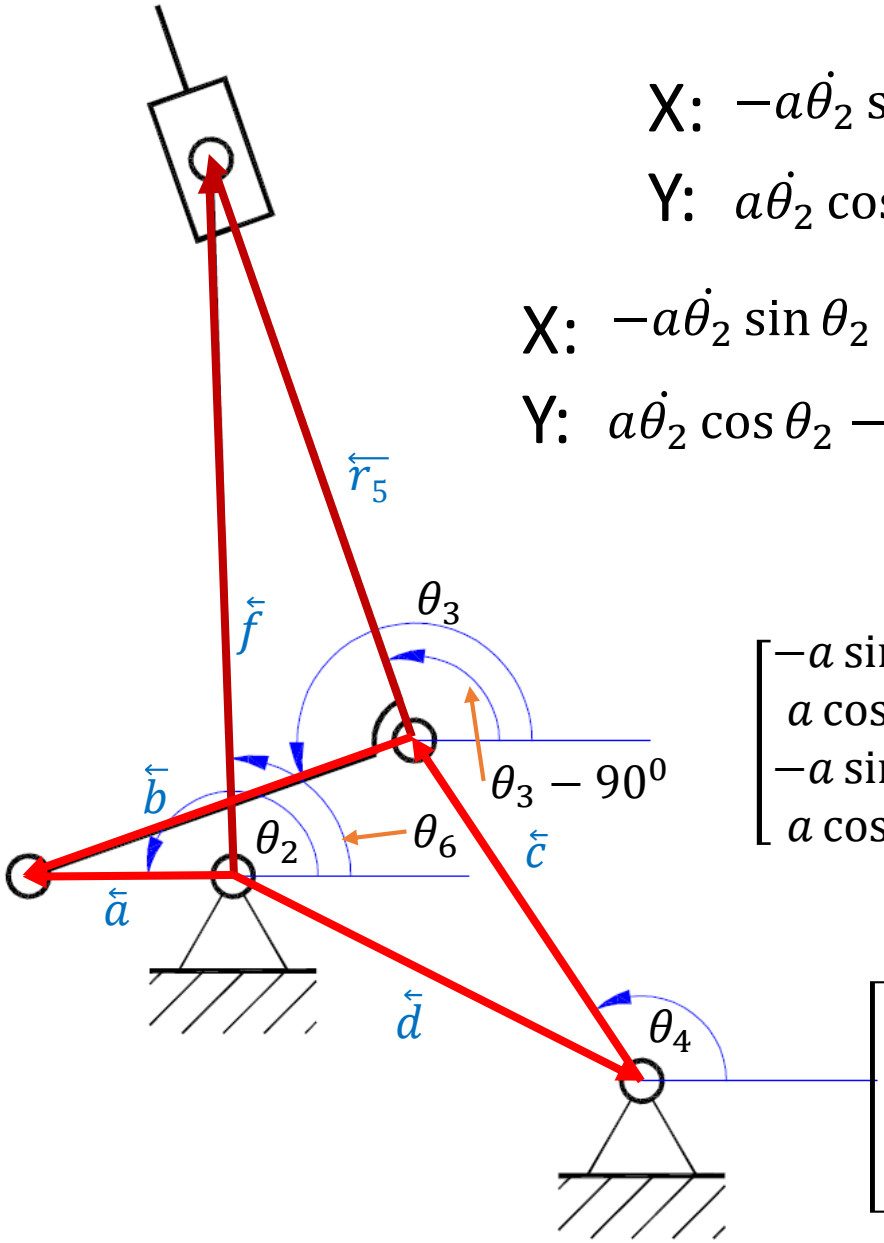
$\sin(\theta_3 - 90^o) = \sin\theta_3 \cos(90^o) - \cos\theta_3 \sin(90^o) = -\cos\theta_3$

$\cos(\theta_3 - 90^o) = \cos\theta_3 \cos(90^o) + \sin\theta_3 \sin(90^o) = \sin\theta_3$

$-a\dot{\theta}_2 \sin\theta_2 + b\dot{\theta}_3 \sin\theta_3 + r_5\dot{\theta}_3 \cos\theta_3 + \dot{r_5} \sin\theta_3 + f\dot{\theta}_6 \sin\theta_6 = 0$

$a\dot{\theta}_2 \cos\theta_2 - b\dot{\theta}_3 \cos\theta_3 + r_5\dot{\theta}_3 \sin\theta_3 - \dot{r_5} \cos\theta_3 - f\dot{\theta}_6 \cos\theta_6 = 0$

Projections of all loops : velocities – matrix form



$$X: \quad -a\dot{\theta}_2 \sin\theta_2 + b\dot{\theta}_3 \sin\theta_3 - c\dot{\theta}_4 \sin\theta_4 = 0$$

$$Y: \quad a\dot{\theta}_2 \cos\theta_2 - b\dot{\theta}_3 \cos\theta_3 + c\dot{\theta}_4 \cos\theta_4 = 0$$

$$X: \quad -a\dot{\theta}_2 \sin\theta_2 + b\dot{\theta}_3 \sin\theta_3 + r_5\dot{\theta}_3 \cos\theta_3 + \dot{r_5} \sin\theta_3 + f\dot{\theta}_6 \sin\theta_6 = 0$$

$$Y: \quad a\dot{\theta}_2 \cos\theta_2 - b\dot{\theta}_3 \cos\theta_3 + r_5\dot{\theta}_3 \sin\theta_3 - \dot{r_5} \cos\theta_3 - f\dot{\theta}_6 \cos\theta_6 = 0$$

$$\begin{bmatrix} -a\sin\theta_2 \\ a\cos\theta_2 \\ -a\sin\theta_2 \\ a\cos\theta_2 \end{bmatrix} \dot{\theta}_2 + \begin{bmatrix} b\sin\theta_3 & -c\sin\theta_4 & 0 & 0 \\ -b\cos\theta_3 & c\cos\theta_2 & 0 & 0 \\ b\sin\theta_3 + r_5\cos\theta_3 & 0 & \sin\theta_3 & f\sin\theta_6 \\ -b\cos\theta_3 + r_5\sin\theta_3 & 0 & -\cos\theta_3 & -f\cos\theta_6 \end{bmatrix} \begin{bmatrix} \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{r_5} \\ \dot{\theta}_6 \end{bmatrix} = 0$$

$$\begin{bmatrix} \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{r_5} \\ \dot{\theta}_6 \end{bmatrix} = -\begin{bmatrix} b\sin\theta_3 & -c\sin\theta_4 & 0 & 0 \\ -b\cos\theta_3 & c\cos\theta_2 & 0 & 0 \\ b\sin\theta_3 + r_5\cos\theta_3 & 0 & \sin\theta_3 & f\sin\theta_6 \\ -b\cos\theta_3 + r_5\sin\theta_3 & 0 & -\cos\theta_3 & -f\cos\theta_6 \end{bmatrix}^{-1} \begin{bmatrix} -a\sin\theta_2 \\ a\cos\theta_2 \\ -a\sin\theta_2 \\ a\cos\theta_2 \end{bmatrix} \dot{\theta}_2$$

Projections of all loops : accelerations

X: $-a\dot{\theta}_2 \sin\theta_2 + b\dot{\theta}_3 \sin\theta_3 - c\dot{\theta}_4 \sin\theta_4 = 0$

Y: $a\dot{\theta}_2 \cos\theta_2 - b\dot{\theta}_3 \cos\theta_3 + c\dot{\theta}_4 \cos\theta_4 = 0$

X: $-a\dot{\theta}_2 \sin\theta_2 + b\dot{\theta}_3 \sin\theta_3 + r_5\dot{\theta}_3 \cos\theta_3 + \dot{r}_5 \sin\theta_3 + f\dot{\theta}_6 \sin\theta_6 = 0$

Y: $a\dot{\theta}_2 \cos\theta_2 - b\dot{\theta}_3 \cos\theta_3 + r_5\dot{\theta}_3 \sin\theta_3 - \dot{r}_5 \cos\theta_3 - f\dot{\theta}_6 \cos\theta_6 = 0$

X: $-a\ddot{\theta}_2 \sin\theta_2 - a\dot{\theta}_2^{\,2} \cos\theta_2 + b\ddot{\theta}_3 \sin\theta_3 + b\dot{\theta}_3^{\,2} \cos\theta_3 - c\ddot{\theta}_4 \sin\theta_4 - c\dot{\theta}_4^{\,2} \cos\theta_4 = 0$

Y: $a\ddot{\theta}_2 \cos\theta_2 - a\dot{\theta}_2^{\,2} \sin\theta_2 - b\ddot{\theta}_3 \cos\theta_3 + a\dot{\theta}_3^{\,2} \sin\theta_3 + c\ddot{\theta}_4 \cos\theta_4 - c\dot{\theta}_4^{\,2} \sin\theta_4 = 0$

X: $-a\ddot{\theta}_2 \sin\theta_2 - a\dot{\theta}_2^{\,2} \cos\theta_2 + b\ddot{\theta}_3 \sin\theta_3 + b\dot{\theta}_3^{\,2} \cos\theta_3 + r_5\ddot{\theta}_3 \cos\theta_3 - r_5\dot{\theta}_3^{\,2} \sin\theta_3 +$
$+\dot{r}_5\dot{\theta}_3 \cos\theta_3 + \dot{r}_5\dot{\theta}_3 \cos\theta_3 + \ddot{r}_5 \sin\theta_3 + f\ddot{\theta}_6 \sin\theta_6 + f\dot{\theta}_6^{\,2} \cos\theta_6 = 0$

Y: $a\ddot{\theta}_2 \cos\theta_2 - a\dot{\theta}_2^{\,2} \sin\theta_2 - b\ddot{\theta}_3 \cos\theta_3 + b\dot{\theta}_3^{\,2} \sin\theta_3 + r_5\ddot{\theta}_3 \sin\theta_3 + r_5\dot{\theta}_3^{\,2} \cos\theta_3 +$
$+\dot{r}_5\dot{\theta}_3 \sin\theta_3 + \dot{r}_5\dot{\theta}_3 \sin\theta_3 - \ddot{r}_5 \cos\theta_3 - f\ddot{\theta}_6 \cos\theta_6 + f\dot{\theta}_6^{\,2} \sin\theta_6 = 0$

Projections of all loops : accelerations – matrix form

$$X: -a\ddot{\theta}_2 \sin\theta_2 - a\dot{\theta}_2^{\,2}\cos\theta_2 + b\ddot{\theta}_3 \sin\theta_3 + b\dot{\theta}_3^{\,2}\cos\theta_3 - c\ddot{\theta}_4 \sin\theta_4 - c\dot{\theta}_4^{\,2}\cos\theta_4 = 0$$

$$Y: \; a\ddot{\theta}_2 \cos\theta_2 - a\dot{\theta}_2^{\,2}\sin\theta_2 - b\ddot{\theta}_3 \cos\theta_3 + a\dot{\theta}_3^{\,2}\sin\theta_3 + c\ddot{\theta}_4 \cos\theta_4 - c\dot{\theta}_4^{\,2}\sin\theta_4 = 0$$

$$X: -a\ddot{\theta}_2 \sin\theta_2 - a\dot{\theta}_2^{\,2}\cos\theta_2 + b\ddot{\theta}_3 \sin\theta_3 + b\dot{\theta}_3^{\,2}\cos\theta_3 + r_5\ddot{\theta}_3 \cos\theta_3 - r_5\dot{\theta}_3^{\,2}\sin\theta_3 +$$
$$+\dot{r}_5\dot{\theta}_3 cos\,\theta_3 + \dot{r}_5\dot{\theta}_3 cos\,\theta_3 + \ddot{r}_5 \sin\theta_3 + f\ddot{\theta}_6 \sin\theta_6 + f\dot{\theta}_6^{\,2}\cos\theta_6 = 0$$

$$Y: \; a\ddot{\theta}_2 \cos\theta_2 - a\dot{\theta}_2^{\,2}\sin\theta_2 - b\ddot{\theta}_3 \cos\theta_3 + b\dot{\theta}_3^{\,2}\sin\theta_3 + r_5\ddot{\theta}_3 \sin\theta_3 + r_5\dot{\theta}_3^{\,2}\cos\theta_3 +$$
$$+\dot{r}_5\dot{\theta}_3 \sin\theta_3 + \dot{r}_5\dot{\theta}_3 \sin\theta_3 - \ddot{r}_5 \cos\theta_3 - f\ddot{\theta}_6 cos\,\theta_6 + f\dot{\theta}_6^{\,2}\sin\theta_6 = 0$$

$$\begin{bmatrix} -a\sin\theta_2 & -a\cos\theta_2 \\ a\cos\theta_2 & -a\sin\theta_2 \\ -a\sin\theta_2 & -a\cos\theta_2 \\ a\cos\theta_2 & -a\sin\theta_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_2 \\ \dot{\theta}_2^{\,2} \end{bmatrix} + \begin{bmatrix} b\sin\theta_3 & -c\sin\theta_4 & 0 & 0 \\ -b\cos\theta_3 & c\cos\theta_2 & 0 & 0 \\ b\sin\theta_3 + r_5\cos\theta_3 & 0 & \sin\theta_3 & f\sin\theta_6 \\ -b\cos\theta_3 + r_5\sin\theta_3 & 0 & -\cos\theta_3 & -f\cos\theta_6 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_4 \\ \ddot{r}_5 \\ \ddot{\theta}_6 \end{bmatrix}$$

$$+ \begin{bmatrix} b\cos\theta_3 & -c\cos\theta_4 & 0 & 0 \\ b\sin\theta_3 & -c\sin\theta_2 & 0 & 0 \\ b\cos\theta_3 - r_5\sin\theta_3 & 0 & \cos\theta_3 f\cos\theta_6 \\ b\sin\theta_3 + r_5\cos\theta_3 & 0 & \sin\theta_3 f\sin\theta_6 \end{bmatrix} \begin{bmatrix} \dot{\theta}_3^{\,2} \\ \dot{\theta}_4^{\,2} \\ 2\dot{\theta}_3\dot{r}_5 \\ \dot{\theta}_6^{\,2} \end{bmatrix} = 0$$

Projections of all loops : accelerations – matrix form

$$\begin{bmatrix} -a\sin\theta_2 & -a\cos\theta_2 \\ a\cos\theta_2 & -a\sin\theta_2 \\ -a\sin\theta_2 & -a\cos\theta_2 \\ a\cos\theta_2 & -a\sin\theta_2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_2 \\ \dot{\theta}_2{}^2 \end{bmatrix} + \begin{bmatrix} b\sin\theta_3 & -c\sin\theta_4 & 0 & 0 \\ -b\cos\theta_3 & c\cos\theta_2 & 0 & 0 \\ b\sin\theta_3 + r_5\cos\theta_3 & 0 & \sin\theta_3 & f\sin\theta_6 \\ -b\cos\theta_3 + r_5\sin\theta_3 & 0 & -\cos\theta_3 & -f\cos\theta_6 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_4 \\ \ddot{r}_5 \\ \ddot{\theta}_6 \end{bmatrix}$$

$$+\begin{bmatrix} b\cos\theta_3 & -c\cos\theta_4 & 0 & 0 \\ b\sin\theta_3 & -c\sin\theta_2 & 0 & 0 \\ b\cos\theta_3 - r_5\sin\theta_3 & 0 & \cos\theta_3 f\cos\theta_6 \\ b\sin\theta_3 + r_5\cos\theta_3 & 0 & \sin\theta_3 f\sin\theta_6 \end{bmatrix}\begin{bmatrix} \dot{\theta}_3{}^2 \\ \dot{\theta}_4{}^2 \\ 2\dot{\theta}_3\dot{r}_5 \\ \dot{\theta}_6{}^2 \end{bmatrix} = 0$$

$$\begin{bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_4 \\ \ddot{r}_5 \\ \ddot{\theta}_6 \end{bmatrix} = -\begin{bmatrix} b\sin\theta_3 & -c\sin\theta_4 & 0 & 0 \\ -b\cos\theta_3 & c\cos\theta_2 & 0 & 0 \\ b\sin\theta_3 + r_5\cos\theta_3 & 0 & \sin\theta_3 & f\sin\theta_6 \\ -b\cos\theta_3 + r_5\sin\theta_3 & 0 & -\cos\theta_3 & -f\cos\theta_6 \end{bmatrix}^{-1}\left(\begin{bmatrix} b\cos\theta_3 & -c\cos\theta_4 & 0 & 0 \\ b\sin\theta_3 & -c\sin\theta_2 & 0 & 0 \\ b\cos\theta_3 - r_5\sin\theta_3 & 0 & \cos\theta_3 f\cos\theta_6 \\ b\sin\theta_3 + r_5\cos\theta_3 & 0 & \sin\theta_3 f\sin\theta_6 \end{bmatrix}\begin{bmatrix} \dot{\theta}_3{}^2 \\ \dot{\theta}_4{}^2 \\ 2\dot{\theta}_3\dot{r}_5 \\ \dot{\theta}_6{}^2 \end{bmatrix} + \right.$$

$$\left. + \begin{bmatrix} -a\sin\theta_2 & -a\cos\theta_2 \\ a\cos\theta_2 & -a\sin\theta_2 \\ -a\sin\theta_2 & -a\cos\theta_2 \\ a\cos\theta_2 & -a\sin\theta_2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_2 \\ \dot{\theta}_2{}^2 \end{bmatrix} \right)$$