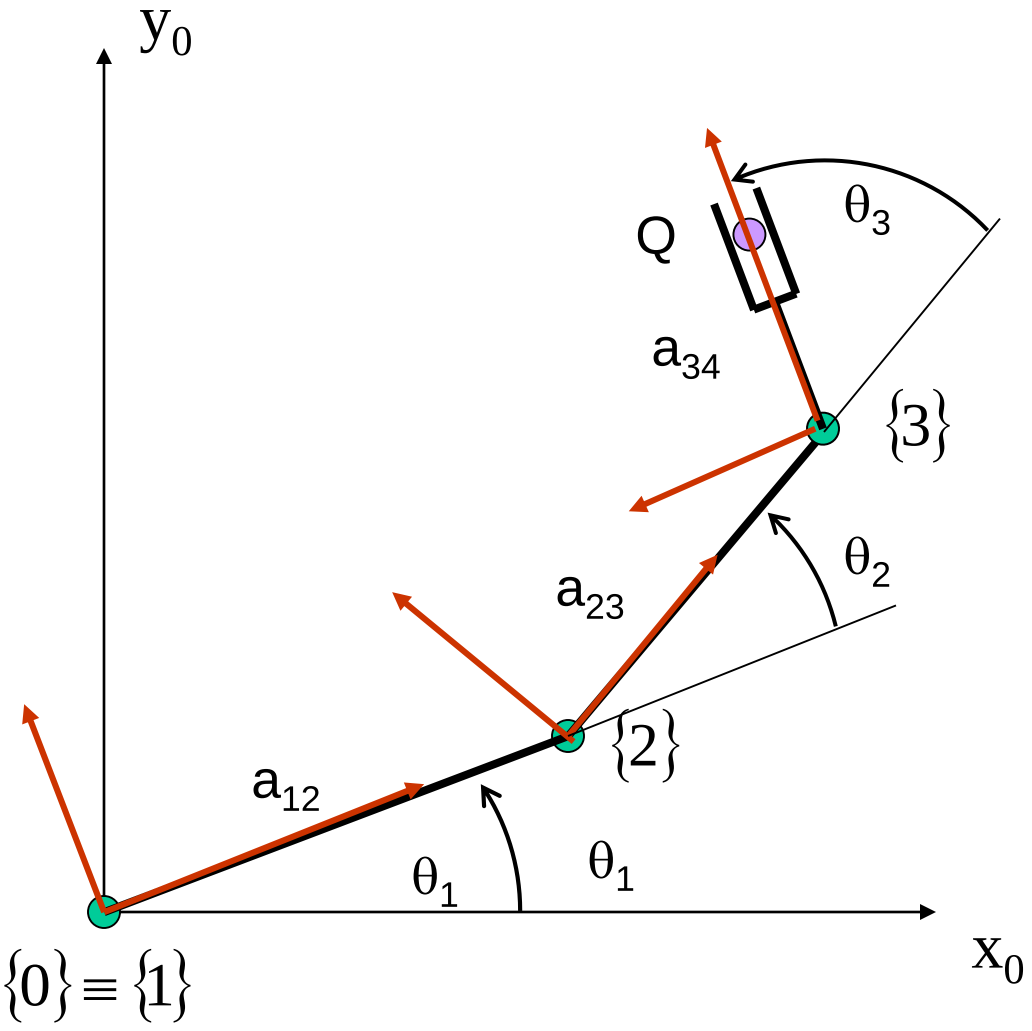
Velocity equations
Jacobian of manipulators

Velocity, jacobian



$$\begin{bmatrix} x_Q \\ y_Q \\ 1 \end{bmatrix} = {}^0\mathbf{A}_1 \cdot {}^1\mathbf{A}_2 \cdot {}^2\mathbf{A}_3 \begin{bmatrix} {}^3x_Q \\ {}^3y_Q \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} {}^3x_Q \\ {}^3y_Q \\ 1 \end{bmatrix} = \begin{bmatrix} a_{34} \\ 0 \\ 1 \end{bmatrix}$$

Velocity, jacobian

$$\begin{bmatrix} x_Q \\ y_Q \\ 1 \end{bmatrix} = {}^0\mathbf{A}_3 \begin{bmatrix} a_{34} \\ 0 \\ 1 \end{bmatrix}$$

$${}^0\mathbf{A}_3 =$$

$$\begin{bmatrix} \cos(\Theta_1+\Theta_2+\Theta_3) & -\sin(\Theta_1+\Theta_2+\Theta_3) & a_{12}\cos\Theta_1 + a_{23}\cos(\Theta_1+\Theta_2) \\ \sin(\Theta_1+\Theta_2+\Theta_3) & \cos(\Theta_1+\Theta_2+\Theta_3) & a_{12}\sin\Theta_1 + a_{23}\sin(\Theta_1+\Theta_2) \\ 0 & 0 & 1 \end{bmatrix}$$

Velocity of point Q

$$\begin{bmatrix} \dot{x}_Q \\ \dot{y}_Q \\ 1 \end{bmatrix} = {}^0\dot{A}_3 \begin{bmatrix} a_{34} \\ 0 \\ 1 \end{bmatrix}$$

Velocity, jacobian

$$
{}^{0}\mathbf{A}_3 =
\left[\begin{array}{c:c:c}
\cos(\Theta_1+\Theta_2+\Theta_3) & -\sin(\Theta_1+\Theta_2+\Theta_3) & a_{12}\cos\Theta_1 + a_{23}\cos(\Theta_1+\Theta_2) \\
\sin(\Theta_1+\Theta_2+\Theta_3) & \cos(\Theta_1+\Theta_2+\Theta_3) & a_{12}\sin\Theta_1 + a_{23}\sin(\Theta_1+\Theta_2) \\
0 & 0 & 1
\end{array}\right]
$$

$$
\begin{bmatrix} \dot{x}_Q \\ \dot{y}_Q \\ 1 \end{bmatrix} = {}^{0}\dot{A}_3 \begin{bmatrix} a_{34} \\ 0 \\ 1 \end{bmatrix}
$$

$$
\dot{x}_Q = -a_{34}(\sin(\theta_1+\theta_2+\theta_3))\left(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3\right) \\
-a_{12}\sin(\theta_1)\,\dot{\theta}_1 - a_{23}(\sin(\theta_1+\theta_2))\left(\dot{\theta}_1+\dot{\theta}_2\right)
$$

$$
\dot{y}_Q = a_{34}(\cos(\theta_1+\theta_2+\theta_3))\left(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3\right) \\
+a_{12}\cos(\theta_1)\,\dot{\theta}_1 + a_{23}(\cos(\theta_1+\theta_2))\left(\dot{\theta}_1+\dot{\theta}_2\right)
$$

$$
\dot{\gamma} = \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3
$$

Velocity, jacobian

Matrix form:

$$\begin{bmatrix} \dot{x}_Q \\ \dot{y}_Q \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

$$j_{11} = -a_{34} \sin(\Theta_1 + \Theta_2 + \Theta_3) - a_{12} \sin \Theta_1 - a_{23} \sin(\Theta_1 + \Theta_2)$$

$$j_{12} = -a_{34} \sin(\Theta_1 + \Theta_2 + \Theta_3) - a_{23} \sin(\Theta_1 + \Theta_2)$$

$$j_{13} = -a_{34} \sin(\Theta_1 + \Theta_2 + \Theta_3)$$

$$j_{21} = \ldots$$

$$\ldots$$

$$j_{31} = j_{32} = j_{33} = 1$$

Velocity, jacobian

$$\begin{bmatrix} \dot{x}_Q \\ \dot{y}_Q \\ \dot{\gamma} \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

J – jacobian of manipulator

$$V = J\dot{\theta}$$

← direct kinematics

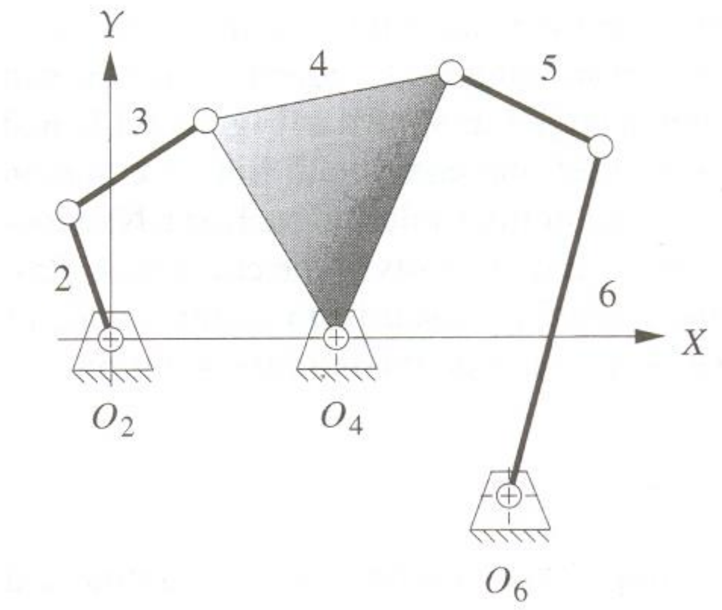
$$\dot{\theta} = J^{-1}V$$

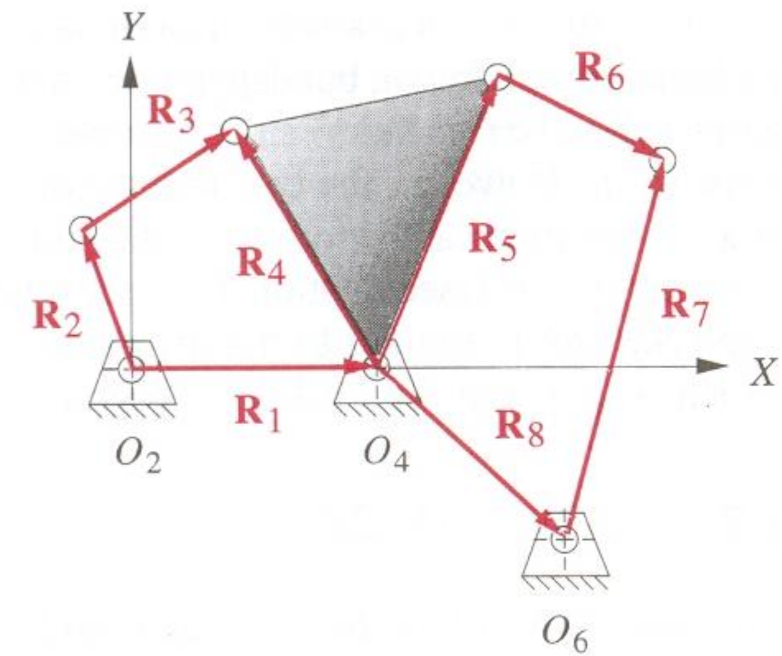
← inverse kinematics

Parallel manipulators
like any mechanism

can be considered as a chain of vectors

What is the chain of vectors ?





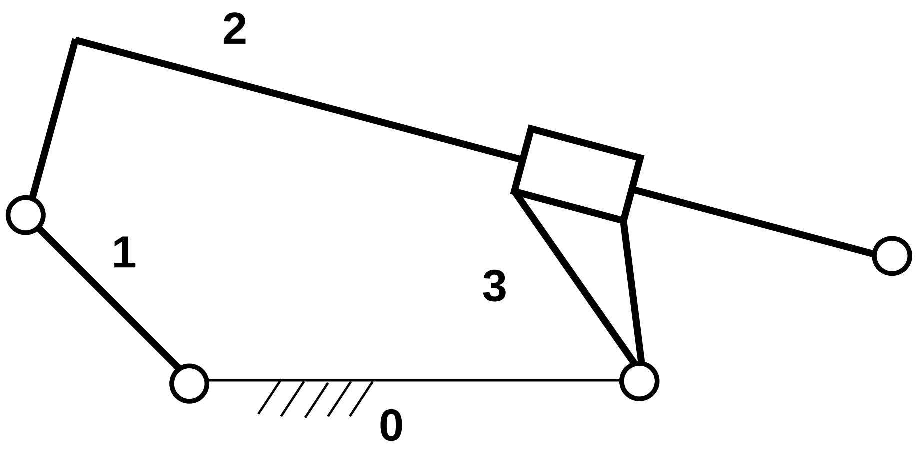
2 loops:

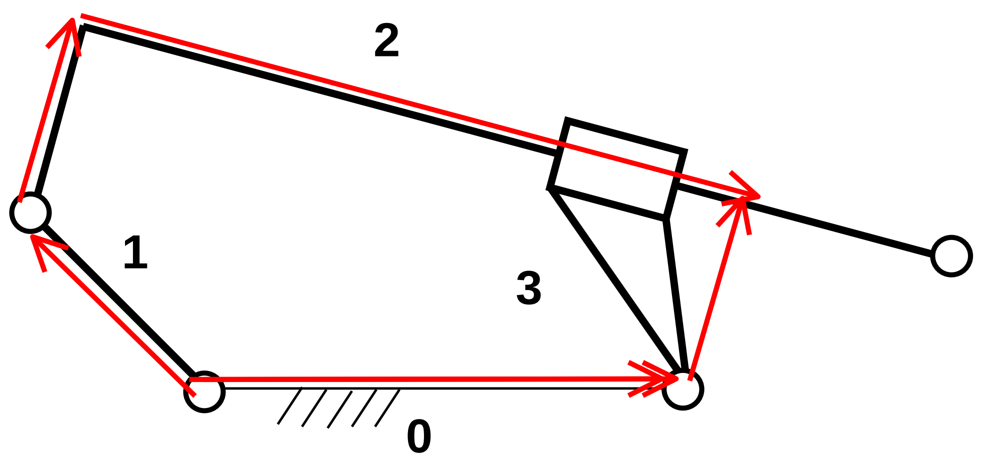
$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_1 - \mathbf{R}_4 = \mathbf{0}$$

$$\mathbf{R}_5 + \mathbf{R}_6 - \mathbf{R}_8 - \mathbf{R}_7 = \mathbf{0}$$

→ 4 projections (equations)

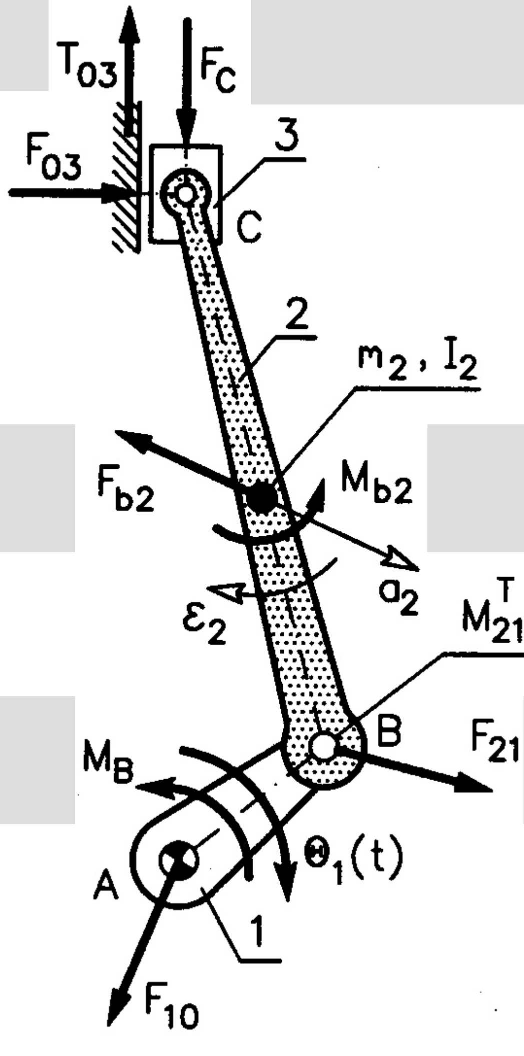
MECHANISM → POLYGON OF VECTORS





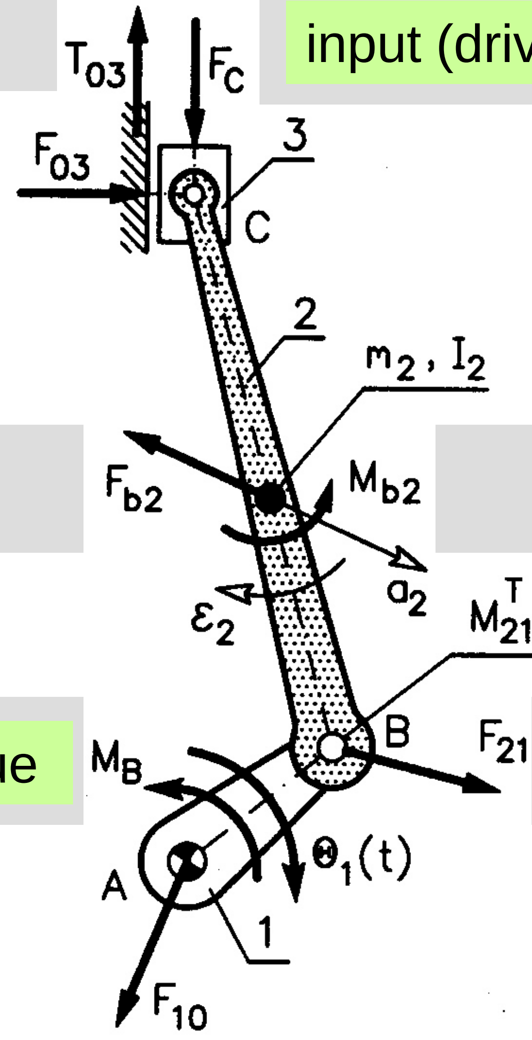
DYNAMICS
OF KINEMATIC SYSTEMS

Forces in Kinematic Systems



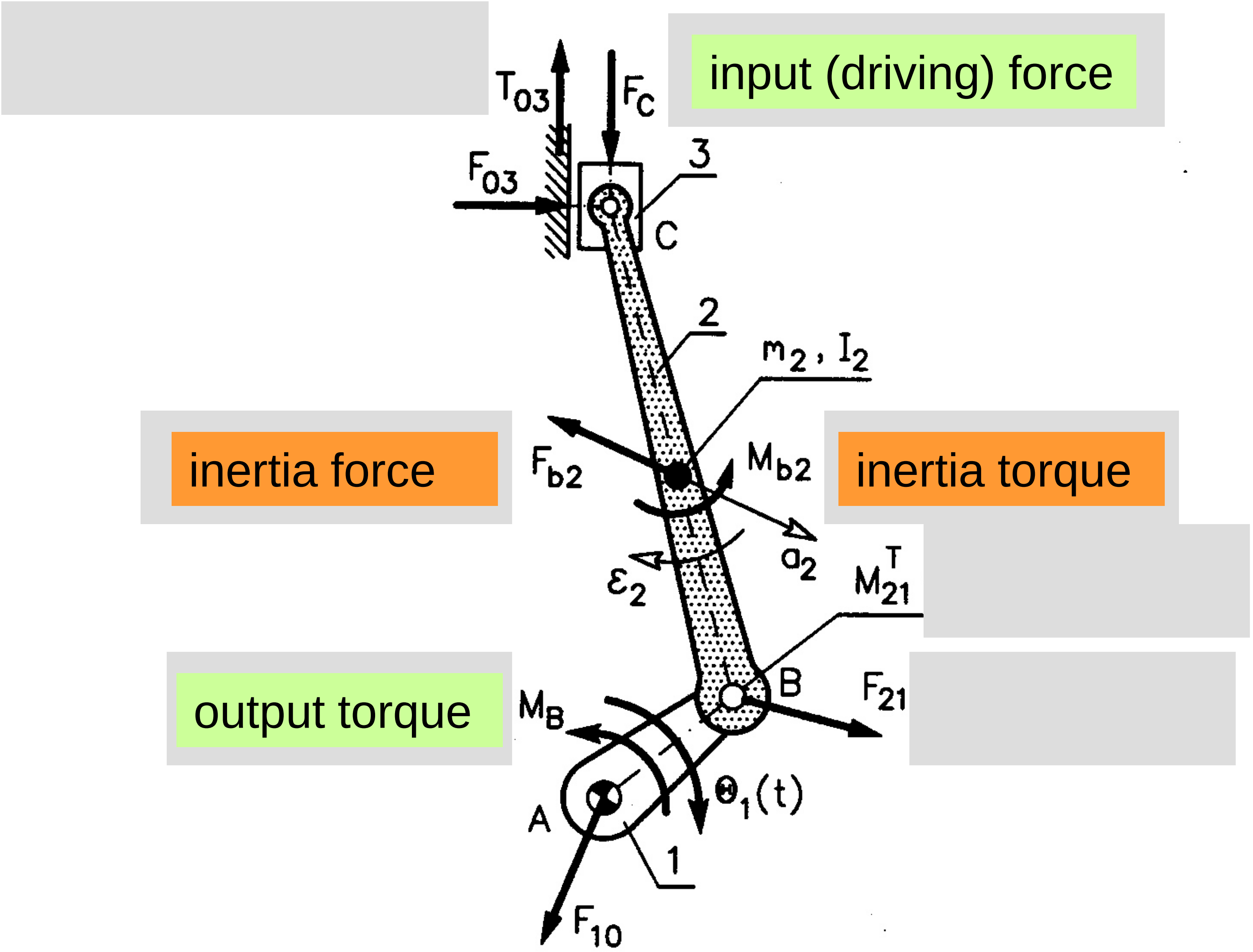
Forces in Kin Systems

input (driving) force

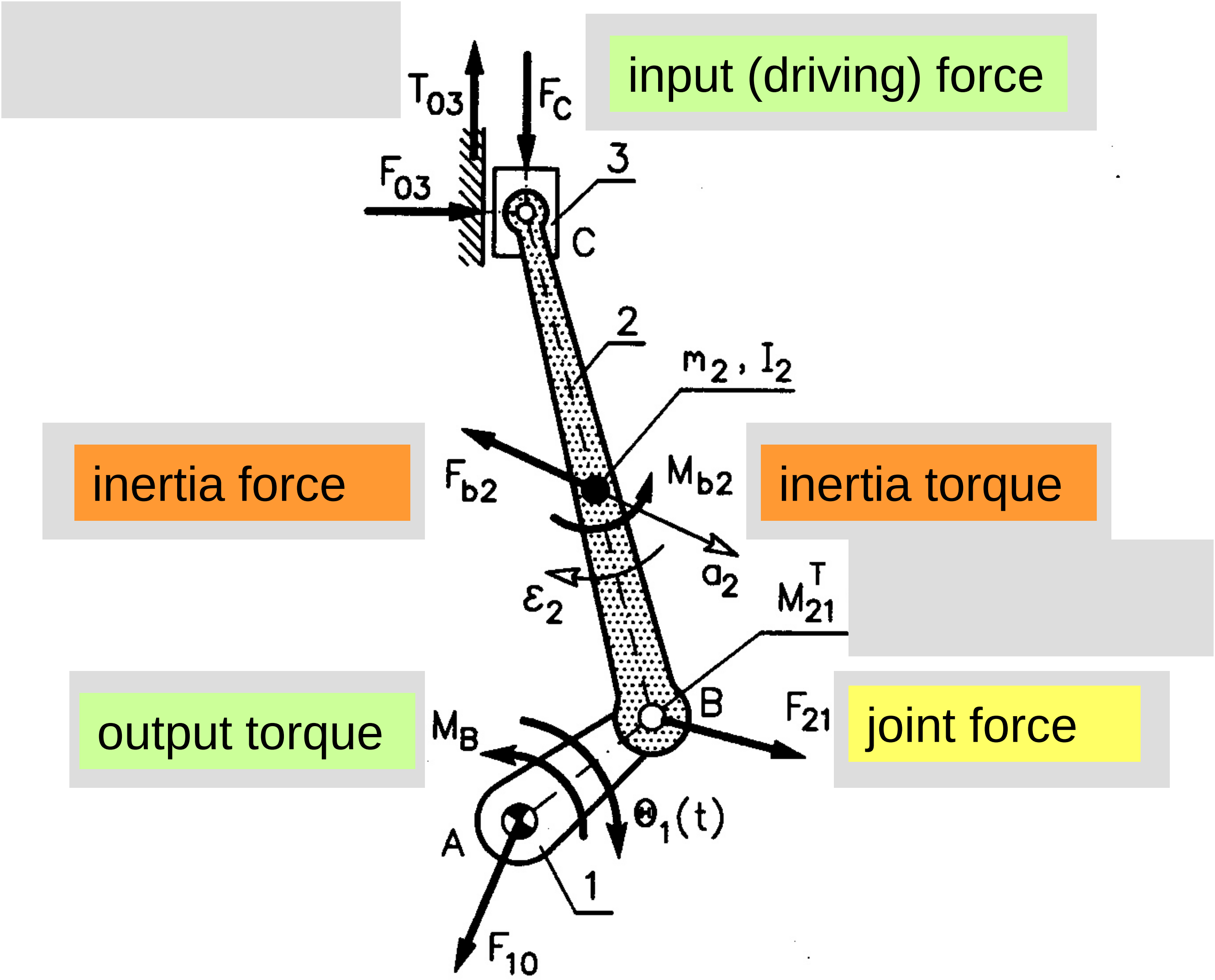


output torque

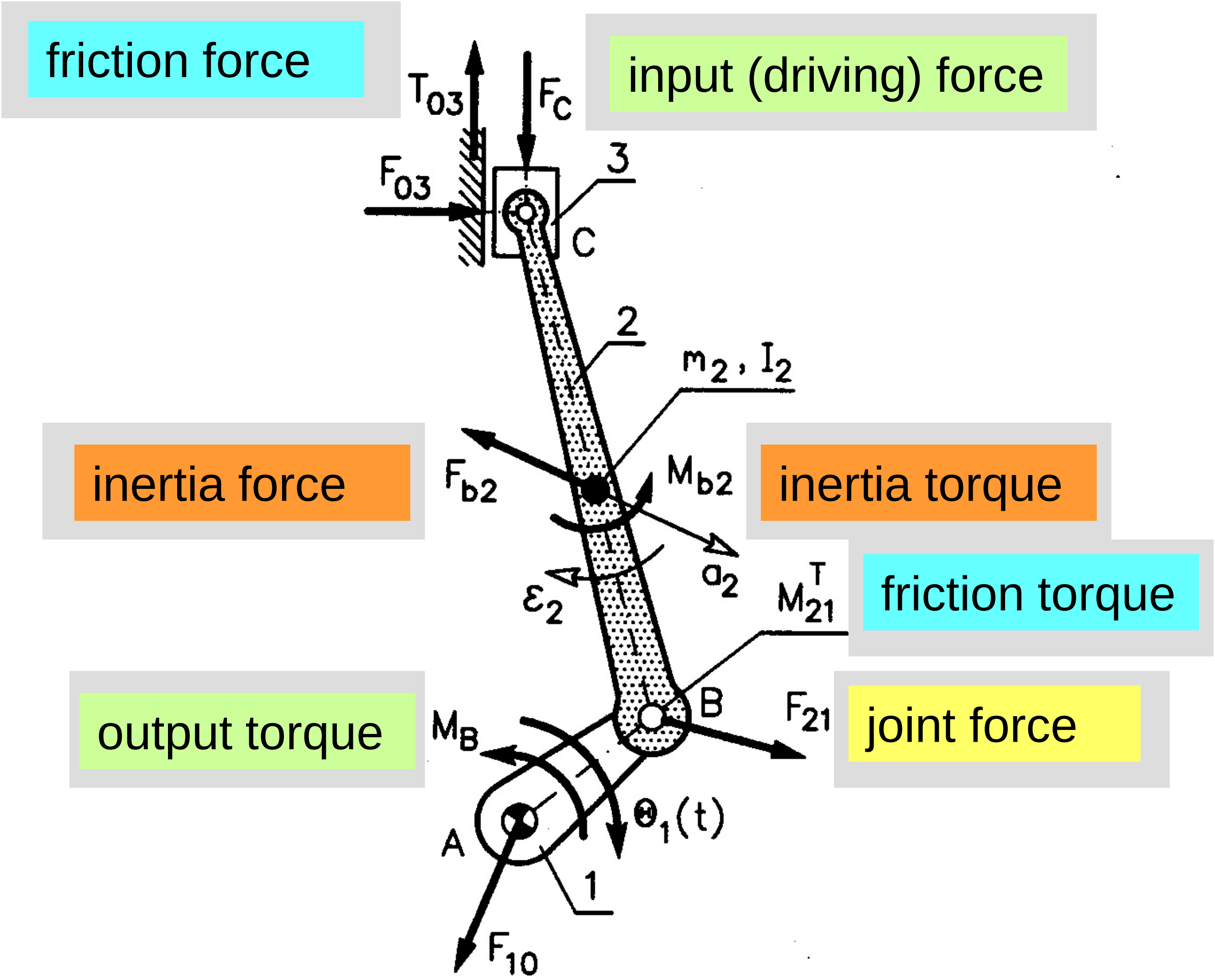
Forces in Kin Systems



Forces in Kin Systems



Forces in Kin Systems



Forces in Kinematic Systems

- Input, output force
- Gravity force
- Inertia force, moment
- Joint force
- Friction force, ...

Dynamics considers relation between:

- motion - $\mathbf{q}(t)$, $d\mathbf{q}(t)/dt$, $d\mathbf{q}(t)/dt^2$

- input forces $\mathbf{F}_C$, output forces $\mathbf{F}_B$, (friction forces $\mathbf{F}_\mu$),

- link masses (including mass distribution $\tilde{\mathbf{M}}$),

- time *t* & links' geometry **w**

$$\mathbf{f}(\mathbf{q}(t), \mathbf{F}_C, \mathbf{F}_B, \mathbf{F}_\mu, \tilde{\mathbf{M}}, t, \mathbf{w}) = 0$$

Having function **f** we can answer two questions:

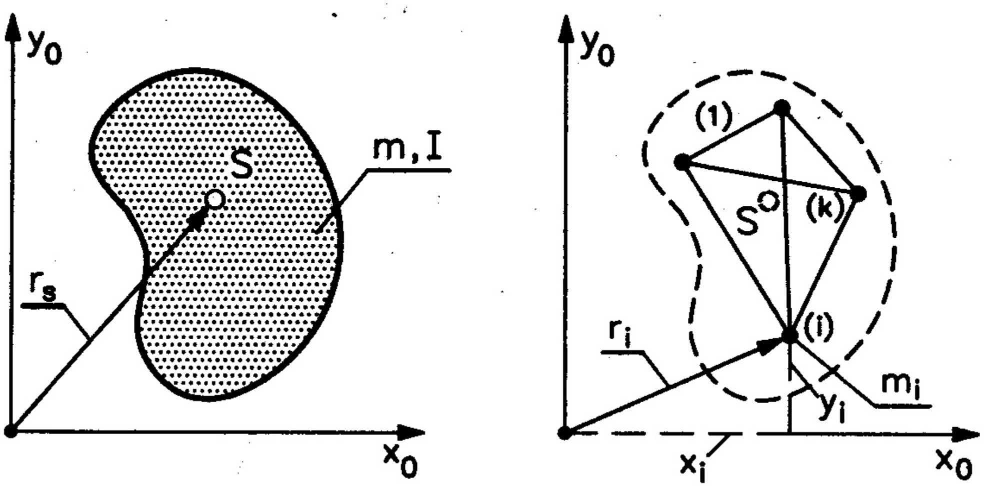
What is the motion for given input $\mathbf{F}_C$ and output $\mathbf{F}_B$ forces ?

Forward dynamics

What is the input force $\mathbf{F}_C$ to obtain required (given) motion $\mathbf{q}(t)$?

INVERSE DYNAMICS = KINETOSTATICS

Link $\rightarrow$ model of mass points



LINK ⟺ MODEL

Conditions of equivalency:

Mass of both the link and model have the same values

Centers of mass are coincident

Mass moments of inertia have the same values

Mass of both the link and model have the same values

$$\sum_{1}^{k} m_i = m$$

Centers of mass are coincident

$$\sum_{1}^{k} m_i x_i = m x_S \qquad \sum_{1}^{k} m_i y_i = m y_S$$

Mass moments of inertia have the same values

$$\sum_{1}^{k} m_i (x_i^2 + y_i^2) = m(x_S^2 + y_S^2) + I_S$$

Mass points (general):

a point mass is described by: *m, x, y*

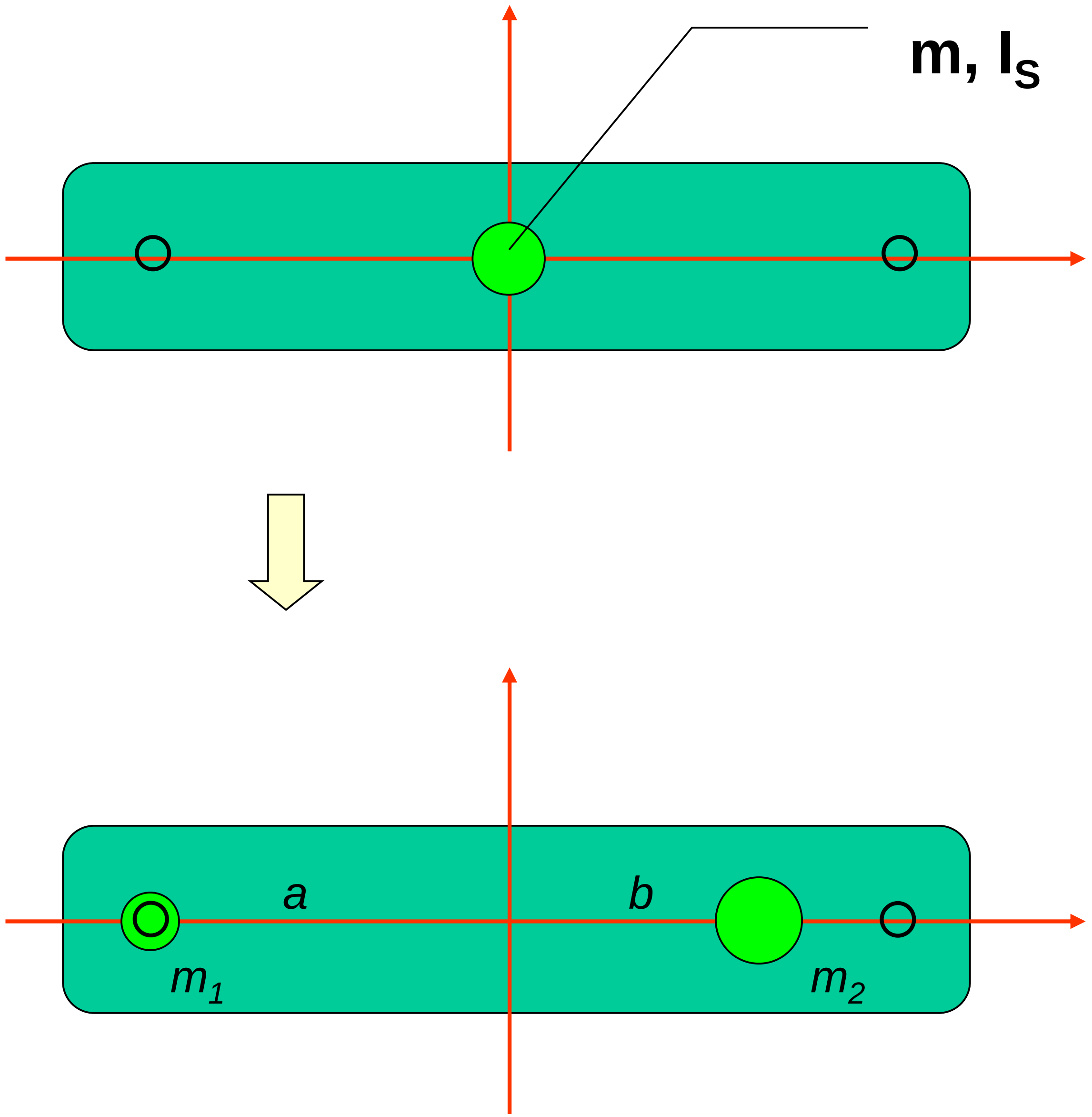
k points means 3*k* parameters

4 equations $\rightarrow$ we can calculate 4 parameters describing mass points

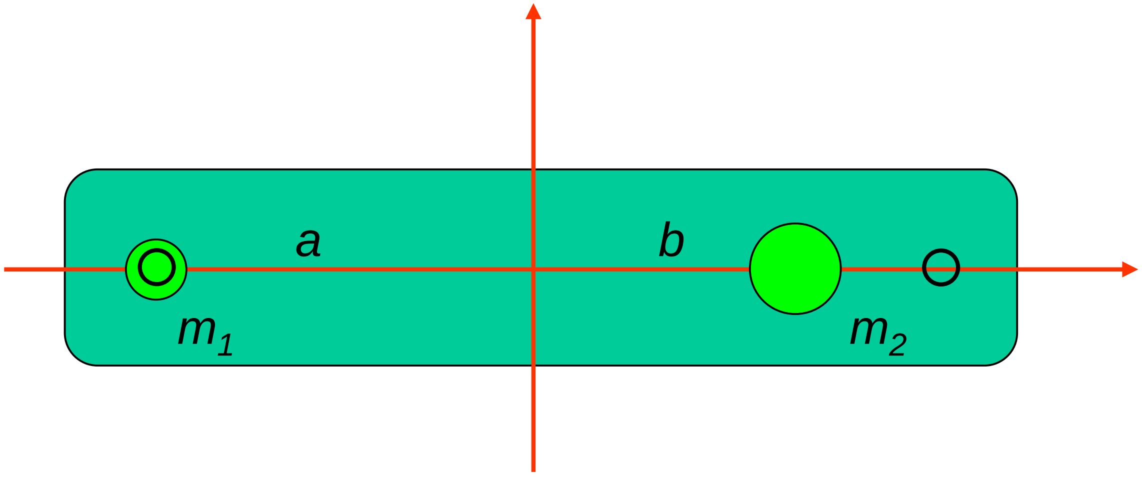
among *3k* parameters we can assume *p*

$$p = 3k - 4$$

Mass points - example



Mass points - example



$$m_1 + m_2 = m$$

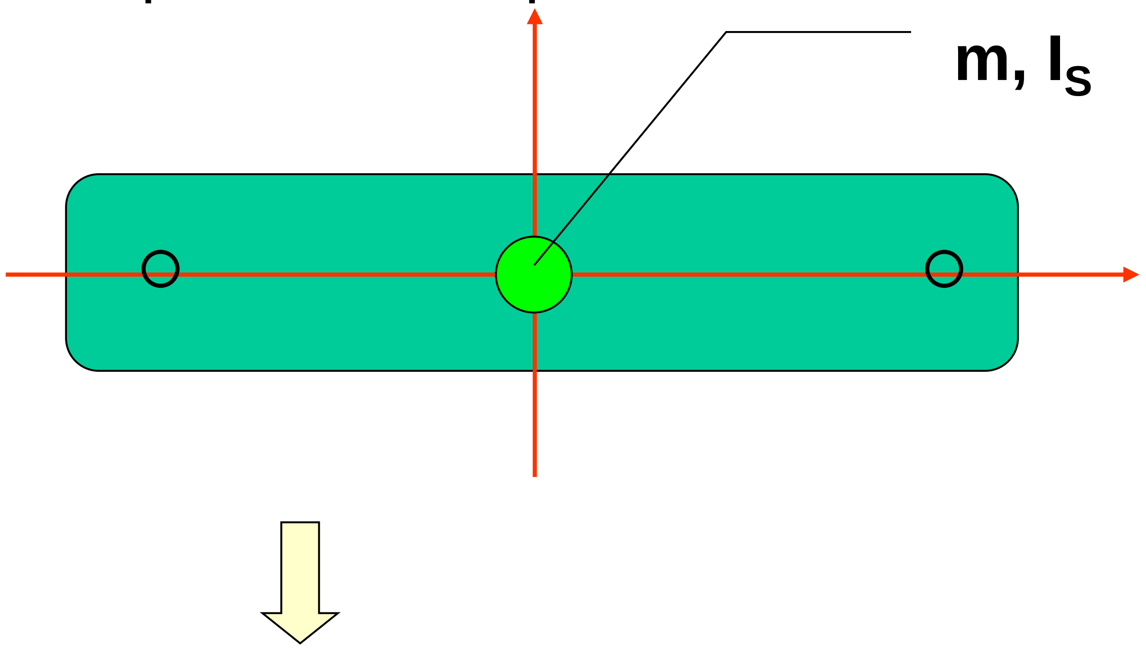
$$m_1 a - m_2 b = 0$$

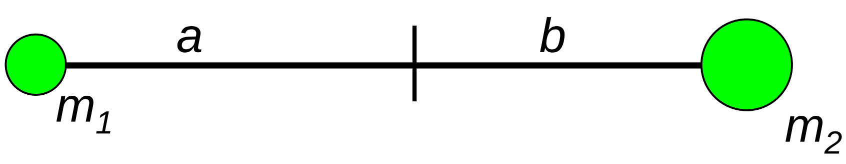
$$m_1 a^2 + m_2 b^2 = I_S$$

Parameters:

$$m_1,\ m_2,\ a,\ b = ?$$

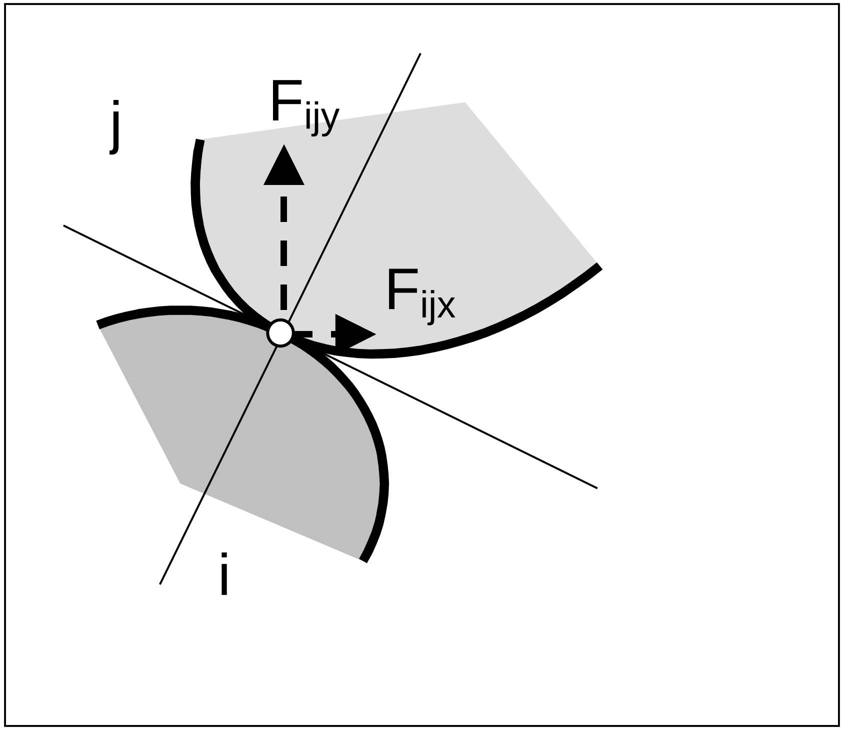
Mass points - example





Joint Forces in Planar Systems

Cam pair K – II class (p_2)

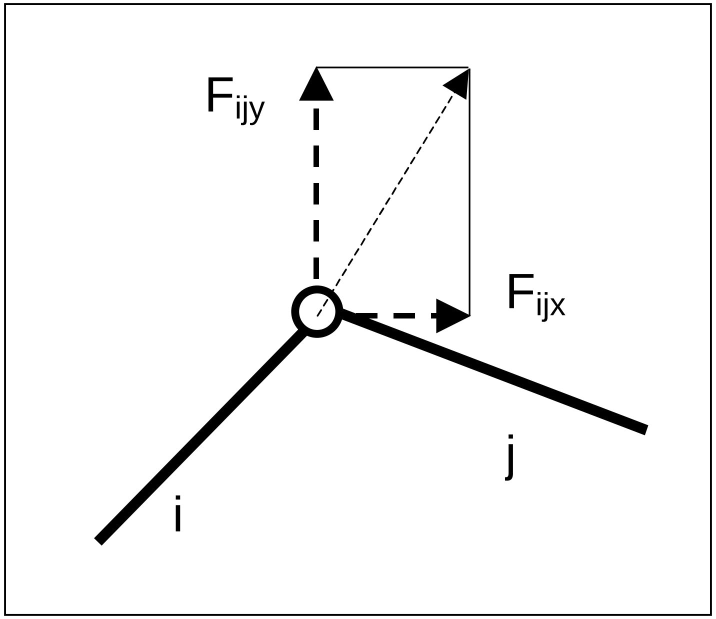


Known direction

Known point of application

One unknown: joint force component F_{ijx} or F_{ijy}

Revolute joint R – I class (p_1)

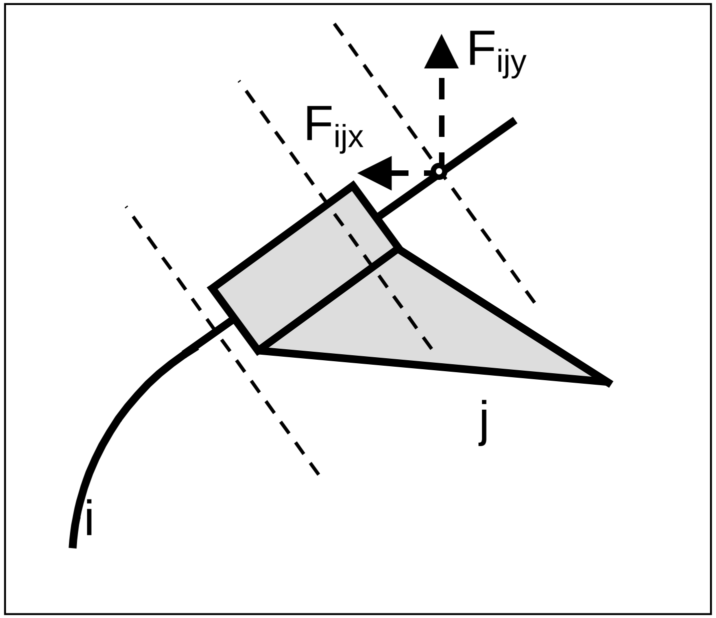


Known point of application

Two unknowns:

components F_{ijx} and F_{ijy}

Prismatic joint T – I class (p_1)



Known direction

two unknowns:

point of application

component F_{ijx} or F_{ijy}

Equilibrium equations

1. Equation of forces - the sum of all forces acting on link equals zero :

$$\sum_i \mathbf{F}_i = 0$$

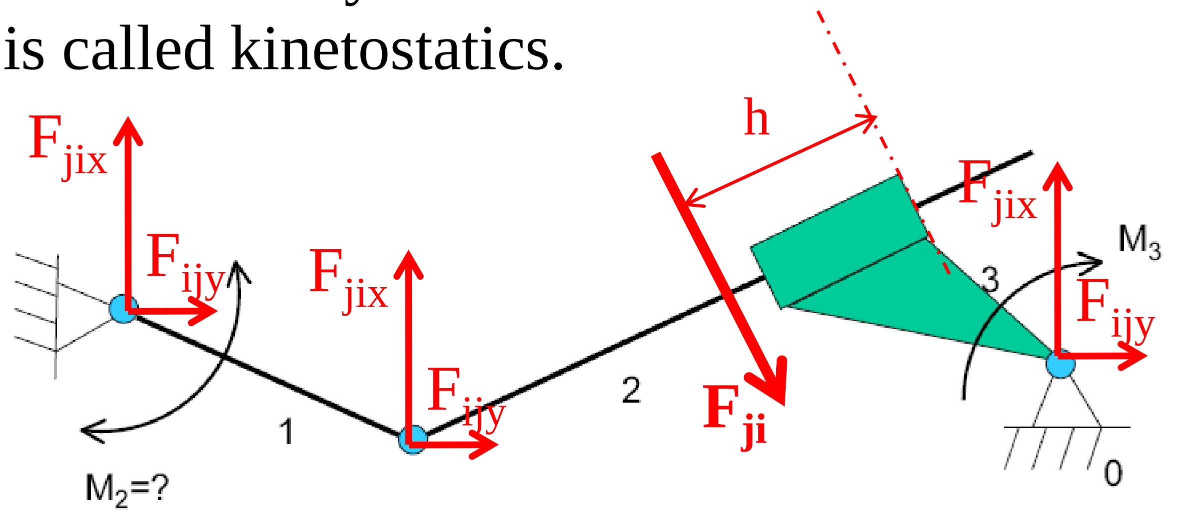
$$\sum_i F_i^x = 0 \qquad \sum_i F_i^y = 0$$

2. The equation of moments (torques)– the sum of all moments and moments from the forces acting on the links in relation to any point equals zero :

$$\sum_j M_j + \sum_i r\ F_i = 0$$

Methods of solution - kinetostatics

If in the load of the links we take into account the forces of inertia, we can solve such a system with the methods of statics - this method is called kinetostatics.



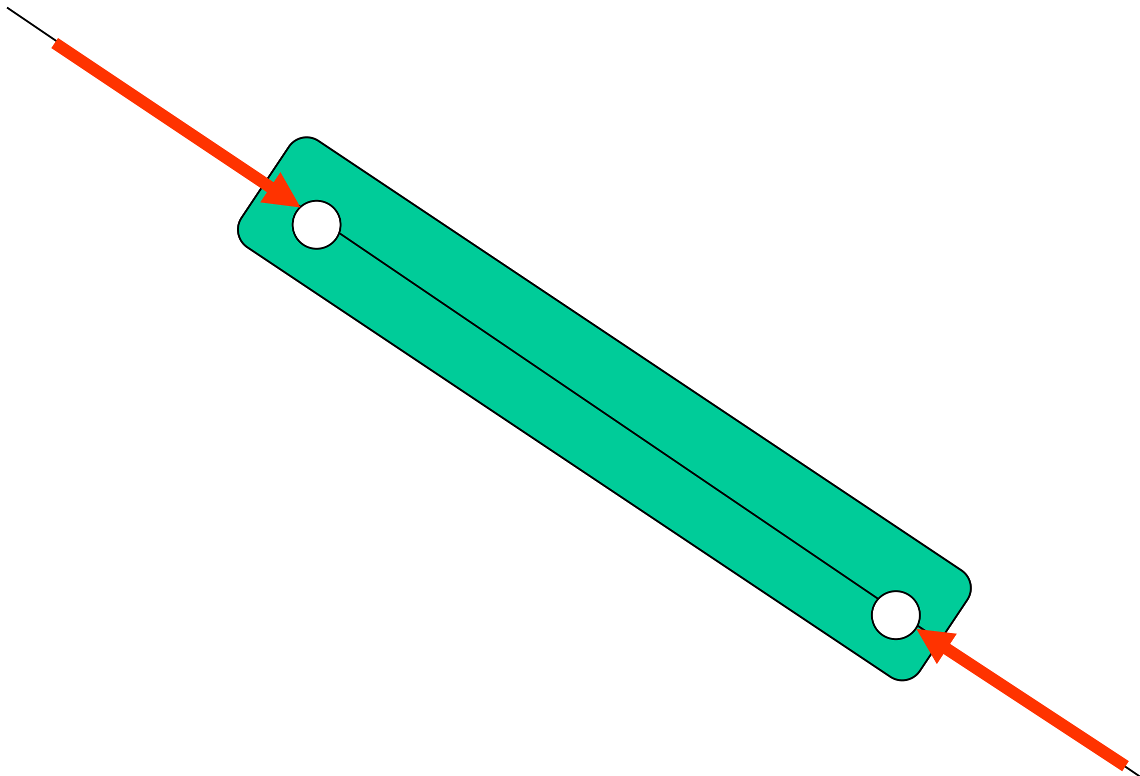
Number of equations = numer of uknown variables

Number of equations = (numer of moveable links) x 3 = 3 x 3=9

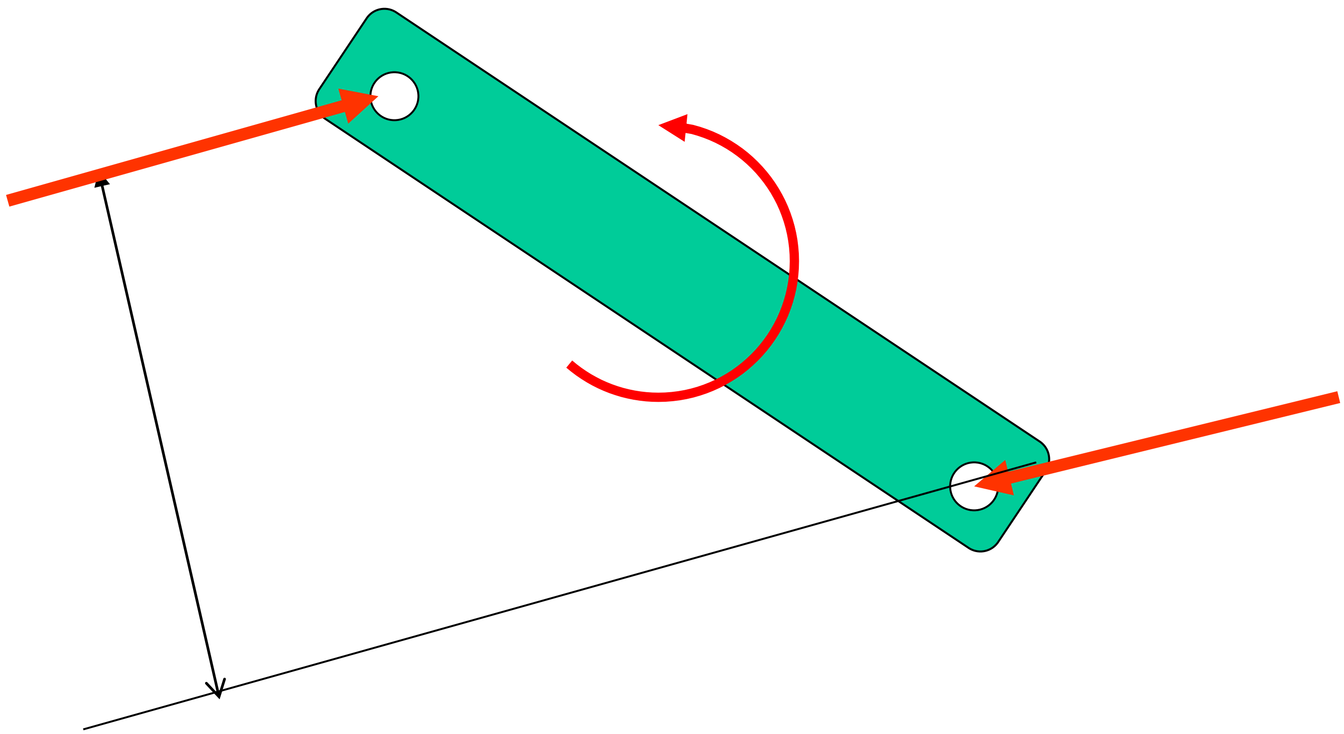
numer of uknown variables = (uknown balancing moment) + (unknown components of forces in kinematic pairs) =

= 1 + 2 + 2 + 2 + 2 =9

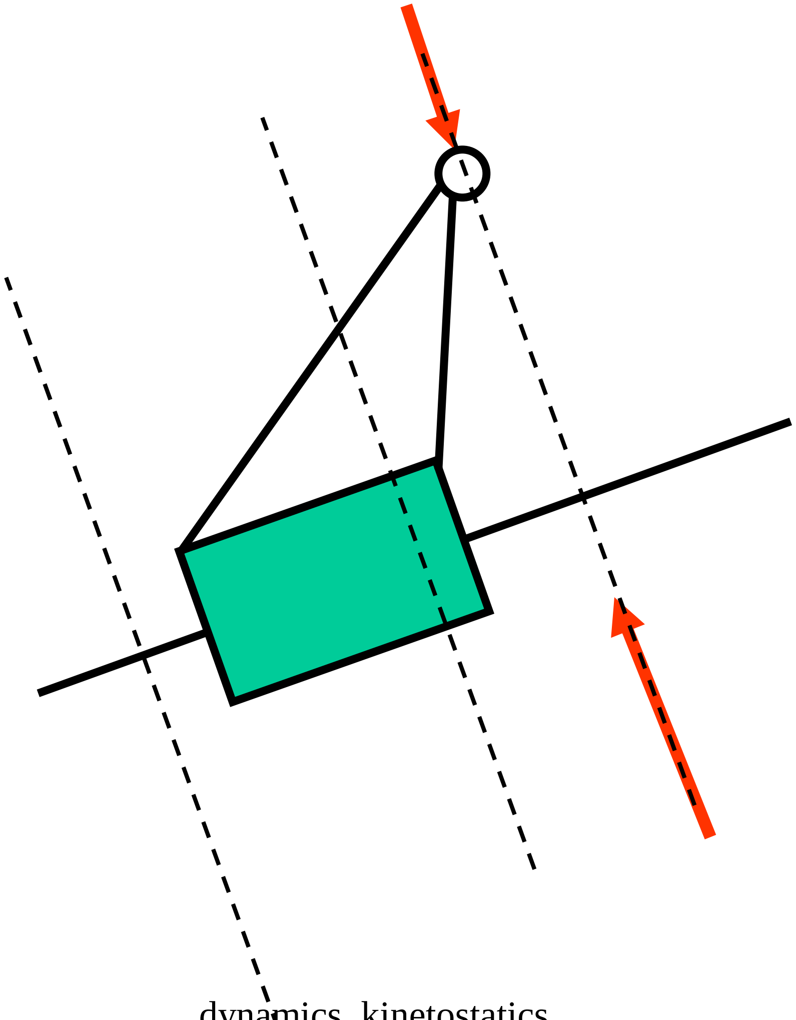
2, 3 & 4 forces equlibrium (1)



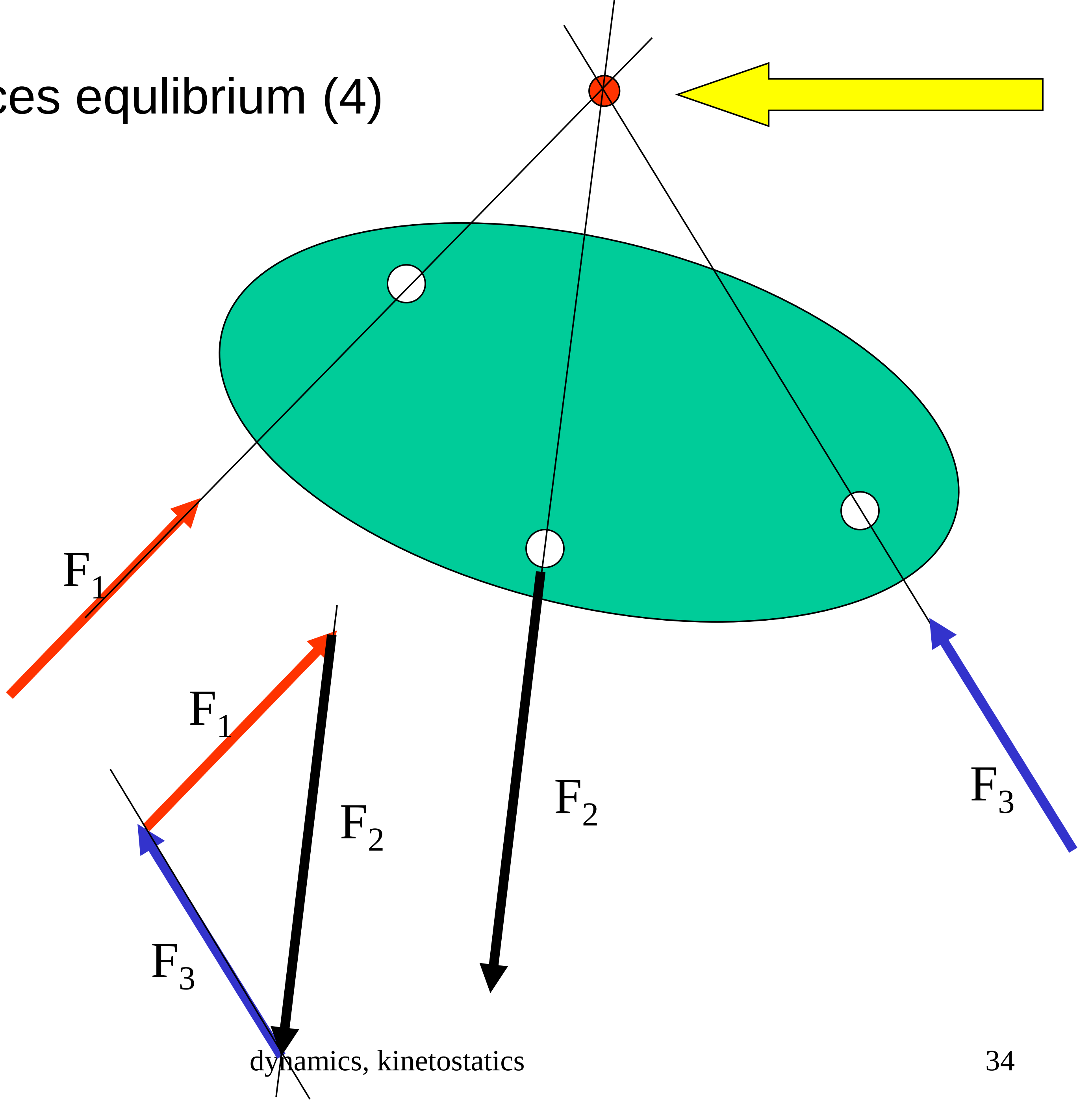
2, 3 & 4 forces equlibrium (2)



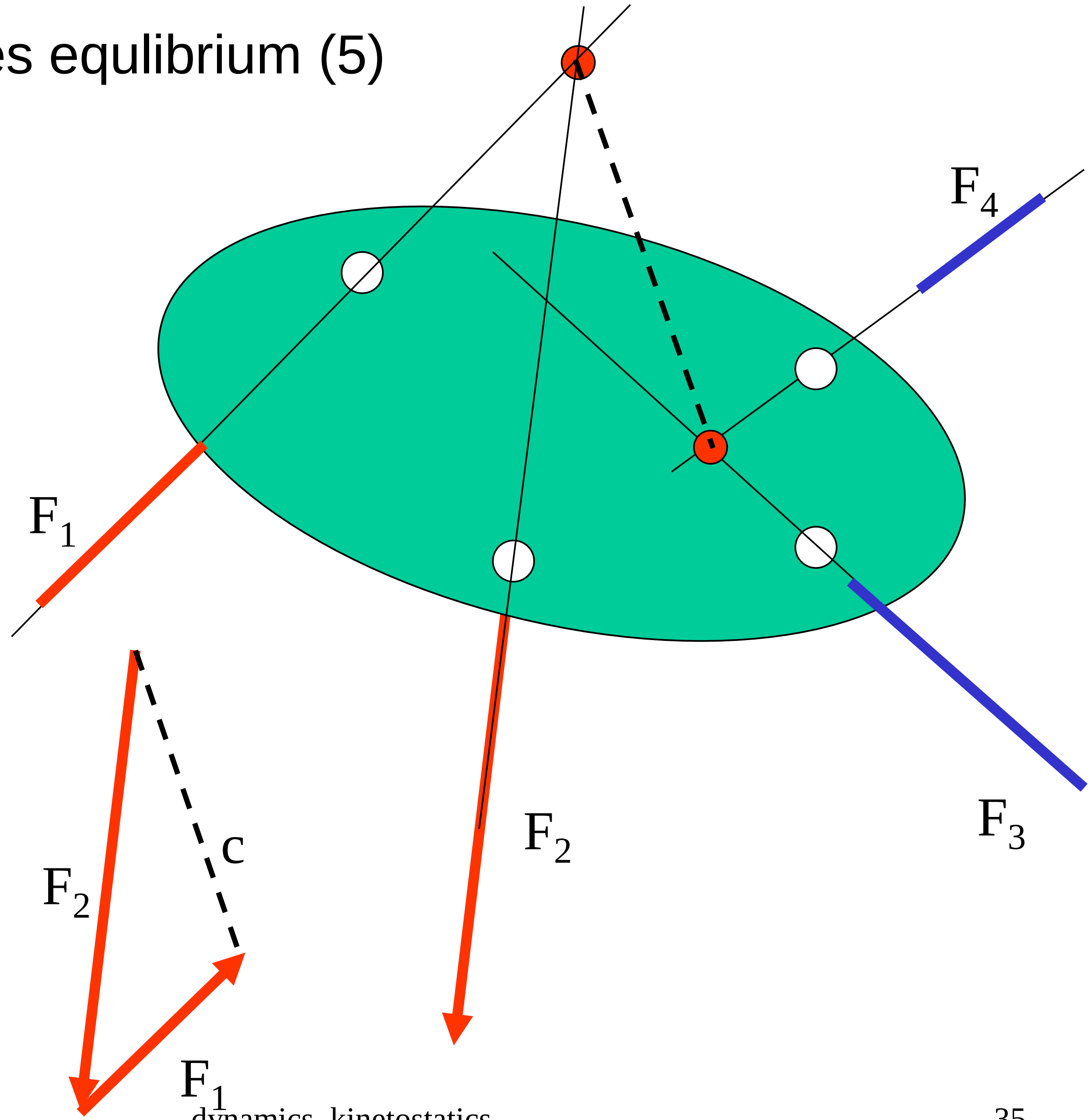
2, 3 & 4 forces equlibrium (3)



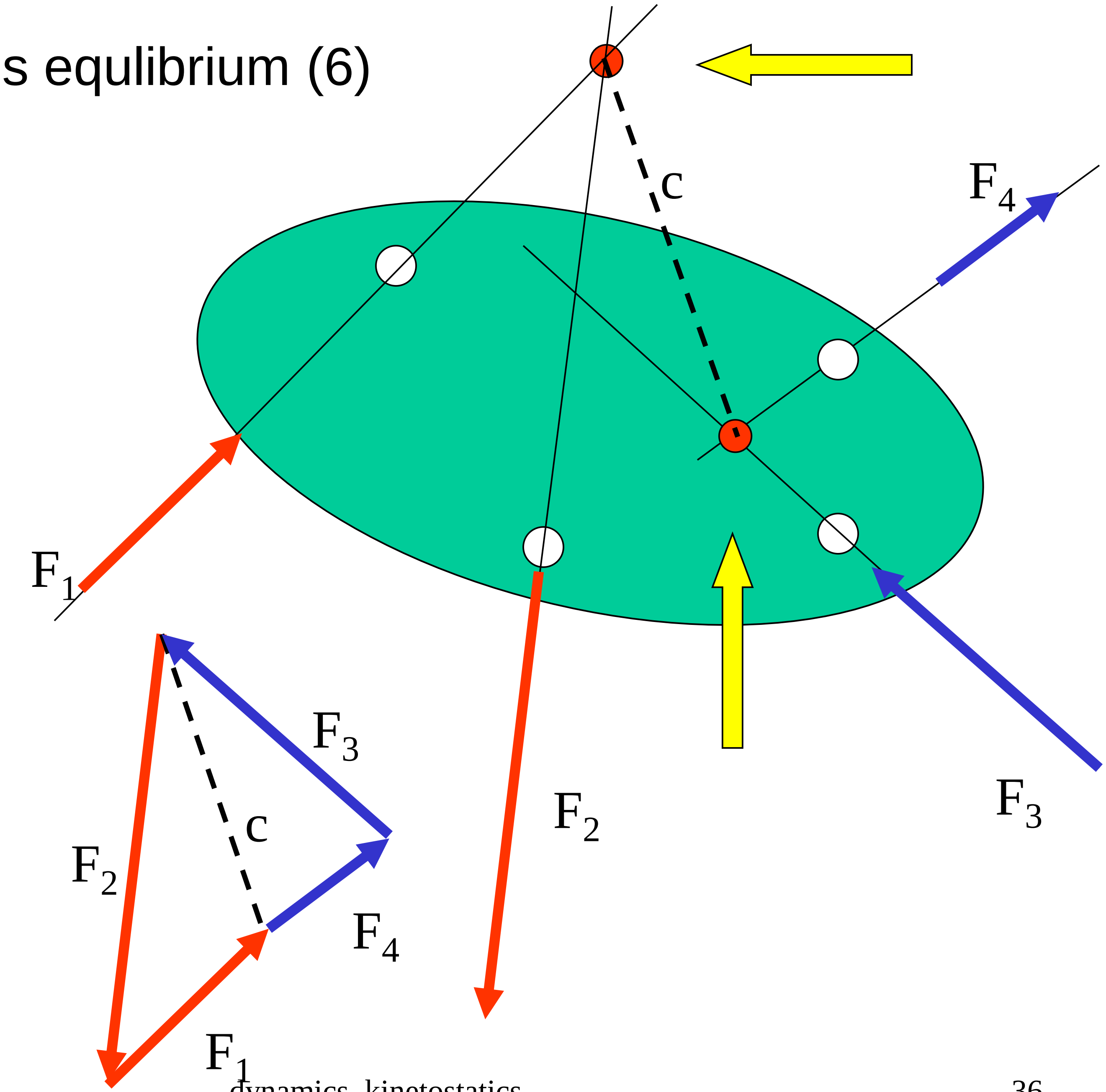
2, 3 & 4 forces equlibrium (4)



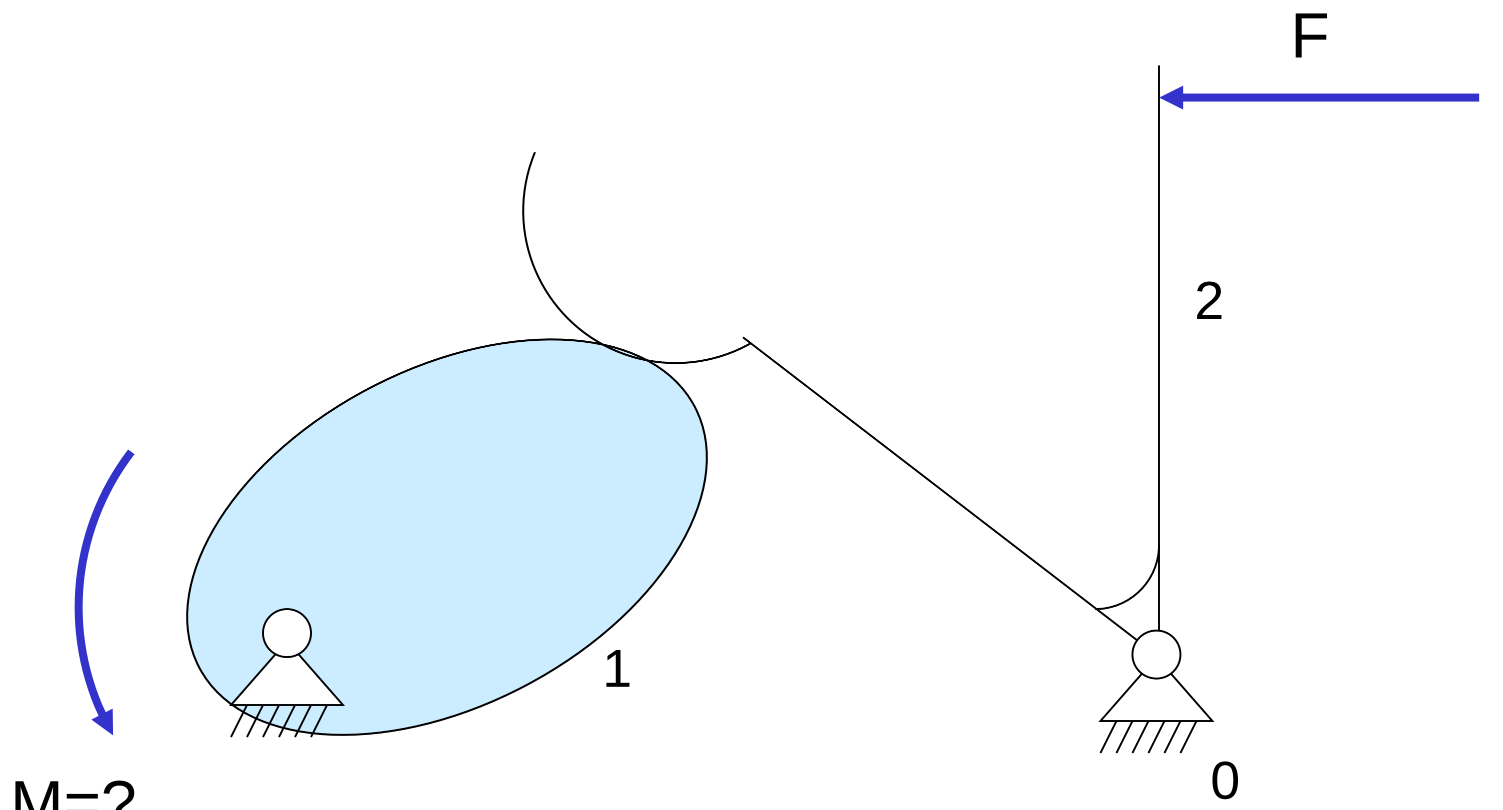
2, 3 & 4 forces equlibrium (5)

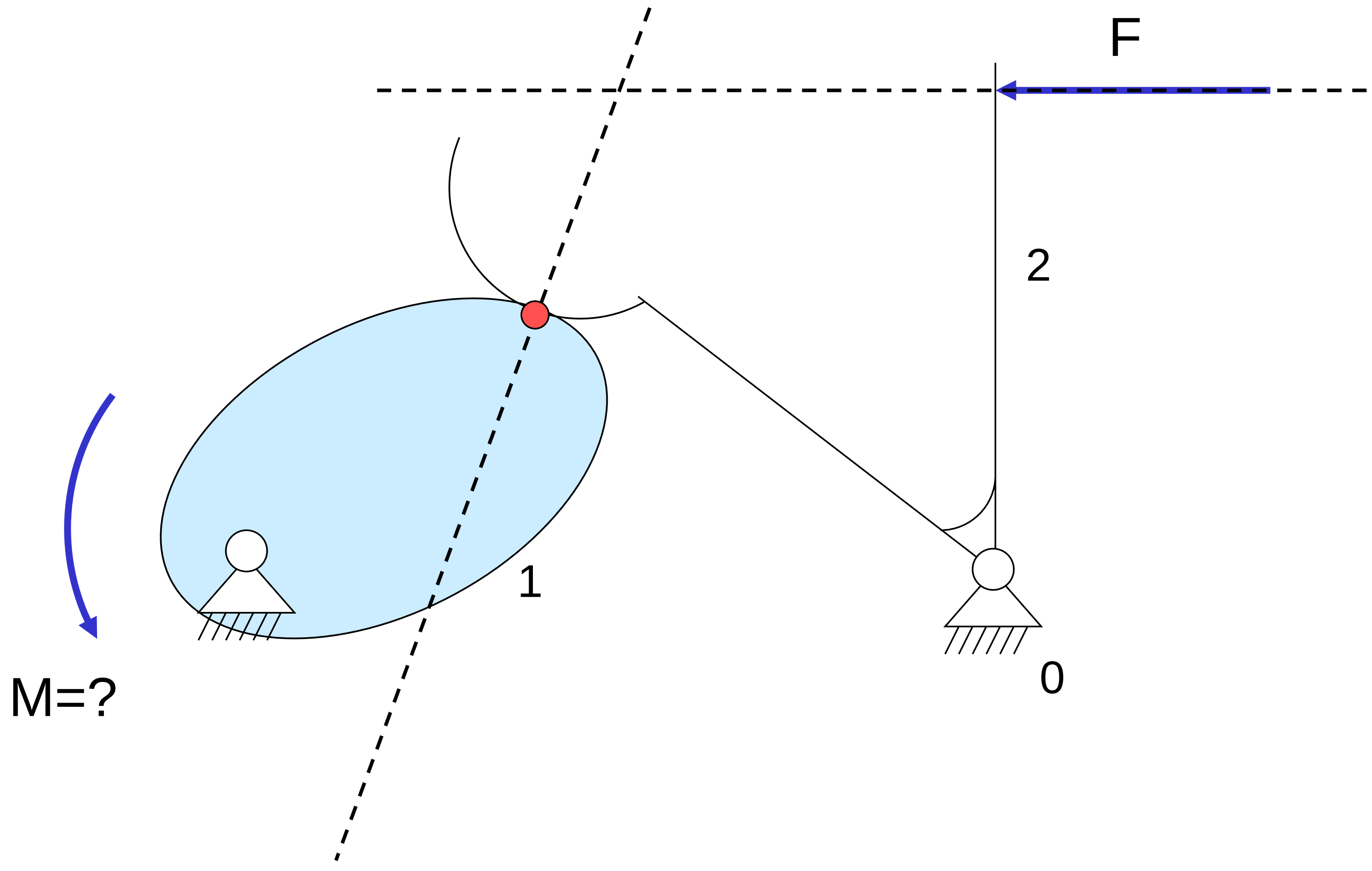


2, 3 & 4 forces equlibrium (6)

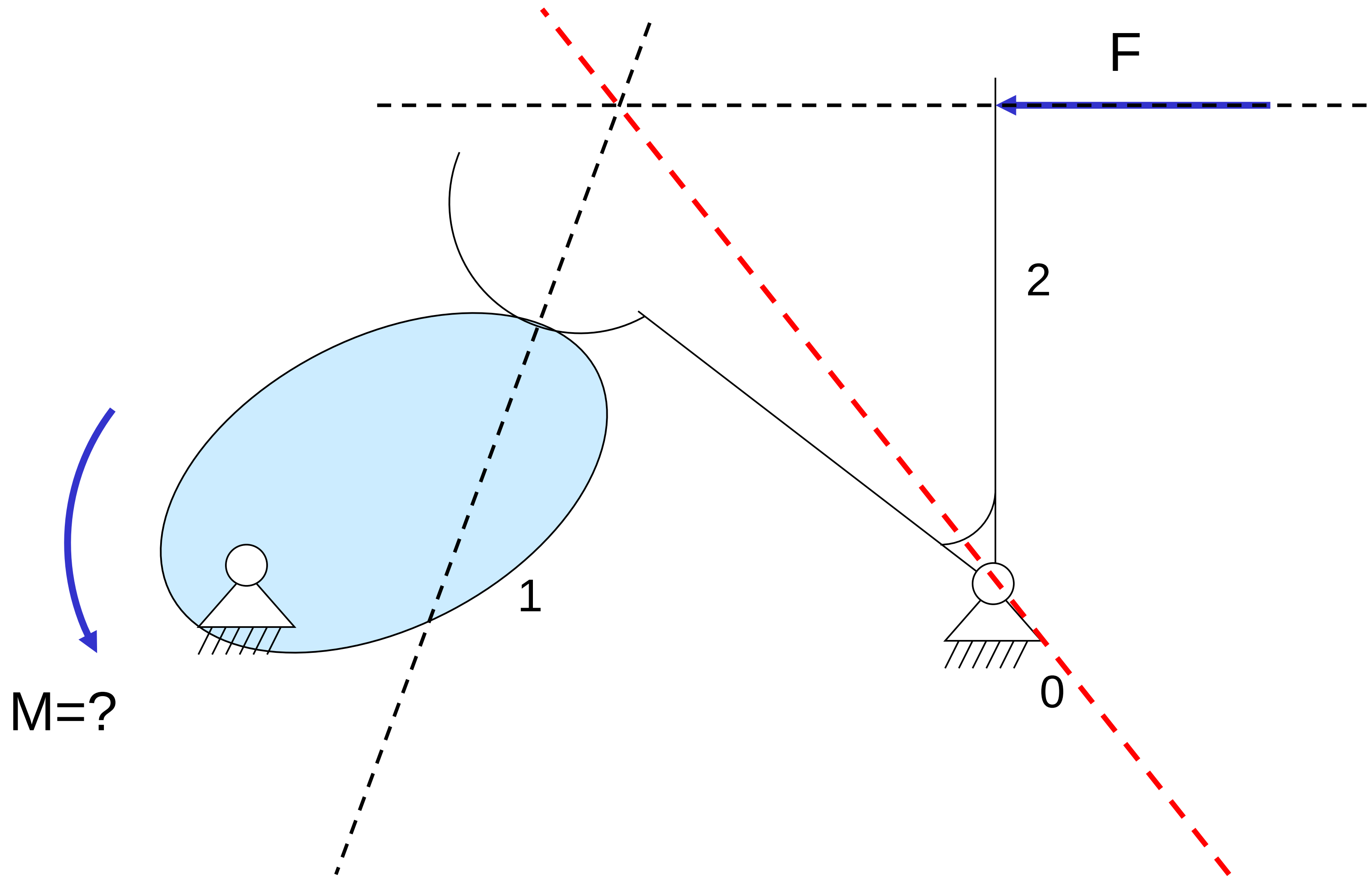


Kinetostatics - example

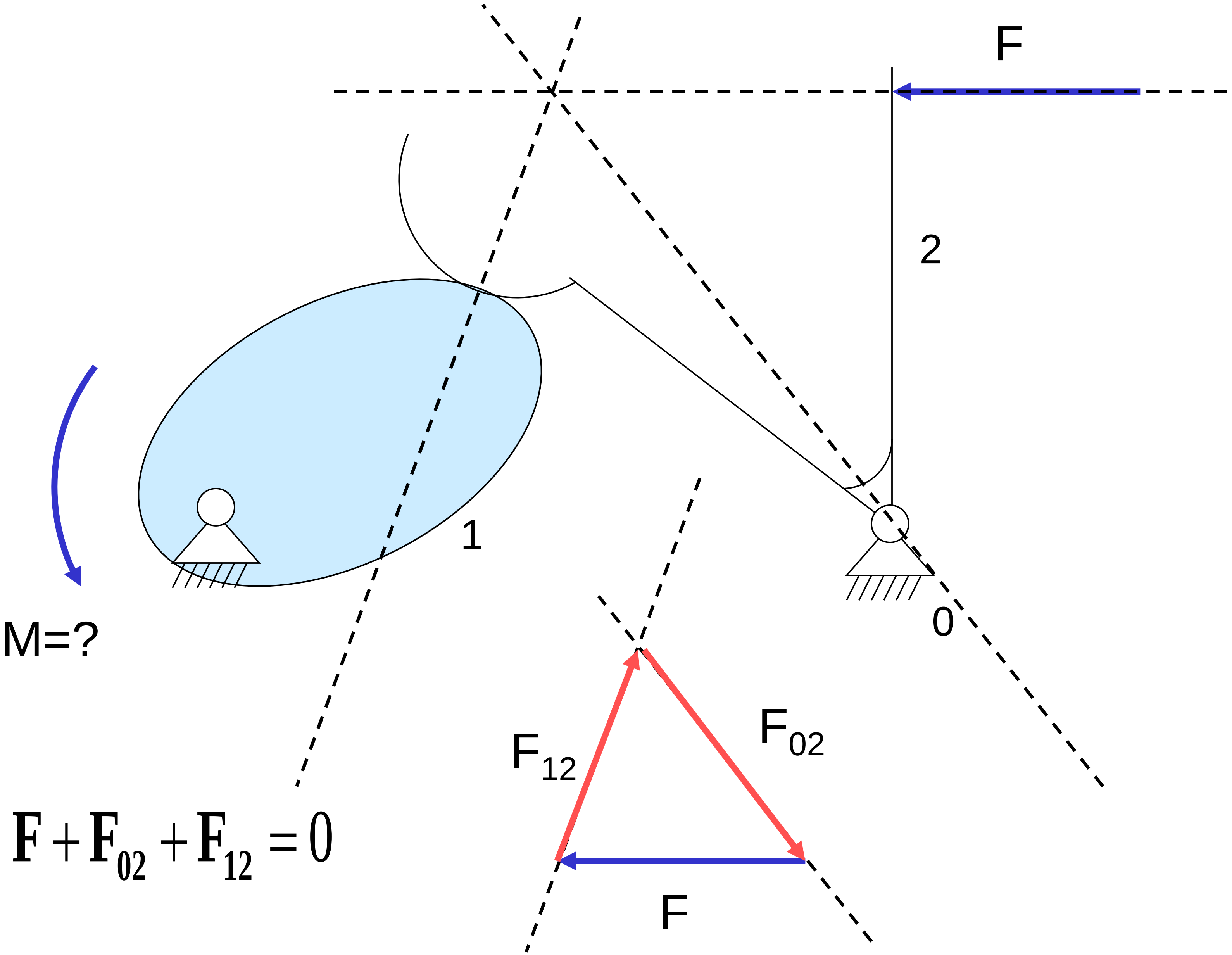




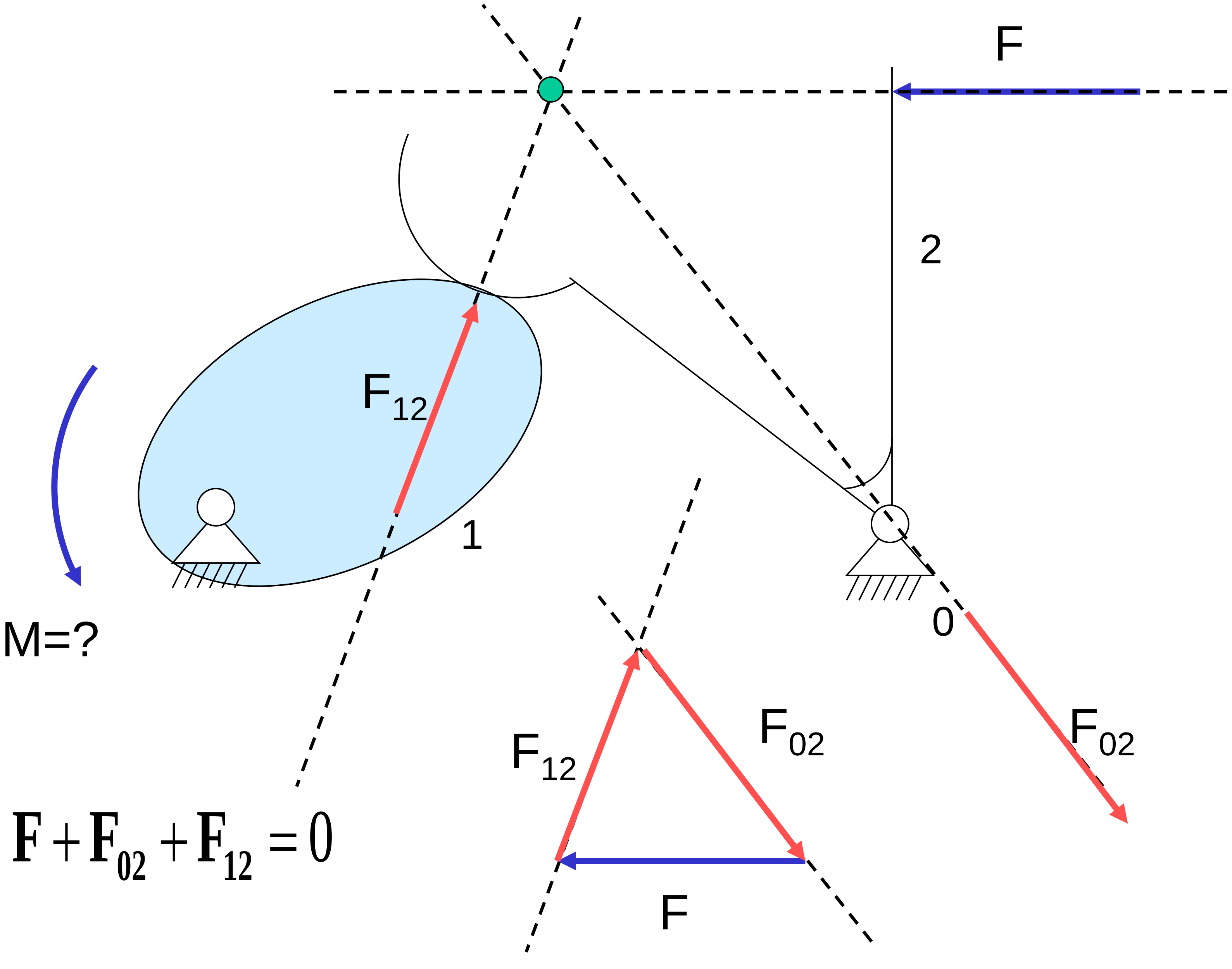
$$\mathbf{F} + \mathbf{F}_{02} + \mathbf{F}_{12} = 0$$



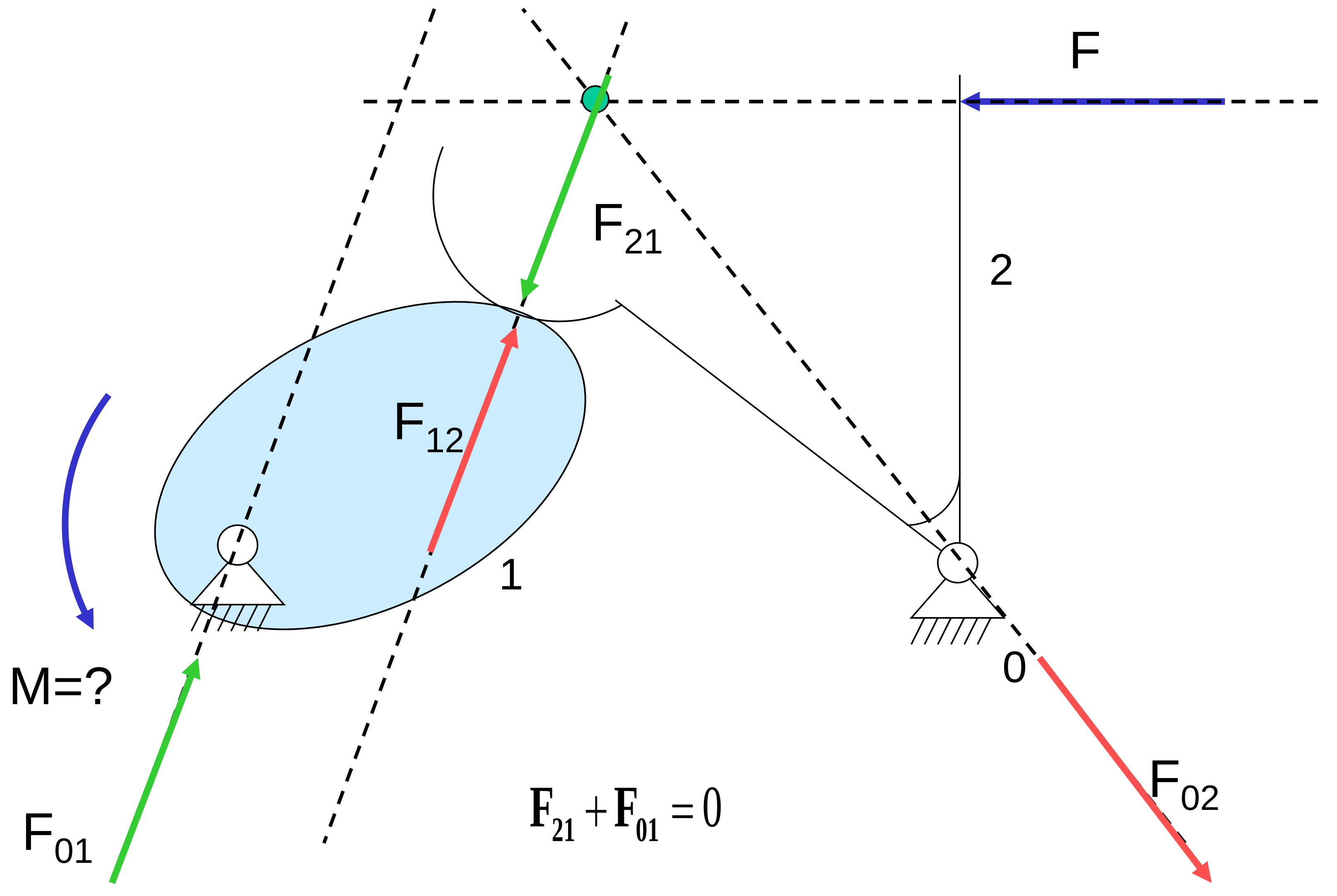
$$\mathbf{F} + \mathbf{F}_{02} + \mathbf{F}_{12} = 0$$



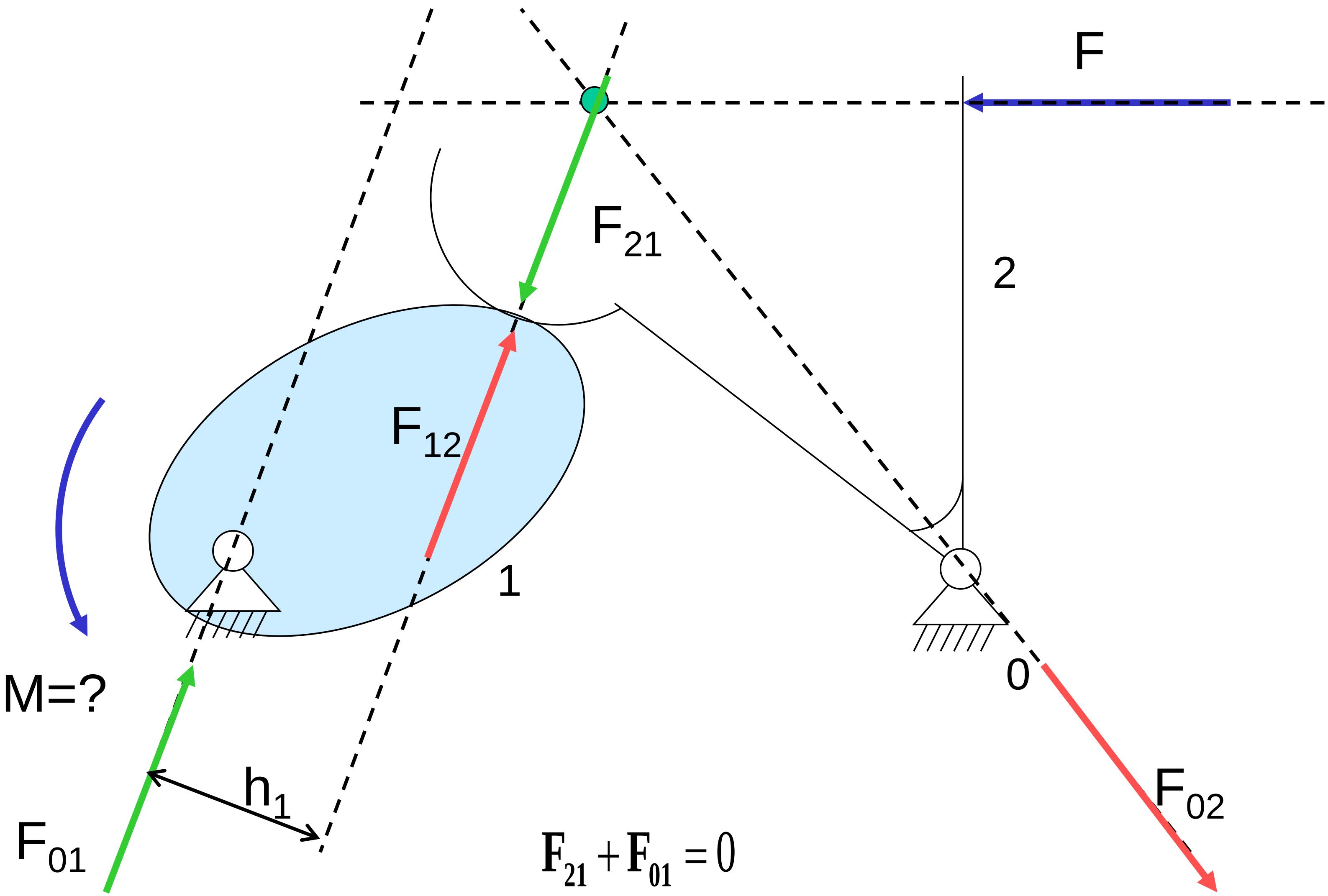
$$\mathbf{F} + \mathbf{F}_{02} + \mathbf{F}_{12} = 0$$



$$\mathbf{F} + \mathbf{F}_{02} + \mathbf{F}_{12} = 0$$



F
F21
2
F12
1
M=?
0
F02
F01
F21 + F01 = 0



$$\mathbf{F}_{21} + \mathbf{F}_{01} = 0$$

$$\sum M_1^A = 0 \rightarrow -F_{21} h_1 + M = 0$$